Examples A in Circles

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Examples A in Circles

Constructing a curve in math, we can put it in a graph. Putting it in a graph though, we need the equation of it. So constructing it, we want to find the equation of it if we are not given the equation, of course. And finding the equation, it is said that we get it, too.

0. Assuming that a line is $\mathbf{2}\boldsymbol{x} + \mathbf{3}\boldsymbol{y} - \mathbf{3} = \mathbf{0}$, construct these:

0.0. A circle that is tangent to the line, and has a radius of 2 and a center where the $\boldsymbol{x}$-coordinate is 1.

0.1. A circle that is tangent to the line, and has a radius of 2 and a center where the $\boldsymbol{y}$-coordinate is 1.

1. Construct a circle that is tangent to a line $\boldsymbol{x} = \boldsymbol{a}$, where $\boldsymbol{a}$ is constant, and has a radius of 2 and a center where the $\boldsymbol{y}$-coordinate is 1.

2. Construct a circle that is tangent to a line $\boldsymbol{y} = \boldsymbol{b}$, where $\boldsymbol{b}$ is constant, and has a radius of 2 and a center where the $\boldsymbol{x}$-coordinate is 1.

Suggestions or Solutions
To the Problem 0 in the Example 0

Construct a circle that is tangent to a line 2*x* + 3*y* – 3 = 0, and has a radius of 2 and a center where the *x*-coordinate is 1.

Suppose ***S*** is the circle to be constructed, ***A*** is the line **2*x* + 3*y* – 3 = 0**, ***B*** is a line parallel to ***A***, and the distance between ***A*** and ***B*** is 2, which is the radius of ***S***.

Then, ***B*** can be put in $y = -\frac{2}{3}x + c$ where ***c*** is constant and $c \neq 1$, for ***B*** is parallel to ***A***.

Meanwhile, we get $2x + 3y - 3 = 0 \Rightarrow y = -\frac{2}{3}x + 1$.

Suppose now, ***t*** is constant, and **(1, *t*)** is a point in ***B***. Then, the center of ***S*** can be put in $(1, -\frac{2}{3} + c)$, the distance from which to ***A*** is the radius of ***S***.

So ***S*** can be for now, put in $(x-1)^2 + \{y - (-\frac{2}{3} + c)\}^2 = 4$.

The distance from a point **(*u*, *v*)** to a line **$px + qy + r = 0$** is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

So the distance from the center to ***A*** is $\frac{|2\cdot1+3(-\frac{2}{3}+c)-3|}{\sqrt{2^2+3^2}} = \frac{|3c-3|}{\sqrt{13}}$, which is 2. Thus, we get

$\frac{3|1-c|}{\sqrt{13}} = 2 \Rightarrow \frac{9(1-c)^2}{13} = 4 \Rightarrow 9(1-c)^2 = 52 \Rightarrow 1-c = \pm\frac{\sqrt{52}}{3} \Rightarrow c = 1 \pm \frac{2\sqrt{13}}{3} = \frac{3\pm2\sqrt{13}}{3}$.

So we get $-\frac{2}{3} + c = -\frac{2}{3} + \frac{3\pm2\sqrt{13}}{3} = \frac{1\pm2\sqrt{13}}{3}$.

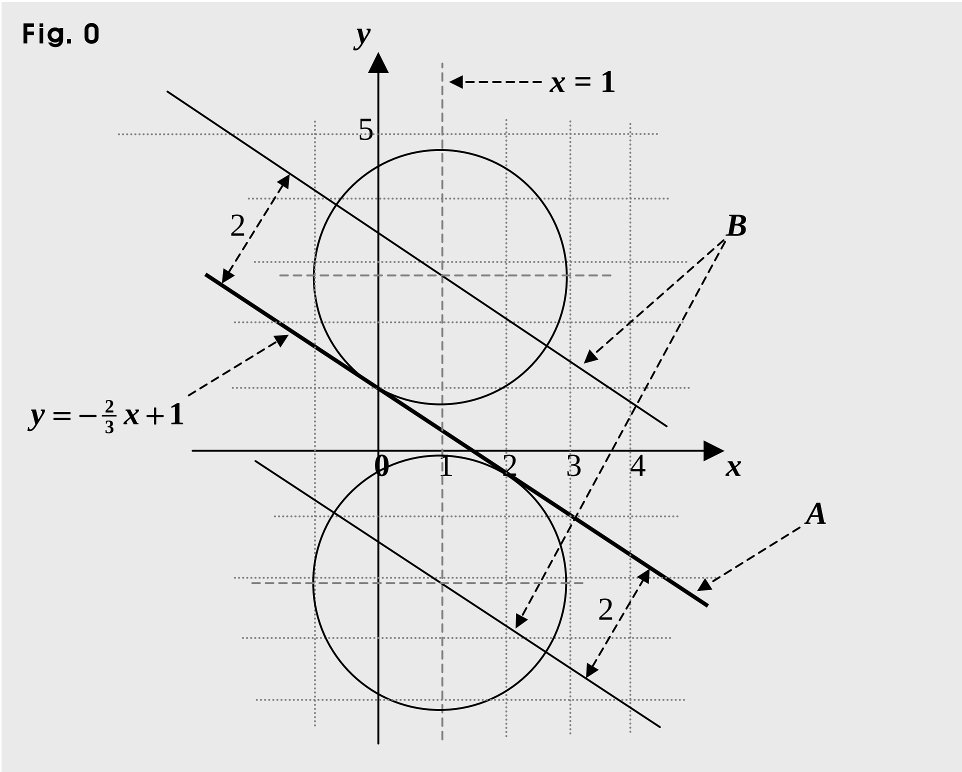
Therefore, the circle ***S*** is as follows. $(x-1)^2 + (y - \frac{1\pm2\sqrt{13}}{3})^2 = 4$.

If not quite sure of the idea behind the processes above, follow the steps below.

Suppose ***A*** is the line **2*x* + 3*y* – 3 = 0**, ***B*** is a line parallel to the line ***A***, and the distance between ***A*** and ***B*** is 2.

Then, if the center of a circle is in the line ***B***, and the circle is tangent to the line ***A***, then the radius is 2. Meanwhile, $2x + 3y - 3 = 0 \Rightarrow y = -\frac{2}{3}x + 1$.

Let's now, put in a graph the ideas above.



Now, we can see that there are two circles that can be the solution to this problem.

Given the center and radius, we can find the circle.
We know that the radius is 2, but the ***x***-coordinate only of the center is given.
So we want to find the ***y***-coordinate of the center.
Anyway, where do we want to look for the center?

The center is in the line ***B***. That is, one of the points in the line ***B*** is the center.
So we want to find the line ***B***, first. How then, do we find the line ***B***?

The line ***B*** is parallel to and 2 away from the line ***A***.
Then, where in the line ***B***, is the center?

We have a fact that at the center, the ***x***-coordinate is 1.
So using the fact, we can find exactly where the center is in the line ***B***.

Then, using the center, together with the radius, which is 2, we can find the circle.

Now, going back to the graph above, we can see that the two lines ***A*** and ***B*** are parallel to each other, so ***B*** can be put in $y = -\frac{2}{3}x + c$ where c is constant and $c \neq 1$, and of course, we want to find ***c***.   How to find it, though? That is, how do we find the line ***B***?

The line ***B*** has a point that is the center of the circle we want. So suppose now, that ***S*** is the circle to be constructed. Then, the center of the circle ***S*** is in the line ***B***.

We know that the ***x***-coordinate at the center is 1.

So we can suppose that the center is **(1, *t*)** where ***t*** is a constant.

Then, the point **(1, *t*)** belongs to the line ***B***, so we can put it into ***B***, which is $y = -\frac{2}{3}x + c$.

Thus, we get $t = -\frac{2}{3} + c$, which is the ***y***-coordinate at the center of the circle ***S***.

So the center of the circle ***S*** can be put this way: $(1, -\frac{2}{3} + c)$. How do we find ***c***, then?

We can find ***c*** using a fact stated earlier, and relevant to the radius. What fact?

The fact is that the distance from ***A*** to the center of ***S*** is the radius, which is 2.
How then, do we use such a fact?

The fact is talking about the radius, so let's now, move on to the radius.

We know the center of ***S*** is in the line ***B***, and the circle ***S*** is tangent to the line ***A***.

So the radius of ***S*** is the distance from the center to the line ***A***. We have a tool for such a distance, and the tool is a template, called a formula for a distance from a point to a line.

• The distance from a point (u, v) to a line $px + qy + r = 0$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

Now, the line $\boldsymbol{A}$ is $\mathbf{2x + 3y - 3 = 0}$, and the center of $\boldsymbol{S}$ is $\mathbf{(1, -\frac{2}{3} + c)}$. Thus, the distance from the center to $\boldsymbol{A}$ is $\frac{|2\cdot 1+3(-\frac{2}{3}+c)-3|}{\sqrt{2^2+3^2}} = \frac{|3c-3|}{\sqrt{13}}$, which is the radius of $\boldsymbol{S}$, which is 2.

So we get $\frac{3|1-c|}{\sqrt{13}} = 2 \Rightarrow \frac{9(1-c)^2}{13} = 4 \Rightarrow 9(1-c)^2 = 52 \Rightarrow 1-c = \pm\frac{\sqrt{52}}{3} \Rightarrow c = 1 \pm \frac{2\sqrt{13}}{3} = \frac{3\pm 2\sqrt{13}}{3}$.

Now, we know that the center of $\boldsymbol{S}$ is $\mathbf{(1, -\frac{2}{3} + c)}$.

So the center is $\mathbf{(1, -\frac{2}{3} + \frac{3\pm 2\sqrt{13}}{3})}$, which means we get two circles. More specifically,

first, if $c = \frac{3+2\sqrt{13}}{3}$, we get $-\frac{2}{3} + c = -\frac{2}{3} + \frac{3+2\sqrt{13}}{3} = \frac{1+2\sqrt{13}}{3} \approx \mathbf{2.737}$.

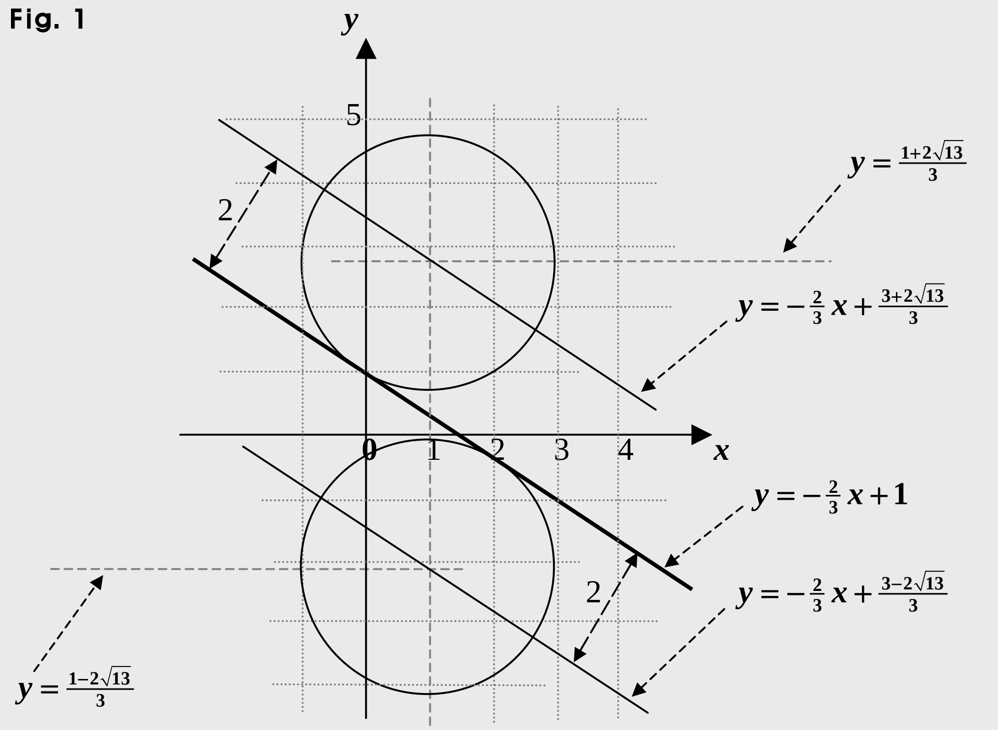
Next, if $c = \frac{3-2\sqrt{13}}{3}$, we get $-\frac{2}{3} + c = -\frac{2}{3} + \frac{3-2\sqrt{13}}{3} = \frac{1-2\sqrt{13}}{3} \approx \mathbf{-2.070}$.

Now, since the radius is 2, using the standard form, we can see that the circle $\boldsymbol{S}$ is

$(x-1)^2 + (y - \frac{1+2\sqrt{13}}{3})^2 = 4$ or $(x-1)^2 + (y - \frac{1-2\sqrt{13}}{3})^2 = 4$.

Of course, we can put the equations above this way, too: $(x-1)^2 + (y - \frac{1\pm 2\sqrt{13}}{3})^2 = 4$.

Fig. 1



Suggestions or Solutions
To the Problem 1 in the Example 0

Construct a circle that is tangent to a line 2*x* + 3*y* – 3 = 0, and has a radius of 2 and a center where the *y*-coordinate is 1.

Suppose first, ***C*** is the circle to be constructed, ***A*** is the line **2*x* + 3*y* – 3 = 0**, ***B*** is a line parallel to the line ***A***, and the distance between the lines ***A*** and ***B*** is 2, which is the radius of ***C***. Then, the center of the circle ***C*** is in the line ***B***.

Next, putting the line ***A*** in the standard form, we get $y=-\frac{2}{3}x+1$, so the line ***B*** can be put in $y=-\frac{2}{3}x+c$, where ***c*** is constant and $c\neq 1$. Suppose now, that **(*s*, 1)** is a point in ***B***.

Then, **(*s*, 1)** is the center of ***C***, and we get $1=-\frac{2}{3}s+c$ since ***B*** is $y=-\frac{2}{3}x+c$.

So we get $1=-\frac{2}{3}s+c\Rightarrow s=\frac{3(c-1)}{2}$, and thus, the center of ***C*** can be put in $(\frac{3(c-1)}{2},1)$.

So the circle ***C*** can be for now, put in $(x-\frac{3(c-1)}{2})^2+(y-1)^2=4$.

The distance from a point **(*u*, *v*)** to a line ***px* + *qy* + *r* = 0** is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

The line ***A*** is **2*x* + 3*y* – 3 = 0**, and therefore, the distance from the center to the line ***A*** is $\frac{|2\cdot\frac{3(c-1)}{2}+3\cdot 1-3|}{\sqrt{2^2+3^2}}=\frac{|3(c-1)|}{\sqrt{13}}$, which is the radius, which is 2. Thus, we get

$\frac{3|1-c|}{\sqrt{13}}=2\Rightarrow\frac{9(1-c)^2}{13}=4\Rightarrow 9(1-c)^2=52\Rightarrow 1-c=\pm\frac{\sqrt{52}}{3}\Rightarrow c=1\pm\frac{2\sqrt{13}}{3}=\frac{3\pm 2\sqrt{13}}{3}$.

So we get $\frac{3(c-1)}{2}=\frac{3(\frac{3\pm 2\sqrt{13}}{3}-1)}{2}=\pm\sqrt{13}$.

Therefore, the circle ***C*** is $(x+\sqrt{13})^2+(y-1)^2=4$, or $(x-\sqrt{13})^2+(y-1)^2=4$.

If not quite sure of the idea behind the processes above, follow the steps below.

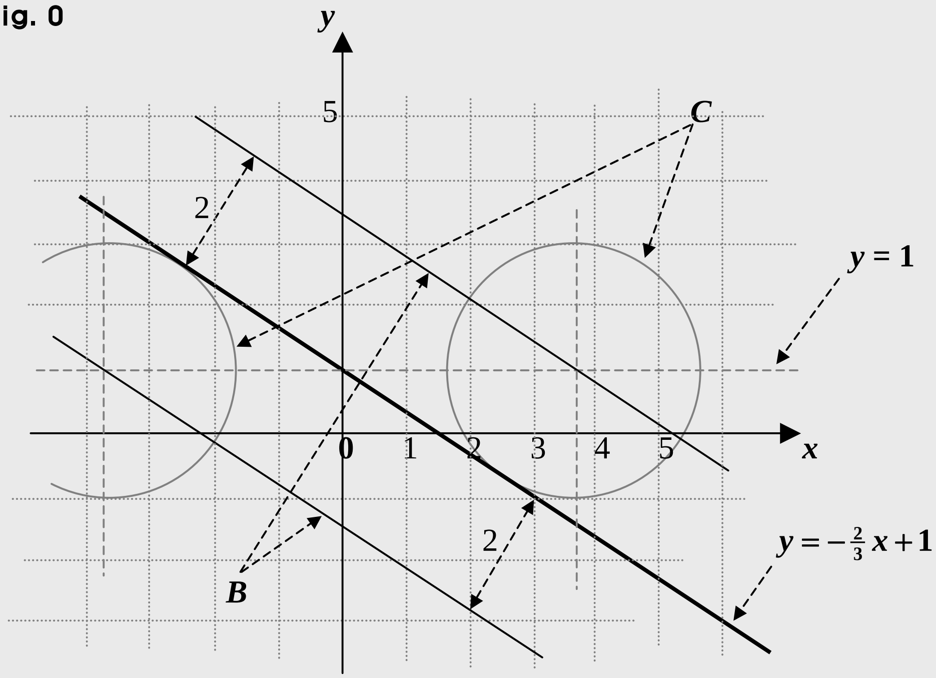
Suppose ***A*** is the line **2*x* + 3*y* – 3 = 0**, and ***B*** is a line parallel to the line ***A***.

Suppose also, the distance between the lines ***A*** and ***B*** is 2, and ***C*** is the circle we are after.

Then, the center of the circle ***C*** is in the line ***B***, and the radius is the same as the distance between ***A*** and ***B***. Meanwhile, $\mathbf{2x + 3y - 3 = 0} \Rightarrow y = -\frac{2}{3}x + 1$, which is the line ***A***.

Now, putting in a graph the ideas above, we can put it the way below.

Fig. 0



Now, we can see that there are two circles that can be the solution to this problem.

• Given the center and radius of a circle, we can find the circle.

We know that the radius is 2, but the ***y***-coordinate only at the center is given.
So we want to find the ***x***-coordinate of the center.
Anyway, whereabouts is the center?

The center is somewhere in the line ***B***. So it is for sure that the line ***B*** passes through the center. So we want to find the line ***B***, first. How then, do we find it?

We know the line ***A***. So what about it?

The line $\boldsymbol{A}$ is parallel to, and is 2 away from the line $\boldsymbol{B}$. And also, the line $\boldsymbol{A}$ is tangent to the circle we want. Then, what point in the line $\boldsymbol{B}$, is the center?

We have a fact that at the center, the $\boldsymbol{y}$-coordinate is 1. So using the fact, along with the line $\boldsymbol{A}$, we can find the exact location of the center in the line $\boldsymbol{B}$.

Then, using the center, along with the radius, which is 2, we can find the circle we want.

Now, the line $\boldsymbol{A}$ is $\boldsymbol{y = -\frac{2}{3}x + 1}$, and is parallel to the line $\boldsymbol{B}$, so the line $\boldsymbol{B}$ can be put in an equation $\boldsymbol{y = -\frac{2}{3}x + c}$, where the slope is the same as $\boldsymbol{A}$'s, and $\boldsymbol{c}$ is constant and $\boldsymbol{\neq 1}$.

Next, we know that the $\boldsymbol{y}$-coordinate at the center of the circle $\boldsymbol{C}$ is 1.

So we can suppose that the center of the circle $\boldsymbol{C}$ is $\boldsymbol{(s, 1)}$, where $\boldsymbol{s}$ is a constant.

Also, we know the center of $\boldsymbol{C}$, that is, $\boldsymbol{(s, 1)}$ is in the line $\boldsymbol{B}$, which is $\boldsymbol{y = -\frac{2}{3}x + c}$.

So we get $\boldsymbol{1 = -\frac{2}{3}s + c \Rightarrow s = \frac{3(c-1)}{2}}$, and thus, the center of $\boldsymbol{C}$ can be put this way: $\boldsymbol{(\frac{3(c-1)}{2}, 1)}$. How then do we find $\boldsymbol{c}$?

The distance from the line $\boldsymbol{A}$ to the center of the circle $\boldsymbol{C}$ is the radius, which is 2.
And we have yet to use the fact above, so let's now, move on to the radius.

First, the center of the circle $\boldsymbol{C}$ is in the line $\boldsymbol{B}$, and the circle $\boldsymbol{C}$ is tangent to the line $\boldsymbol{A}$, so the radius of the circle $\boldsymbol{C}$ is the distance from the center to the line $\boldsymbol{A}$. And next, there is a template, where the distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\boldsymbol{\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}}$.

The line $\boldsymbol{A}$ is $\boldsymbol{2x + 3y - 3 = 0}$, the center of the circle $\boldsymbol{C}$ is $\boldsymbol{(s, 1) = (\frac{3(c-1)}{2}, 1)}$, so the distance from the center of $\boldsymbol{C}$ to $\boldsymbol{A}$ is $\boldsymbol{\frac{|2\cdot\frac{3(c-1)}{2}+3\cdot 1-3|}{\sqrt{2^2+3^2}} = \frac{|3(c-1)|}{\sqrt{13}}}$, which is the radius, which is 2.

So we get $\boldsymbol{\frac{3|1-c|}{\sqrt{13}} = 2 \Rightarrow \frac{9(1-c)^2}{13} = 4 \Rightarrow 9(1-c)^2 = 52 \Rightarrow 1-c = \pm\frac{\sqrt{52}}{3} \Rightarrow c = 1 \pm \frac{2\sqrt{13}}{3} = \frac{3\pm 2\sqrt{13}}{3}}$.

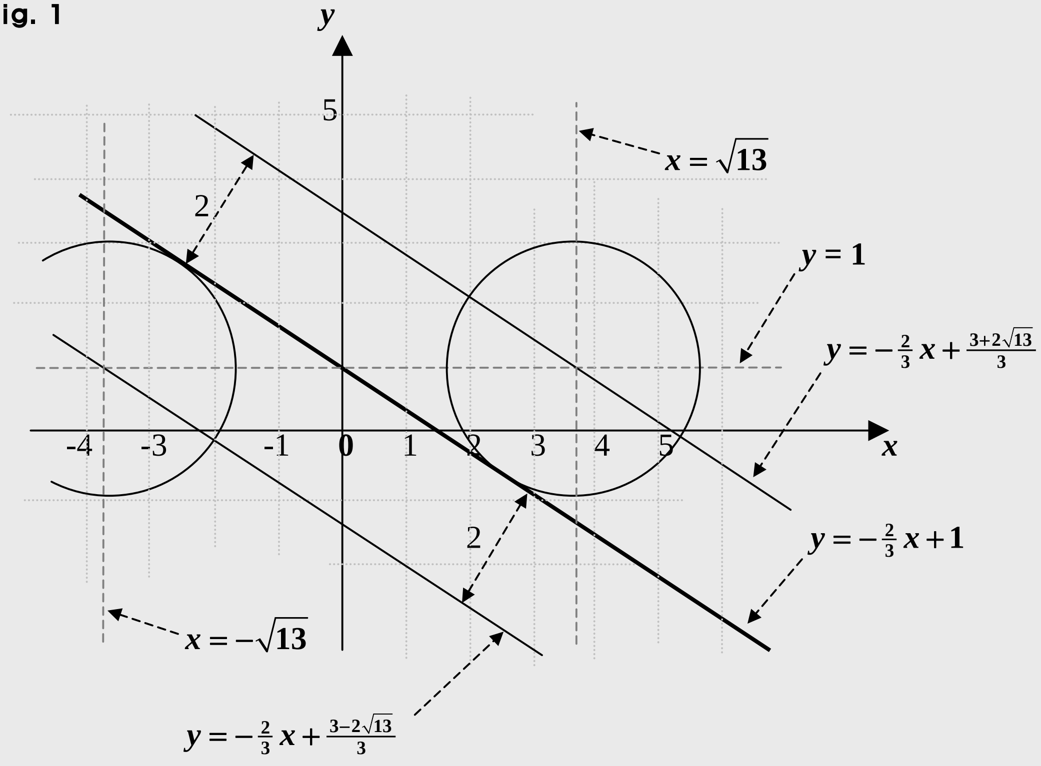
Then, we get $c = \frac{3\pm2\sqrt{13}}{3} \Rightarrow (\frac{3(c-1)}{2},1) = (\frac{3c-3}{2},1) = (\frac{3\pm2\sqrt{13}-3}{2},1) = (\frac{\pm2\sqrt{13}}{2},1) = (\pm\sqrt{13},1)$.

Therefore, the center of the circle C is $(\sqrt{13},1)$ or $(-\sqrt{13},1)$, so we can get two circles.

Now, since the radius is 2, using the standard form, we can see that the circle C is

$(x-\sqrt{13})^2+(y-1)^2 = 4$ or $(x+\sqrt{13})^2+(y-1)^2 = 4$. And we have $\sqrt{13} \approx 3.606$.

Fig. 1



Suggestions or Solutions
To the Problem in the Example 1

Construct a circle that is tangent to a line $x = a$, where a is constant, and has a radius of 2 and a center where the y-coordinate is 1.

Suppose C is the circle to be constructed, A is the line $x = a$, B is another line $x = c$, where c is a constant and $\neq a$, and the distance from A to B is 2, which is the radius of C.

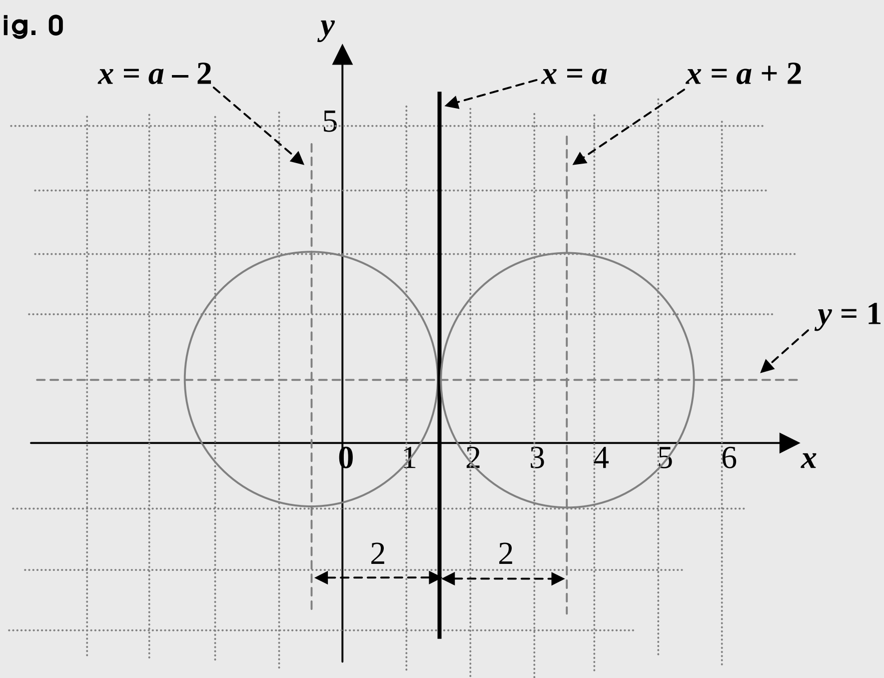
Then, the center of C is $(c, 1)$, and the radius is $|c - a|$, so we get
$|c - a| = 2 \Rightarrow c - a = \pm 2 \Rightarrow c = a + 2$ or $a - 2$.

Therefore, the circle C is $(x - a + 2)^2 + (y - 1)^2 = 2^2$ or $(x - a - 2)^2 + (y - 1)^2 = 2^2$.

If not quite sure of the idea behind the processes above, follow the steps below.

Let's begin with putting in a graph the line and some probable circles.
Then, we can see better whereabouts the solution is.

Fig. 0



Now, we can see that there can be such two circles as the one this problem is asking.

Given the center and radius of a circle, we can find the circle.

We know that the radius is 2, but the $\boldsymbol{y}$-coordinate only of the center is given, and is 1. So we want to find the $\boldsymbol{x}$-coordinate of the center.

Suppose $\boldsymbol{C}$ is the circle to be constructed, $\boldsymbol{A}$ is the line $\boldsymbol{x = a}$, $\boldsymbol{B}$ is a line parallel to the line $\boldsymbol{A}$, and the distance between the two lines $\boldsymbol{A}$ and $\boldsymbol{B}$ is 2.

Then, the center of the circle $\boldsymbol{C}$ is in the line $\boldsymbol{B}$.

The line $\boldsymbol{B}$ is parallel to the line $\boldsymbol{A}$, which is $\boldsymbol{x = a}$, so the line $\boldsymbol{B}$ can be put in an equation where $\boldsymbol{x = c}$, where $\boldsymbol{c}$ is a constant, and $\neq \boldsymbol{a}$. In the line $\boldsymbol{x = c}$ in the $\boldsymbol{x}$-$\boldsymbol{y}$ system, every value of $\boldsymbol{x}$ is $\boldsymbol{c}$ for every value of $\boldsymbol{y}$. In short, $\boldsymbol{x}$ is $\boldsymbol{c}$ for all $\boldsymbol{y}$.

So the center of the circle $\boldsymbol{C}$ is $\boldsymbol{(c, 1)}$, for the center is in $\boldsymbol{B}$ and the $\boldsymbol{y}$-coordinate is 1 there. Besides, the radius of the circle $\boldsymbol{C}$ is 2, so the circle $\boldsymbol{C}$ can be put this way, for now:
$\boldsymbol{(x - c)^2 + (y - 1)^2 = 2^2}$.

Let's now, move on to the radius to find $\boldsymbol{c}$.

First, the distance between the two lines $\boldsymbol{A}$ and $\boldsymbol{B}$ is the radius of the circle $\boldsymbol{C}$.

Next, looking at the graph, we can immediately see that the distance between $\boldsymbol{A}$ and $\boldsymbol{B}$ is $\boldsymbol{|c - a|}$.

So this time, we don't have to use the formula for the distance from a point to a line.

Let's see however, if the formula still works in this case, too.

The distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

The line $\boldsymbol{A}$ is $\boldsymbol{x = a}$, that is, $\boldsymbol{1 \cdot x + 0 \cdot y - a = 0}$, and the center is $\boldsymbol{(c, 1)}$.

So the distance from the line $\boldsymbol{A}$ to the center, that is, the radius is $\frac{|1\cdot c+0\cdot 1-a|}{\sqrt{1^2+0^2}} = |c - a|$.

Now, since the radius is 2, we get $|c - a| = 2 \Rightarrow c - a = \pm 2 \Rightarrow c = a + 2$ or $a - 2$.

So first, if $c = a + 2$, we get $(x - c)^2 + (y - 1)^2 = 2^2 \Rightarrow (x - a - 2)^2 + (y - 1)^2 = 2^2$.

Next, if $c = a - 2$, we get $(x - c)^2 + (y - 1)^2 = 2^2 \Rightarrow (x - a + 2)^2 + (y - 1)^2 = 2^2$.

Thus, the circle C is $(x - a - 2)^2 + (y - 1)^2 = 2^2$ or $(x - a + 2)^2 + (y - 1)^2 = 2^2$, which is a circle that is tangent to a line $x = a$, and has a radius of 2 and a center in which the y-coordinate is 1.

In short:

Suppose C is the circle to be constructed, A is the line $x = a$, B is another line $x = c$, where c is a constant and $\neq a$, and the distance from A to B is 2, which is the radius of C.

Then, the center of C is $(c, 1)$, and the radius is $|c - a|$, so we get

$|c - a| = 2 \Rightarrow c - a = \pm 2 \Rightarrow c = a + 2$ or $a - 2$.

Therefore, the circle C is $(x - a + 2)^2 + (y - 1)^2 = 2^2$ or $(x - a - 2)^2 + (y - 1)^2 = 2^2$.

Suggestions or Solutions
To the Problem in the Example 2

Construct a circle that is tangent to a line $y = b$, where b is constant, and has a radius of 2 and a center where the x-coordinate is 1.

Suppose $\boldsymbol{C}$ is the circle to be constructed, $\boldsymbol{A}$ is the line $\boldsymbol{y = b}$, $\boldsymbol{B}$ is a line $\boldsymbol{y = c}$, where $\boldsymbol{c}$ is constant and $\neq \boldsymbol{b}$, and the distance from $\boldsymbol{A}$ to $\boldsymbol{B}$ is 2, which is the radius of the circle $\boldsymbol{C}$.

Then, the center of the circle $\boldsymbol{C}$ is $\mathbf{(1,}\ \boldsymbol{c}\mathbf{)}$, and the radius of $\boldsymbol{C}$ is $|\boldsymbol{c - b}|$, so we get

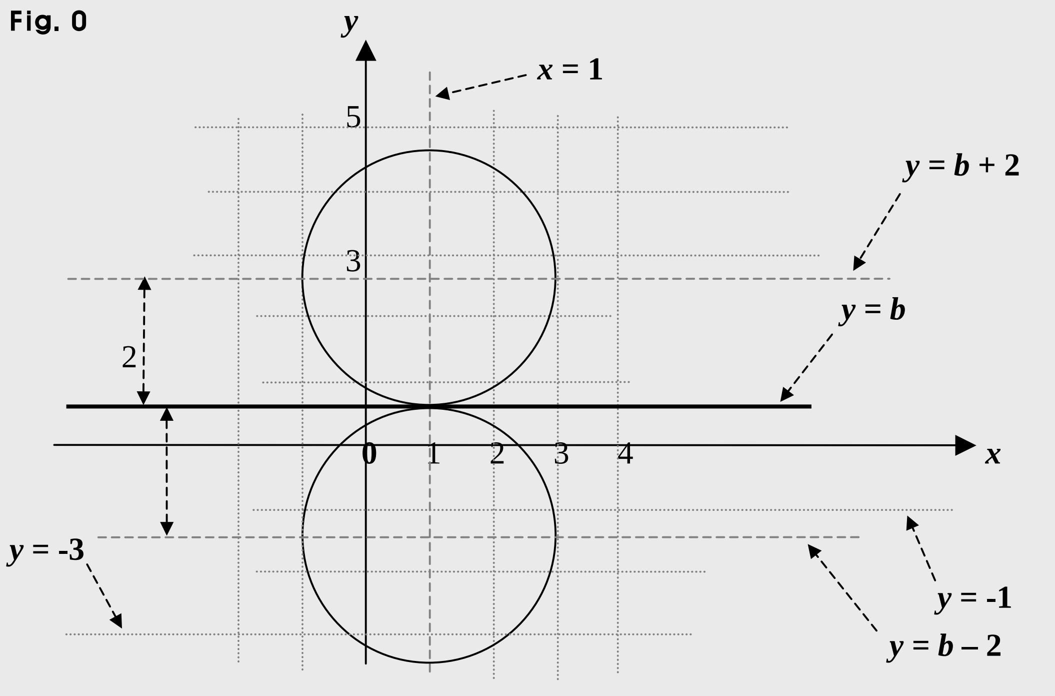
$|\boldsymbol{c - b}| = 2 \Rightarrow \boldsymbol{c - b} = \pm 2 \Rightarrow \boldsymbol{c = b} + 2$ or $\boldsymbol{b} - 2$.

Therefore, the circle $\boldsymbol{C}$ is $(\boldsymbol{x} - 1)^2 + (\boldsymbol{y - b} + 2)^2 = 2^2$ or $(\boldsymbol{x} - 1)^2 + (\boldsymbol{y - b} - 2)^2 = 2^2$.

If not quite sure of the idea behind the processes above, follow the steps below.

Let's begin with putting the problem in a graph. Then, we can see better whereabouts the solution is.

Fig. 0



Now, to this problem, too, we can see that two circles can be the solution.

Suppose ***C*** is the circle we want, ***A*** is the line $\boldsymbol{y = b}$, ***B*** is a line parallel to the line ***A***, and the distance from the line ***A*** to the line ***B*** is 2, which is the radius of the circle ***C***.

Then, the center of the circle ***C*** is in the line ***B***, for the radius is the distance above.

Since the two lines ***A*** and ***B*** are parallel to each other, the line ***B*** can be indicated by an equation of a line $\boldsymbol{y = c}$, where $\boldsymbol{c}$ is constant and $\neq \boldsymbol{b}$.

In the line $\boldsymbol{y = c}$ in the $\boldsymbol{x}$-$\boldsymbol{y}$ system, every value of $\boldsymbol{y}$ is $\boldsymbol{c}$ for every value of $\boldsymbol{x}$.
In short, $\boldsymbol{y}$ is $\boldsymbol{c}$ for all $\boldsymbol{x}$.

Since the $\boldsymbol{x}$-coordinate of the center of the circle ***C*** is 1, we can suppose the center of the circle ***C*** is $\boldsymbol{(1, t)}$, where $\boldsymbol{t}$ is constant. Then, we want to find $\boldsymbol{t}$.

The center $\boldsymbol{(1, t)}$ is in the line ***B***. So we get $\boldsymbol{t = c}$ since the line ***B*** is a line $\boldsymbol{y = c}$.
Thus, the center of the circle ***C*** is $\boldsymbol{(1, c)}$.

Let's now, move on to the radius.

First, the distance between the two lines ***A*** and ***B*** is the radius of the circle ***C***.

Next, looking at the graph, we can immediately see that the distance from the line ***A*** to the line ***B*** is $\boldsymbol{|c - b|}$. So this time, either, we don't have to use the formula for the distance from a point to a line. Let's see however, if the formula still works in this case, too.

• The distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

The line ***A*** is $\boldsymbol{y = b}$, that is, $\boldsymbol{0 \cdot x + 1 \cdot y - b = 0}$, and the center is $\boldsymbol{(1, c)}$.

So the distance from the line ***A*** to the center, that is, the radius is $\frac{|0\cdot 1+1\cdot c-a|}{\sqrt{0^2+1^2}} = |c - b|$.

Now, since the radius of the circle ***C*** is 2, $\boldsymbol{|c - b| = 2 \Rightarrow c - b = \pm 2 \Rightarrow c = b + 2}$ or $\boldsymbol{b - 2}$.

Why $\boldsymbol{c - b = \pm 2}$, though?

$|v|$ indicates the absolute value of v, that is, the magnitude of v.

So for instance, we get $|w| = 1 \Rightarrow w = 1$ or -1, that is, $w = \pm 1$.

Now, since the radius of the circle C is 2 and the center is $(1, c)$, the circle C can be put this way, for now: $(x - 1)^2 + (y - c)^2 = 2^2$.

So first, if $c = b + 2$, we get $(x - 1)^2 + (y - c)^2 = 2^2 \Rightarrow (x - 1)^2 + (y - b - 2)^2 = 2^2$.

Next, if $c = b - 2$, we get $(x - 1)^2 + (y - c)^2 = 2^2 \Rightarrow (x - 1)^2 + (y - b + 2)^2 = 2^2$.

Thus, the circle C is $(x - 1)^2 + (y - b - 2)^2 = 2^2$ or $(x - 1)^2 + (y - b + 2)^2 = 2^2$, which is a circle that is tangent to the line $y = b$ and has a radius of 2 and a center where the x-coordinate is 1.

We can put the circle C this way, too, of course: $(x - 1)^2 + (y - b \pm 2)^2 = 2^2$.

In short:

Suppose C is the circle to be constructed, A is the line $y = b$, B is a line $y = c$, where c is constant and $\neq b$, and the distance from A to B is 2, which is the radius of the circle C.

Then, the center of the circle C is $(1, c)$, and the radius of C is $|c - b|$, so we get

$|c - b| = 2 \Rightarrow c - b = \pm 2 \Rightarrow c = b + 2$ or $b - 2$.

Therefore, the circle C is $(x - 1)^2 + (y - b \pm 2)^2 = 2^2$.