Examples 3 in Circles

Click this to start.

Examples 3 in Circles

A circle is one of curved line segments simply closed. It is in fact, the simplest of all line segments closed. Why though, is it that simple?

Either curved or not, a line segment is a collection of points in a plane.
Every point in a line segment called a circle is the same distance away from a particular point called the center, and the same distance is called the radius.

So saying a circle, we mean the radius and the center.
And a circle is that simple. When it comes to a problem though, it is not that simple.

Working with a curve, we put it in an equation if we can. So working with a circle, we work with the equation of it. And putting it in a graph, we can work with it more readily since we can actually see it. And the same is true for any other curves, too, of course.
How then, do we put a circle in an equation?

We can do it by means of the definition for circles, which is no other than the distance formula, called Pythagorean theorem, too. So using the formula, we get an equation called a standard equation. And the standard equation is $(x - a) + (y - b) = c^2$.

In the equation above, (a, b) is the center, and c is the radius, of course.
So plugging in the center and the radius, we get the equation, so we can call it a template.

And simplifying the standard equation, we get an equivalent equation called a general equation, which takes this form: $x^2 + y^2 + Ax + By + C = 0$, which is called therefore, the general equation of a circle.

Now, doing the examples in this set and subsequent sets, we get to find a lot more about the idea called a circle.

Find the circle in each case below assume the circle is in the ***x-y*** plane.

0. Two points **(2, 1)** and **(-3, 2)** are endpoints of a diameter of the circle.

1. Three points **(2, 1)**, **(1, 3)**, and **(-3, 2)** are in the circle.

2. A point **(2, 1)** is in the circle centered at **(-3, 4)**.

3. A point **(2, 1)** is in the circle where the radius is 2.

Suggestions or Solutions
To the Problem in the Example 0

Two points (2, 1) and (-3, 2) are endpoints of a diameter of the circle.

To begin with, the center is $\left(\frac{2+(-3)}{2},\ \frac{1+2}{2}\right) = \left(-\frac{1}{2},\ \frac{3}{2}\right)$.
Next, assuming d is the diameter, and r is the radius, we get
$d^2 = \{2-(-3)\}^2 + (1-2)^2 = 25 + 1 = 26 \Rightarrow d^2 = 26 \Rightarrow r = \frac{d}{2} \Rightarrow r^2 = \frac{d^2}{4} = \frac{26}{4} = \frac{13}{2}$.
Therefore, the circle is $(x+\frac{1}{2})^2 + (y-\frac{3}{2})^2 = \frac{13}{2}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Given the center and the radius in a circle, we can find the circle, of course.
In this problem however, they are not given, are they?

Of course, they are, but hidden somewhere in the problem.
So we've got to find them. Where can we find them, though?

They are in a diameter.
What diameter? Are there many diameters in a circle?

We can have two interpretations of a diameter in a circle.
One is a line segment connecting two points facing each other in a circle.
The other is the length of such a line segment.

Usually, saying *the* diameter, we mean the length, and saying just a diameter though, we mean such a line segment.
So a circle has infinitely many diameters, each of which has the same length, of course.

And the same is true for a radius, too.

We can have therefore, two interpretations of a radius in a circle.

One is a line segment connecting two points, one of the two is the center, and the other is a point in the circle.
The other interpretation is the length of such a line segment.

So usually, saying *the* radius, we mean the length, and saying just a radius however, we mean such a line segment.
Thus, a circle has infinitely many radii, each of which has the same length, of course.

Now, in this problem, a diameter is given, so we may want to begin with it.

A diameter has the center and radius, and can be specified by its two endpoints.

The distance between the two endpoints is the diameter, which is twice the radius.

So the center is the midpoint between the two endpoints, **(2, 1)** and **(-3, 2)**, and the radius is half the diameter, of course. So what should we find first?

We may want to find the midpoint first, which is the center.
So suppose now, that the midpoint is $\boldsymbol{(s, t)}$.

Then, we get $\boldsymbol{s = \frac{2+(-3)}{2} = -\frac{1}{2}}$, and $\boldsymbol{t = \frac{1+2}{2} = \frac{3}{2}}$. So the center is $\boldsymbol{(-\frac{1}{2},\ \frac{3}{2})}$.
So next, what should we move on to?

It is the radius, which is half the diameter, of course. So what do we want to begin with?

We want to begin with the two points **(2, 1)** and **(-3, 2)**, since both are the endpoints of a diameter of the circle.

Thus, the diameter is the distance between **(2, 1)** and **(-3, 2)**. So first, assuming the diameter is $\boldsymbol{d}$, we get $\boldsymbol{d^2 = (\Delta x)^2 + (\Delta y)^2 = \{2 - (-3)\}^2 + (1 - 2)^2 = 25 + 1 = 26 \Rightarrow d^2 = 26}$.

Next, assuming the radius is ***r***, we get $\boldsymbol{r = \frac{d}{2} \Rightarrow r^2 = \frac{d^2}{4} = \frac{26}{4} = \frac{13}{2}}$. Why $\boldsymbol{r^2}$, though?

We are going to use the standard form, $\boldsymbol{(x - a)^2 + (y - b)^2 = r^2}$, in which we have $\boldsymbol{r^2}$.

Now that we have the center and radius, we can use the standard form to get the circle.

Thus, we get $\boldsymbol{\{x - (-\frac{1}{2})\}^2 + (y - \frac{3}{2})^2 = r^2 = \frac{13}{2}}$, so the circle is $\boldsymbol{(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = \frac{13}{2}}$.

In short:

To begin with, the center is $\boldsymbol{(\frac{2+(-3)}{2}, \frac{1+2}{2}) = (-\frac{1}{2}, \frac{3}{2})}$.

Next, assuming ***d*** is the diameter, and ***r*** is the radius, we get

$\boldsymbol{d^2 = \{2 - (-3)\}^2 + (1 - 2)^2 = 25 + 1 = 26 \Rightarrow d^2 = 26 \Rightarrow r = \frac{d}{2} \Rightarrow r^2 = \frac{d^2}{4} = \frac{26}{4} = \frac{13}{2}}$.

Therefore, the circle is $\boldsymbol{(x + \frac{1}{2})^2 + (y - \frac{3}{2})^2 = \frac{13}{2}}$.

Also, even if given two points only, we can still find a particular circle if one of the two is the center, and the other is in the circle. How?

We can find the radius, and then, the circle. How can we find the radius?

We can find it using the distance formula. Why?

Every point in a circle is the radius away from the center, so if we are given two points, one is the center, and the other is in the circle, then we can find the radius using the distance formula, since the distance between the two is the radius.

Suggestions or Solutions
To the Problem in the Example 1

Three points (2, 1), (1, 3), and (-3, 2) are in the circle.

Suppose the circle is $\boldsymbol{x^2 + y^2 + ax + by + c = 0}$ where $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are constant.
Then, we get $\mathbf{(2, 1) \Rightarrow 4 + 1 + 2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} = 0}$, $\mathbf{(1, 3) \Rightarrow 1 + 9 + \boldsymbol{a} + 3\boldsymbol{b} + \boldsymbol{c} = 0}$, and
$\mathbf{(-3, 2) \Rightarrow 9 + 4 - 3\boldsymbol{a} + 2\boldsymbol{b} + \boldsymbol{c} = 0}$.

So we get $\mathbf{2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + 5 - (\boldsymbol{a} + 3\boldsymbol{b} + \boldsymbol{c} + 10) = 0 \Rightarrow \boldsymbol{a} - 2\boldsymbol{b} - 5 = 0}$, and
$\mathbf{\boldsymbol{a} + 3\boldsymbol{b} + \boldsymbol{c} + 10 - (-3\boldsymbol{a} + 2\boldsymbol{b} + \boldsymbol{c} + 13) \Rightarrow 4\boldsymbol{a} + \boldsymbol{b} - 3 = 0}$.

Thus, we get $\mathbf{4(\boldsymbol{a} - 2\boldsymbol{b} - 5) - (4\boldsymbol{a} + \boldsymbol{b} - 3) = 0 \Rightarrow -9\boldsymbol{b} - 17 = 0 \Rightarrow \boldsymbol{b} = -\frac{17}{9} \Rightarrow}$
$\mathbf{4\boldsymbol{a} + \boldsymbol{b} - 3 = 4\boldsymbol{a} - \frac{17}{9} - 3 = 0 \Rightarrow 4\boldsymbol{a} = \frac{44}{9} \Rightarrow \boldsymbol{a} = \frac{11}{9}}$.
So we get $\mathbf{2\boldsymbol{a} + \boldsymbol{b} + \boldsymbol{c} + 5 = 2 \cdot \frac{11}{9} - \frac{17}{9} + \boldsymbol{c} + 5 = \boldsymbol{c} - \frac{5}{9} + 5 = \boldsymbol{c} + \frac{40}{9} = 0 \Rightarrow \boldsymbol{c} = -\frac{40}{9}}$.
Therefore, the circle is $\boldsymbol{x^2 + y^2 + \frac{11}{9}x - \frac{17}{9}y - \frac{40}{9} = 0}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Three particular points in a plane can determine a circle if the three are not in a line, of course. So given three points not in a line, we should be able to find a particular circle. How do we know though, if the three points are not in a line?

We can construct a line using two of the three, and then, can see if the other is in the line by plugging it into the line.

So first, using **(2, 1)** and **(1, 3)**, we can get the slope, which is $\mathbf{\frac{3-1}{1-2} = -2}$, and thus, the line is as follows: $\boldsymbol{y - 1 = -2(x - 2)}$.

(If not sure of how to get a line this way, refer to **CONICS 1**.)

Next, putting **(-3, 2)** into the line above, we get

On the left hand side, **2 – 1 = 1**, and on the right hand side **-2(–3 – 2) = 10**.

We can see that both sides are not equal, so the three points are not in a line.

So we can find the circle passing through the three. How?

We can find it using the general form, $x^2 + y^2 + ax + by + c = 0$ where a, b, and c are constant. (We can use the standard form, too, of course.)

Putting each of the three points into the general form above, we get an equation for the three constants a, b, and c, and therefore, we can set up a system of three equations for the three constants.

First, putting **(2, 1)** into the form, we get $4 + 1 + 2a + b + c = 0 \Rightarrow 2a + b + c + 5 = 0$.

Next, putting **(1, 3)** into it, we get $1 + 9 + a + 3b + c = 0 \Rightarrow a + 3b + c + 10 = 0$.

Finally, putting **(-3, 2)** into it, we get $9 + 4 - 3a + 2b + c = 0 \Rightarrow -3a + 2b + c + 13 = 0$.

Then, eliminating c, we get

First, $2a + b + c + 5 - (a + 3b + c + 10) = 0 \Rightarrow a - 2b - 5 = 0$.

Next, $a + 3b + c + 10 - (-3a + 2b + c + 13) \Rightarrow 4a + b - 3 = 0$.

Eliminating a, we get $4(a - 2b - 5) - (4a + b - 3) = 0 \Rightarrow -9b - 17 = 0 \Rightarrow b = -\frac{17}{9}$.

So next, we get $b = -\frac{17}{9} \Rightarrow 4a + b - 3 = 4a - \frac{17}{9} - 3 = 0 \Rightarrow 4a = \frac{44}{9} \Rightarrow a = \frac{11}{9}$.

Thus, we get $2a + b + c + 5 = 2 \cdot \frac{11}{9} - \frac{17}{9} + c + 5 = c - \frac{5}{9} + 5 = c + \frac{40}{9} = 0 \Rightarrow c = -\frac{40}{9}$.

So the circle is $x^2 + y^2 + \frac{11}{9}x - \frac{17}{9}y - \frac{40}{9} = 0$, where we can't see however, the center and the radius.

So let's put the circle in the standard form. We don't have to for this problem, of course. Just checking though

Putting the equation above into a pair of complete squares, we get the standard version.

So first, for x, we get $x^2 + \frac{11}{9}x = x^2 + \frac{11}{9}x + (\frac{11}{2\cdot 9})^2 - (\frac{11}{2\cdot 9})^2 = (x + \frac{11}{18})^2 - (\frac{11}{18})^2$.

And next, for y, we get $y^2 - \frac{17}{9}y = y^2 - \frac{17}{9}y + (\frac{17}{2\cdot 9})^2 - (\frac{17}{2\cdot 9})^2 = (y - \frac{17}{18})^2 - (\frac{17}{18})^2$.

So we get

$$x^2 + y^2 + \frac{11}{9}x - \frac{17}{9}y - \frac{40}{9} = 0 \Rightarrow (x + \frac{11}{18})^2 - (\frac{11}{18})^2 + (y - \frac{17}{18})^2 - (\frac{17}{18})^2 - \frac{40}{9} = 0$$

$$\Rightarrow (x + \frac{11}{18})^2 + (y - \frac{17}{18})^2 - (\frac{11}{18})^2 - (\frac{17}{18})^2 - \frac{2\cdot 40}{2\cdot 9} = 0 \Rightarrow (x + \frac{11}{18})^2 + (y - \frac{17}{18})^2 - \frac{11^2 + 17^2 + 80\cdot 18}{18^2} = 0$$

$$\Rightarrow (x + \frac{11}{18})^2 + (y - \frac{17}{18})^2 = \frac{11^2 + 17^2 + 80\cdot 18}{18^2} = \frac{1850}{18^2} = \frac{5^2\cdot 74}{18^2} = \frac{1850}{324} \Rightarrow (x + \frac{11}{18})^2 + (y - \frac{17}{18})^2 = \frac{1850}{324},$$

which indicates a circle of radius $\frac{5\sqrt{74}}{18}$ centered at $(-\frac{11}{18}, \frac{17}{18})$.

In short:

Suppose the circle is $x^2 + y^2 + ax + by + c = 0$ where a, b, and c are constant.

Then, we get $(2, 1) \Rightarrow 4 + 1 + 2a + b + c = 0$, $(1, 3) \Rightarrow 1 + 9 + a + 3b + c = 0$, and $(-3, 2) \Rightarrow 9 + 4 - 3a + 2b + c = 0$.

So we get $2a + b + c + 5 - (a + 3b + c + 10) = 0 \Rightarrow a - 2b - 5 = 0$, and $a + 3b + c + 10 - (-3a + 2b + c + 13) \Rightarrow 4a + b - 3 = 0$.

Thus, we get $4(a - 2b - 5) - (4a + b - 3) = 0 \Rightarrow -9b - 17 = 0 \Rightarrow b = -\frac{17}{9} \Rightarrow$

$4a + b - 3 = 4a - \frac{17}{9} - 3 = 0 \Rightarrow 4a = \frac{44}{9} \Rightarrow a = \frac{11}{9}$.

So we get $2a + b + c + 5 = 2\cdot\frac{11}{9} - \frac{17}{9} + c + 5 = c - \frac{5}{9} + 5 = c + \frac{40}{9} = 0 \Rightarrow c = -\frac{40}{9}$.

Therefore, the circle is $x^2 + y^2 + \frac{11}{9}x - \frac{17}{9}y - \frac{40}{9} = 0$.

Suggestions or Solutions
To the Problem in the Example 2

A point (2, 1) is in the circle centered at (-3, 4).

Suppose the circle is $(x - a) + (y - b) = c^2$, where (a, b) is the center and c is the radius.
The center is at **(-3, 4**, and the point **(2, 1)** is in the circle.
So we get $(\Delta x)^2 + (\Delta y)^2 = c^2 \Rightarrow (-3 - 2)^2 + (4 - 1)^2 = 25 + 9 = 34 = c^2$.

Therefore, the circle is $(x + 3)^2 + (y - 4)^2 = 34$.

If not quite sure of the idea behind the processes above, follow the steps below.

Having the center and the radius, we can get a particular circle.
We are given a point **(2, 1)** in the circle, together with the center, which is at **(-3, 4)**.
Where then, is the radius?

It is between the center and the point given.
Every point in a circle is the same distance away from the center of the circle, and the same distance is the radius. So what's the radius?

The radius in the circle we want is the distance from the center **(-3, 4)** to the point **(2, 1)**.

We can get the distance by the distance formula, often called Pythagorean theorem, too.

And next, putting the center and the radius into the template, the standard form where $(x - a) + (y - b) = c^2$, where (a, b) is the center and c is the radius, we can get the circle.

So first, applying the distance formula to the center and the point, we get
$(\Delta x)^2 + (\Delta y)^2 = c^2 \Rightarrow (-3 - 2)^2 + (4 - 1)^2 = 25 + 9 = 34 = c^2$, which is the radius squared.

Therefore, the circle we are after is $\mathbf{(x + 3)^2 + (y - 4)^2 = 34}$, where the radius is $\sqrt{\mathbf{34}}$.

Putting it in the general form, we just expand the version standard above, so we get

$$\mathbf{(x + 3)^2 + (y - 4)^2 = x^2 + 6x + 9 + y^2 - 8y + 16 = x^2 + y^2 + 6x - 8y + 25 = 34}$$
$$\Rightarrow \mathbf{x^2 + y^2 + 6x - 8y - 9 = 0}.$$

In short:

Suppose the circle is $\mathbf{(x - a) + (y - b) = c^2}$, where $\mathbf{(a, b)}$ is the center and $\mathbf{c}$ is the radius.

The center is at **(-3, 4**, and the point **(2, 1)** is in the circle.

So we get $\mathbf{(\Delta x)^2 + (\Delta y)^2 = c^2 \Rightarrow (-3 - 2)^2 + (4 - 1)^2 = 25 + 9 = 34 = c^2}$.

Therefore, the circle is $\mathbf{(x + 3)^2 + (y - 4)^2 = 34}$.

Suggestions or Solutions
To the Problem in the Example 3

A point (2, 1) is in the circle where the radius is 2.

Since the radius is 2, we can put the circle in such an equation as follows.
$(x - a)^2 + (y - b)^2 = 2^2$ where (a, b) is the center.

Then, since the point **(2, 1)** is in the circle, we get
$(2 - a)^2 + (1 - b)^2 = 2^2$, which is the same as $(a - 2)^2 + (b - 1)^2 = 2^2$.

Therefore, the solution to this problem is every circle that satisfies an equation below.
$(x - a)^2 + (y - b)^2 = 2^2$ where a and b satisfy $(a - 2)^2 + (b - 1)^2 = 2^2$.

If not quite sure of the idea behind the processes above, follow the steps below.

Given the center and the radius, we may want to use the template (standard form) below.
$(x - a)^2 + (y - b)^2 = c^2$, where (a, b) is the center, and c is the radius.

The radius is given, and is 2, but the center is not given. Where then, is the center?

Every point in a circle is the radius away from the center, so the center of the circle we are after is 2 away from the point **(2, 1)**.

Is there only one point though, that is 2 away from the point **(2, 1)**? Can we have only one point the distance from which to the point **(2, 1)** is 2?

No, it’s not the case.
There can be many points that can be the center of the circle we are after, and thus, the solution to this problem can be more than one circle. Why?

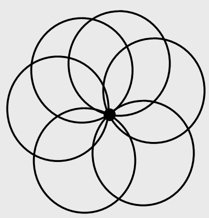
There are a lot of things can be 2 away from the point. We can have in fact, many things that can be the same distance away from a particular point in a plane. Many lines can be the same distance away from a point in a plane.

And the same is true for points, too. So there can be many points the same distance away from a particular point in a plane. That is in fact, what a circle is about, isn't it?

So many circles can have the same radii, and also, can pass through the same point.
That is, many circles of a particular radius can share a particular point.

For instance, many circles of radius 1 can pass through a point (1, 2).

Fig. 0



All the circles above have the same radii, and share the same point.

Now, in this problem, the particular radius is 2, and the particular point is **(2, 1)**.

So all circles of radius 2 share the point **(2, 1)**.
That is, all circles of radius 2 meet altogether at **(2, 1)**.

In fact, infinitely many can be such, so the solution is a group of infinitely many circles.
Thus, we want to produce an equation representing a group of circles.

So let's put them in an equation using the template, $(x - a)^2 + (y - b)^2 = c^2$.

First of all, we know that the radius is 2, so c is 2 in the template (standard) above.

So we get $(x - a)^2 + (y - b)^2 = 2^2$, for now.

Next, the point **(2, 1)** is in the circle, so putting the point into the equation above, we get such an equation as follows: $(2 - a)^2 + (1 - b)^2 = 2^2$. What equation is it, though?

Suppose now, ***C*** is the circle we are after. Then, the circle ***C*** is $(x - a)^2 + (y - b)^2 = 2^2$.

And (a, b) is the center of the circle ***C***, and is a point, too, of course.

Also, ***a*** and ***b*** are assumed to be constants that can take a pair of real numbers at a time.

So every point (a, b) satisfying the equation $(2 - a)^2 + (1 - b)^2 = 2^2$ can be the center of the circle ***C***.

Then, the equation above is the connective expression between the coordinates of (a, b), which represents all points that can be the center of ***C***. Where are all those points, then?

They are in a circle $(x - 2)^2 + (y - 1)^2 = 2^2$, and are all the points in the circle, so each of all the points in the circle is the center of the circle ***C***.   Why?

The connective expression $(2 - a)^2 + (1 - b)^2 = 2^2$ is the same as $(a - 2)^2 + (b - 1)^2 = 2^2$.

So taking for variables, ***a*** and ***b*** in the expression above, the expression can be taken for an equation of a circle in an ***a-b*** plane, where the ***a***-axis is perpendicular to the ***b***-axis as in the case where the ***x-y*** plane is set up.

Thus, the equation $(a - 2)^2 + (b - 1)^2 = 2^2$ indicates a circle where the center is the point $(2, 1)$ and the radius is 2 in the ***a-b*** plane. So (a, b) can be an arbitrary point in the circle.

Now, putting in the ***x-y*** plane, the circle $(a - 2)^2 + (b - 1)^2 = 2^2$, we just replace ***a*** with ***x***, and ***b*** with ***y***.
Then, we get $(x - 2)^2 + (y - 1)^2 = 2^2$, which is the circle where every point is the center of the circle ***C***.   Why?

Suppose now, ***D*** is the circle $(x - 2)^2 + (y - 1)^2 = 2^2$, where the center is a point $(2, 1)$.
Suppose also, ***A*** is a circle where the radius is 2 and the center is a point in the circle ***D***.

Then, ***A*** passes through the center of ***D***, which is the point $(2, 1)$.   Why?

The distance from every point in ***D*** to the center of ***D*** is 2 since the radius of ***D*** is 2.

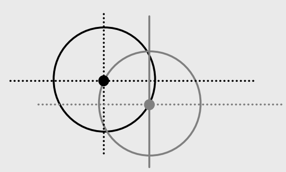
The same is true for the circle ***A***, too, and the center of ***A*** is in ***D***. So the center of ***D*** is in ***A***, and thus, ***A*** passes through the center of ***D***, which is the point **(2, 1)**.

And the same is true for each of all the other points in the circle ***D***, too.

So each point in the circle ***D*** can be a center of a circle of radius 2 centered at **(2, 1)**.

If two circles share the same radius, and one of the two passes through the center of the other, the center of the one is in the other, and the other passes through the center of the one, and has its center in the one. So both are in the same situation.

Fig. 1



Thus, we get a group of circles, each of which passes through the point **(2, 1)**, and has a radius of 2, and a center that is one of all the points in the circle ***D***.

So each point (a, b) satisfying the expression $(a - 2)^2 + (b - 1)^2 = 2^2$ is the center of each of all the circles in the group.

We know a circle of radius 2 centered at (a, b) is indicated by $(x - a)^2 + (y - b)^2 = 2^2$.

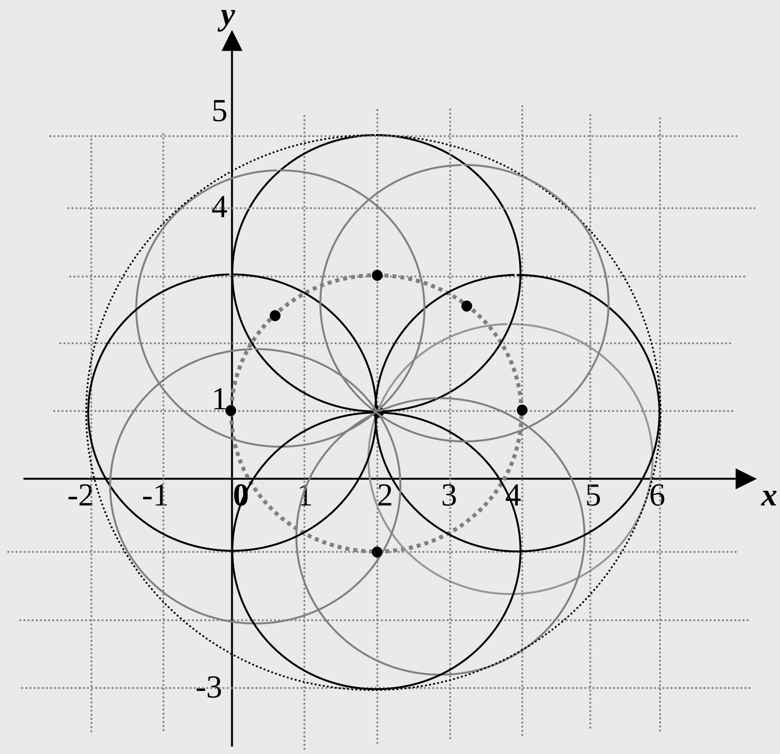
Therefore, the solution is every circle that satisfies the equation as follows.

$(x - a)^2 + (y - b)^2 = 2^2$ where a and b satisfy $(a - 2)^2 + (b - 1)^2 = 2^2$.

So "$(x - a)^2 + (y - b)^2 = 2^2$ where a and b satisfy $(a - 2)^2 + (b - 1)^2 = 2^2$." can represent a group of circles, and for each pair of values of a and b, $(x - a)^2 + (y - b)^2 = 2^2$ indicates a circle of radius 2 centered at (a, b).

And putting in a graph some of the circles in the group, we can get

Fig. 2



We can see all the circles above are within a circle of radius 4 centered at the point **(2, 1)**.

Also, $(a-2)^2+(b-1)^2=2^2$ can be called the equation of the trace of the centers of all the circles in the group.

In short:

Since the radius is 2, we can put the circle in such an equation as follows.
$(x-a)^2+(y-b)^2=2^2$ where **(*a*, *b*)** is the center.

Then, since the point **(2, 1)** is in the circle, we get
$(2-a)^2+(1-b)^2=2^2$, which is the same as $(a-2)^2+(b-1)^2=2^2$.

Therefore, the solution to this problem is every circle that satisfies an equation below.
$(x-a)^2+(y-b)^2=2^2$ where ***a*** and ***b*** satisfy $(a-2)^2+(b-1)^2=2^2$.

Note:

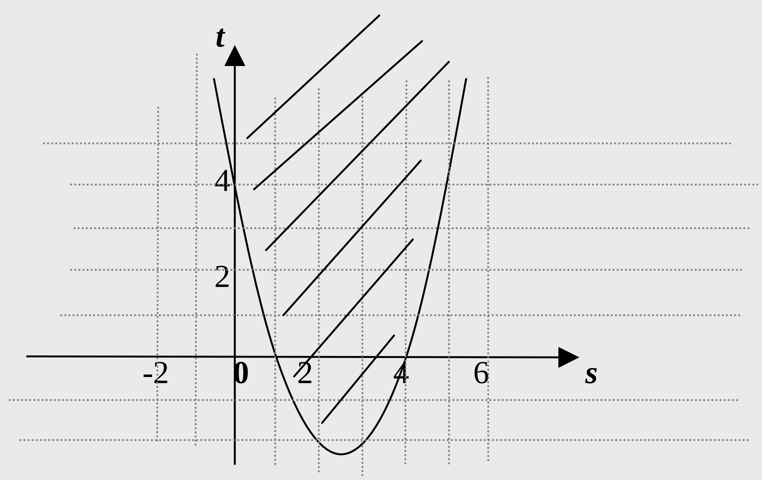
A connective expression shows a relationship between variables. So does an equation.

However, some connective expressions are equations, and some others are not.

For instance, a connective expression can be $s^2 - 3s < t - 2$, which is not an equation.

Then, putting the expression in a graph, we can get the one shown below.

Fig. 3



So the expression, $s^2 - 3s < t - 2$ explains the relationship between the coordinates of each of all the points in the area above the parabola in the ***s-t*** plane above.

All the points in the parabola itself are not included in the area, of course.