Examples B in Circles

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Examples B in Circles

Constructing a curve in math, we can put it in a graph. Putting it in a graph though, we need the equation of it. So constructing it, we want to find the equation of it if we are not given the equation, of course. And finding the equation, it is said that we get it, too.

Assuming a line $\boldsymbol{A}$ is $\boldsymbol{y = 2x + 1}$,

0. Construct a circle that is tangent to the line $\boldsymbol{A}$ and has a radius of 2 and a center where the $\boldsymbol{x}$-coordinate is 1.

1. Construct every circle that is tangent to the line $\boldsymbol{A}$, and has a radius of 2.

2. Find all the circles that have the same radii and are tangent to the line $\boldsymbol{A}$.

Suggestions or Solutions
To the Problem in the Example 0

Construct a circle that is tangent to a line $y = 2x + 1$, and has a radius of 2 and a center where the x-coordinate is 1.

Suppose first, $\boldsymbol{S}$ is the circle we want, $\boldsymbol{A}$ is the line $\boldsymbol{y = 2x + 1}$, $\boldsymbol{B}$ is a line $\boldsymbol{y = 2x + c}$, where $\boldsymbol{c}$ is constant, but $\neq \mathbf{1}$, and the distance between $\boldsymbol{A}$ and $\boldsymbol{B}$ is 2, which is the radius of $\boldsymbol{S}$.
Suppose next, $\boldsymbol{(1, t)}$ is the center of $\boldsymbol{S}$, and is a point in $\boldsymbol{B}$, since the $\boldsymbol{x}$-coordinate of the center is 1.

Then, we get $\boldsymbol{t = 2 + c}$, and the center of $\boldsymbol{S}$ can be put this way: $\boldsymbol{(1, 2 + c)}$.

So the circle $\boldsymbol{S}$ can be put for now, this way: $\boldsymbol{(x-1)^2 + \{y-(2+c)\}^2 = 4}$.

The distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

The line $\boldsymbol{A}$ is $\boldsymbol{y = 2x + 1}$, so $\boldsymbol{A}$ can be indicated by $\boldsymbol{2x - y + 1 = 0}$, too.

So the distance from the center $\boldsymbol{(1, 2 + c)}$ to the line $\boldsymbol{A}$ is $\frac{|2\cdot 1+(-1)(2+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2-2-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

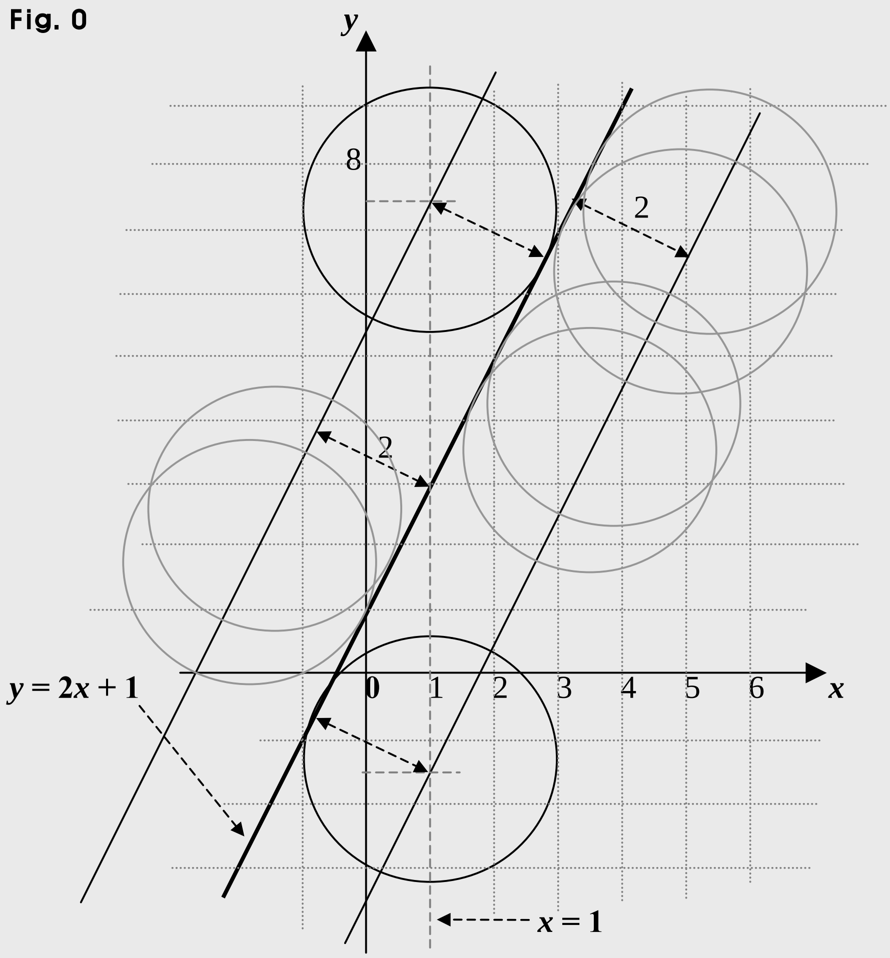
Thus, we get $\frac{|1-c|}{\sqrt{5}} = 2 \Rightarrow \frac{(1-c)^2}{5} = 4 \Rightarrow (1-c)^2 = 20 \Rightarrow 1-c = \pm 2\sqrt{5} \Rightarrow c = 1 \pm 2\sqrt{5}$.

Therefore, the circle $\boldsymbol{S}$ is $\boldsymbol{(x-1)^2 + (y-3 \pm 2\sqrt{5})^2 = 4}$.

If not quite sure of the idea behind the processes above, follow the steps below.

It's a good idea to put the problem in a graph.
So let's begin with putting this problem in a graph.
Putting this problem in a graph, put in a graph the line given and some circles probable.
Then, you can see better where the solution can be around.

Fig. 0



Now, in the graph above, is there any circle satisfying the conditions in the problem?

We can see that there are two such circles, and they are in black. How do we find them?

• Given the center and radius, we can readily find the circle. Then, the equation of it will be in the standard form.

We know that the radius is 2, but the $\boldsymbol{x}$-coordinate only of the center is given.

So we want to find the $\boldsymbol{y}$-coordinate of the center.

Suppose $\boldsymbol{A}$ is the line $\boldsymbol{y = 2x + 1}$, and $\boldsymbol{B}$ is a line $\boldsymbol{y = 2x + c}$, where $\boldsymbol{c}$ is constant and $\boldsymbol{\neq 1}$.

Suppose also, the distance between the two lines ***A*** and ***B*** is 2.

Then, a circle tangent to the line ***A*** and centered at a point in the line ***B*** has a radius of 2, which is the distance between the two lines ***A*** and ***B***.

So we want to find the value of ***c*** that makes the circle above have a radius of 2.

Suppose now, that ***S*** is the circle this problem is asking.

Then, we want the circle ***S*** to have a center where the ***x***-coordinate is 1.

So suppose for now, that the center of the circle ***S*** is $(1, t)$ where ***t*** is constant.

Then, the center $(1, t)$ is in ***B***, which is $y = 2x + c$, since ***S*** is centered at a point in ***B***.

So we get $t = 2 + c$, and can say that the center is $(1, 2 + c)$.

Let's now, move on to the radius.

The distance between the lines ***A*** and ***B*** is the distance from the center of ***S*** to the line ***A***, so the distance is the radius. And we know the center is $(1, 2 + c)$.

Besides, we have a formula for a distance from a point to a line.

• The distance from a point (u, v) to a line $px + qy + r = 0$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

We can put the line ***A***, $y = 2x + 1$, in this way, too: $2x - y + 1 = 0$.

Then, the distance, that is, the radius is as follows. $\frac{|2\cdot 1+(-1)(2+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2-2-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

So the circle ***S*** can be for now, put this way: $(x-1)^2 + \{y-(2+c)\}^2 = \frac{(1-c)^2}{5}$.

Now, since the radius is 2 in the circle ***S***, we get

$\frac{|1-c|}{\sqrt{5}} = 2 \Rightarrow \frac{(1-c)^2}{5} = 4 \Rightarrow (1-c)^2 = 20 \Rightarrow 1-c = \pm 2\sqrt{5} \Rightarrow c = 1 \pm 2\sqrt{5}$.

Then, first, if $c = 1 + 2\sqrt{5}$, we get

$$(x-1)^2 + \{y-(2+c)\}^2 = (x-1)^2 + (y-2-1-2\sqrt{5})^2 = 4.$$

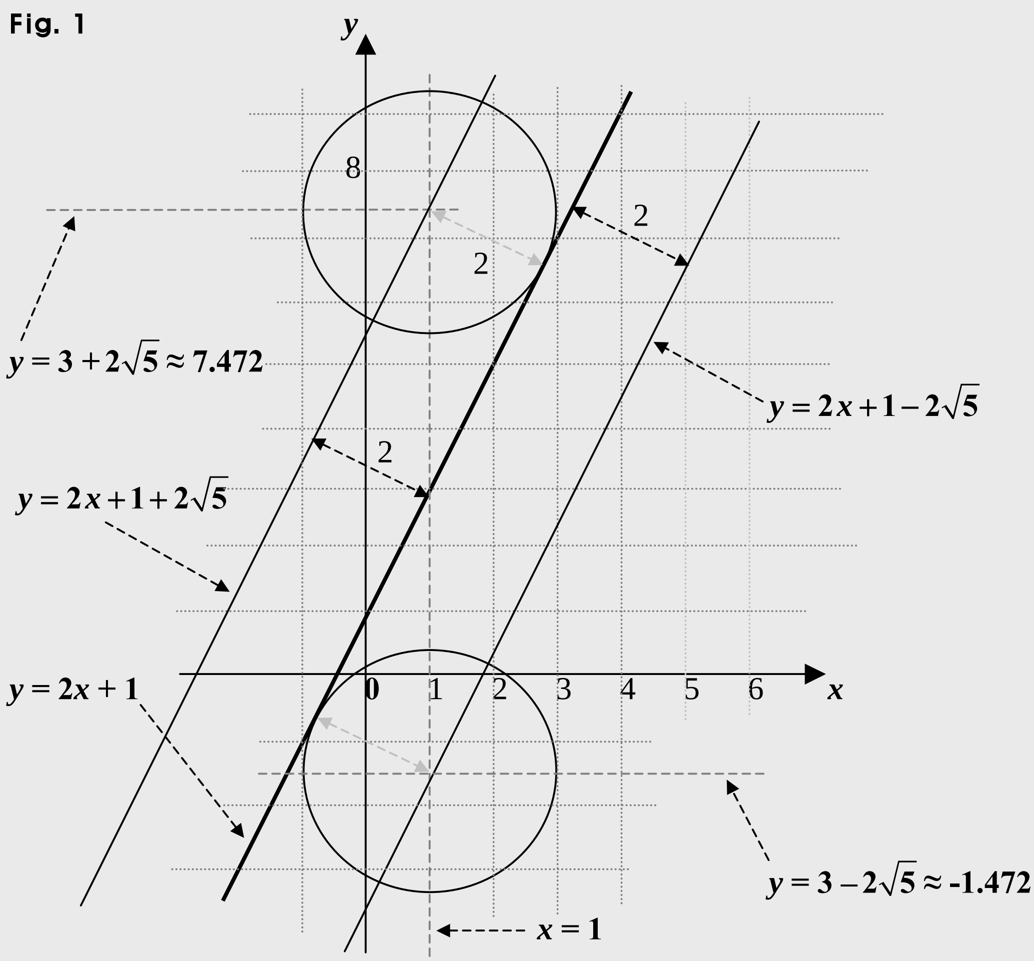
And next, if $c = 1 - 2\sqrt{5}$, we get

$$(x-1)^2 + \{y-(2+c)\}^2 = (x-1)^2 + (y-2-1+2\sqrt{5})^2 = 4.$$

Therefore, the circle S is as follows.

$(x-1)^2 + (y-3-2\sqrt{5})^2 = 4$ or $(x-1)^2 + (y-3+2\sqrt{5})^2 = 4$.

Fig. 1



Suggestions or Solutions
To the Problem in the Example 1

Construct every circle that is tangent to a line $y = 2x + 1$, and has a radius of 2.

Suppose $\boldsymbol{C}$ is the circle we want, $\boldsymbol{A}$ is the line $\boldsymbol{y = 2x + 1}$, $\boldsymbol{B}$ is a line $\boldsymbol{y = 2x + c}$, where $\boldsymbol{c}$ is constant, but $\neq \mathbf{1}$, and the distance between the lines $\boldsymbol{A}$ and $\boldsymbol{B}$ is 2.

Then, the line $\boldsymbol{B}$ is parallel to the line $\boldsymbol{A}$, and any circle tangent to the line $\boldsymbol{A}$ and centered at any point in the line $\boldsymbol{B}$ has a radius of 2.

The distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

Suppose now, that $\boldsymbol{(s, t)}$ is a point in $\boldsymbol{B}$, which is $\boldsymbol{y = 2x + c}$.

Then, the center of $\boldsymbol{C}$ can be put in $\boldsymbol{(s, 2s + c)}$.

Thus, the circle $\boldsymbol{C}$ can be put for now, in such a way as follows.

$(x-s)^2 + \{y-(2s+c)\}^2 = 4$, where $\boldsymbol{s}$ and $\boldsymbol{c}$ are constant.

Since the line $\boldsymbol{A}$ is $\boldsymbol{y = 2x + 1}$, $\boldsymbol{A}$ can be indicated by $\boldsymbol{2x - y + 1 = 0}$, too.

So the distance from the center to $\boldsymbol{A}$ is as follows. $\frac{|2s+(-1)(2s+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2s-2s-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

The distance is the radius of the circle $\boldsymbol{C}$, and the radius is 2.

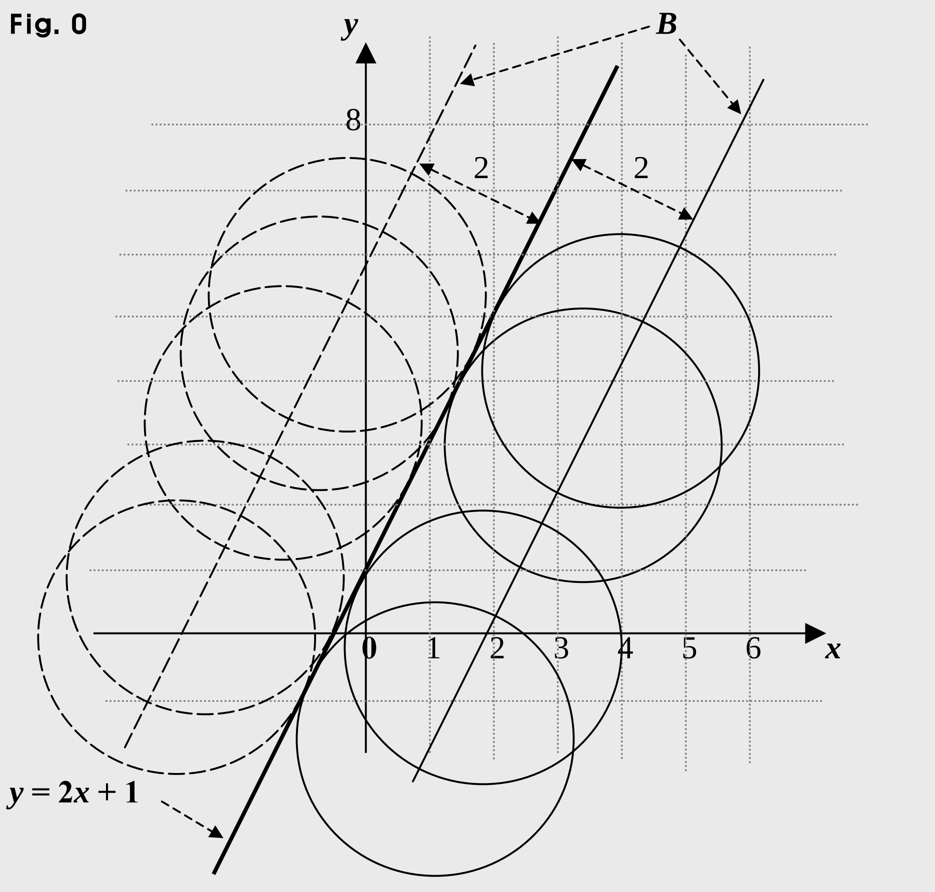
So we get $\frac{|1-c|}{\sqrt{5}} = 2 \Rightarrow \frac{(1-c)^2}{5} = 4 \Rightarrow (1-c)^2 = 20 \Rightarrow 1-c = \pm 2\sqrt{5} \Rightarrow c = 1 \pm 2\sqrt{5}$.

Therefore, the circle $\boldsymbol{C}$ is $(x-s)^2 + (y-2s-1 \pm 2\sqrt{5})^2 = 4$, where $\boldsymbol{s}$ is constant.

If not quite sure of the idea behind the processes above, follow the steps below.

Putting the problem in a graph, that is, putting in a graph the line given and some circles probable, which are of radius 2, we can see better whereabouts the solution can be.

Fig. 0



Now, finding a circle, we need the center as well as the radius. The radius is 2, but the center is not specified. The information on the center is implicit, that is, hidden.
So we may want to begin with the center's whereabouts.   How then, do we get there?

The center in this case has much to do with the radius and the line tangent to the circle.

Suppose ***A*** is the line $\boldsymbol{y = 2x + 1}$, and ***B*** is a line $\boldsymbol{y = 2x + c}$, where ***c*** is constant, but $\neq \mathbf{1}$, and ***C*** is the circle this problem is asking.

Then, the line ***B*** is parallel to the line ***A***. Thus, if a circle is tangent to the line ***A*** and centered at a point in the line ***B***, the distance between the lines ***A*** and ***B*** is the radius.

Thus, every circle tangent to A and centered at a point in B has the same radius.

So if the distance between A and B is 2, we can see a lot of Cs. That is, every circle centered at a point in the line B and tangent to the line A can be the circle C.

So the center of C is everywhere in the line B, and thus, infinitely many circles can be the circle C. Each radius is 2, but the center is everywhere in the line B. What do we do about such centers, then?

We can come up with an equation that can represent all circles where the radii are 2, and the centers are in the line B.

Thus, we want to find first, the line B, which is $y = 2x + c$ where c is constant, but $\neq 1$.

So we want to find the value of c so that the distance between A and B is 2.

Then, no matter what point we may choose in the line B, the point will be the center of a circle of radius 2, which is tangent to the line A.

So we want to get an equation of a circle tangent to A and centered at an arbitrary point in the line B.

Then, the equation represents all circles that can be the circle C, and thus, is the solution to this problem.

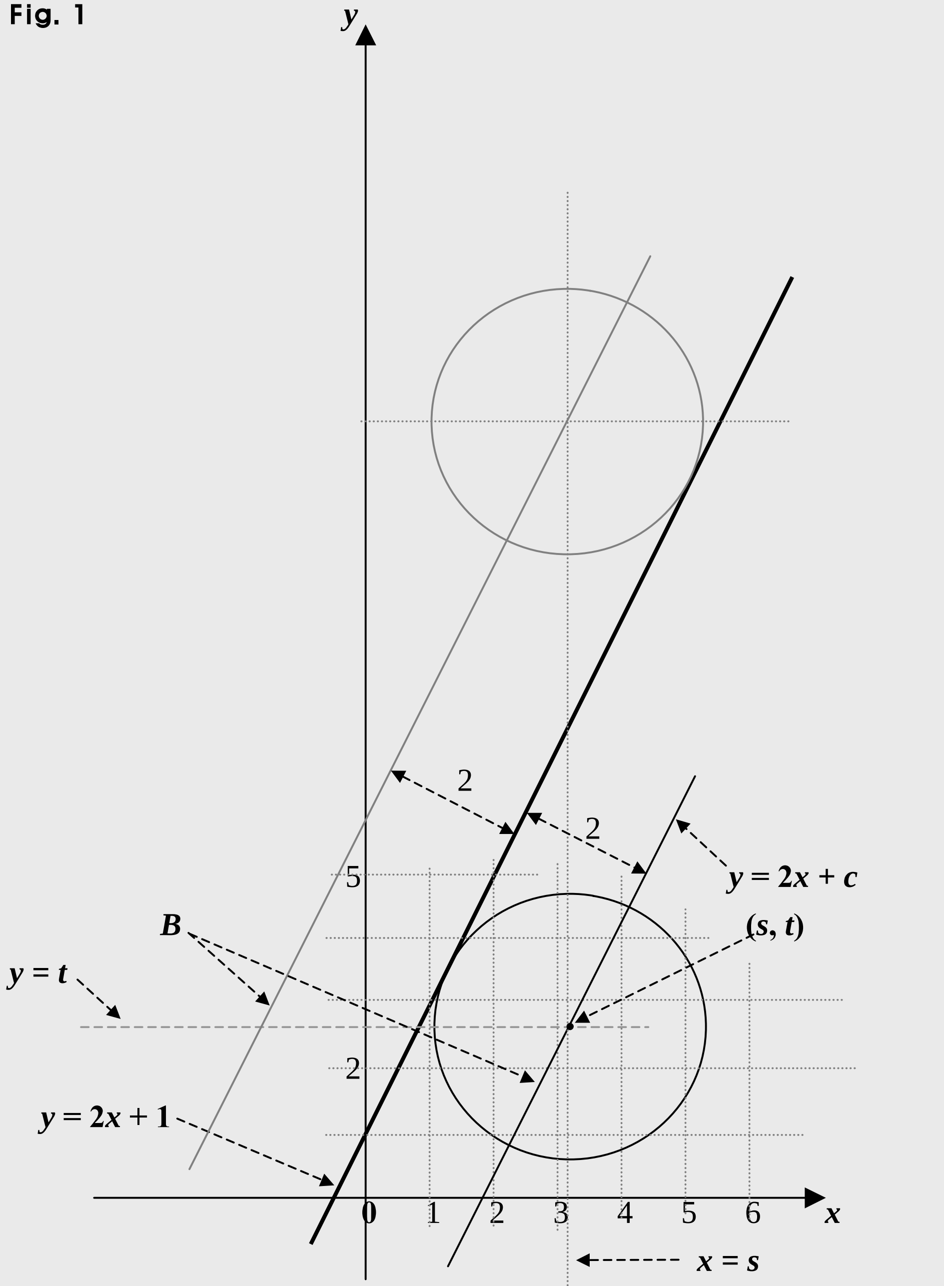
So for instance, in the equation, which is in the standard form, the center will be specified by (s, t), which is an arbitrary point in the line B.

Then, specifying only the x-coordinate, that is s, of a point in B, we can get a circle centered at the point and tangent to A.

That is, we have only to specify the x-coordinate at the center, and the y-coordinate will be determined, and then, we will get the circle.

Note however, that two lines can be the line B. One is above the line A, and below is the other. So we can get two circles at a time. One is above the line A, and below is the other.

Fig. 1



Suppose now, that a point ***(s, t)*** is in the line ***B***.
Suppose also, that the point ***(s, t)*** is the center of a circle tangent to the line ***A***.

Then, ***s*** is the ***x***-coordinate of the center, so the center is in the line ***x = s*** as shown above.
Of course, two centers are in the line since we can get two circles at a time.

Now, we are ready to find the value of $\boldsymbol{c}$.

To begin with, since $\mathbf{(}\boldsymbol{s}\mathbf{,}\ \boldsymbol{t}\mathbf{)}$ is in the line $\boldsymbol{B}$, which is $\boldsymbol{y} = \mathbf{2}\boldsymbol{x} + \boldsymbol{c}$, we get $\boldsymbol{t} = \mathbf{2}\boldsymbol{s} + \boldsymbol{c}$.
So the center can be put this way: $(\boldsymbol{s}, \mathbf{2}\boldsymbol{s} + \boldsymbol{c})$.

Next, the distance between the lines $\boldsymbol{A}$ and $\boldsymbol{B}$ is the distance from the center $(\boldsymbol{s}, \mathbf{2}\boldsymbol{s} + \boldsymbol{c})$ to the line $\boldsymbol{A}$, and is the very radius, which is 2.
Besides, we have a formula for a distance from a point to a line in a plane.

- The distance from a point $(\boldsymbol{u}, \boldsymbol{v})$ to a line $\boldsymbol{px} + \boldsymbol{qy} + \boldsymbol{r} = \mathbf{0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

Since the line $\boldsymbol{A}$ is $\boldsymbol{y} = \mathbf{2}\boldsymbol{x} + \mathbf{1}$, $\boldsymbol{A}$ can be indicated by $\mathbf{2}\boldsymbol{x} - \boldsymbol{y} + \mathbf{1} = \mathbf{0}$, too.
The center is $(\boldsymbol{s}, \mathbf{2}\boldsymbol{s} + \boldsymbol{c})$, and the distance from the center to the line $\boldsymbol{A}$ is the radius.

So the radius can be put in such a way as follows. $\frac{|2s+(-1)(2s+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2s-2s-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

Thus, we get $\frac{|1-c|}{\sqrt{5}} = \mathbf{2}$ since the radius is 2.

So we get $\frac{|1-c|}{\sqrt{5}} = 2 \Rightarrow \frac{(1-c)^2}{5} = 4 \Rightarrow (1-c)^2 = 20 \Rightarrow 1-c = \pm 2\sqrt{5} \Rightarrow c = 1 \pm 2\sqrt{5}$.

Then, first, if $c = 1 + 2\sqrt{5} \approx 5.472$, we get
$(x-s)^2 + \{y-(2s+c)\}^2 = (x-s)^2 + (y-2s-1-2\sqrt{5})^2 = 4$.

Next, if $c = 1 - 2\sqrt{5} \approx -3.472$, we get
$(x-s)^2 + \{y-(2s+c)\}^2 = (x-s)^2 + (y-2s-1+2\sqrt{5})^2 = 4$.

Therefore, the equation for circles we are after is as follows.
$(x-s)^2 + (y-2s-1-2\sqrt{5})^2 = 4$ or $(x-s)^2 + (y-2s-1+2\sqrt{5})^2 = 4$, where $\boldsymbol{s}$ is a constant.

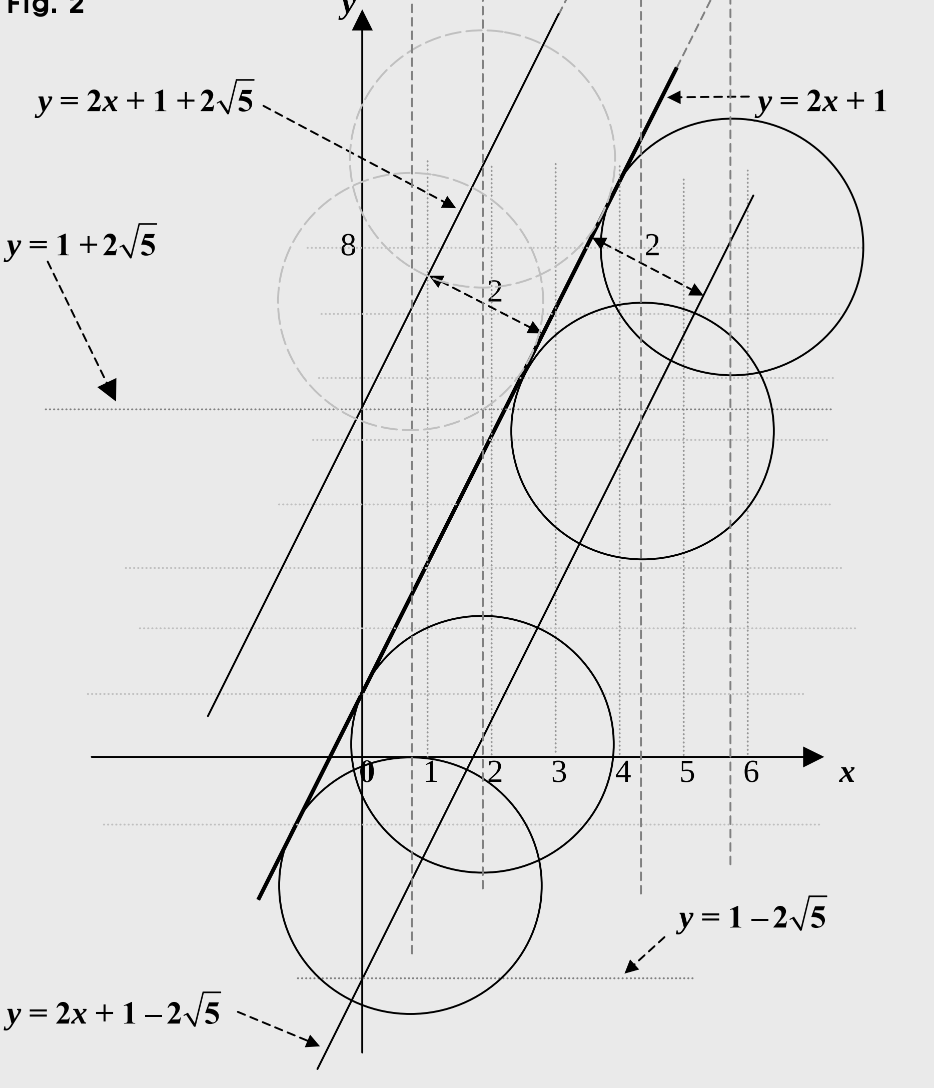
We can put together the two above the way below, too.
$(x-s)^2 + (y-2s-1 \pm 2\sqrt{5})^2 = 4$ where $\boldsymbol{s}$ is a constant.

Now, the equation above indicates every circle of radius 2 that is tangent to the line $\boldsymbol{A}$.
All the centers are all the points in the line $\boldsymbol{B}$.

So if we choose a point in the line $\boldsymbol{B}$, a circle centered at the point and tangent to the line $\boldsymbol{A}$ gets determined. Choosing the center, we have only to specify the $\boldsymbol{x}$-coordinate of a point in the line $\boldsymbol{B}$. In fact, for each choice of the $\boldsymbol{x}$-coordinate, which is $\boldsymbol{s}$, we can get two circles of radius 2, each of which is tangent to the given line $\boldsymbol{A}$, and is centered at each of the two points, $(s, 2s+1+2\sqrt{5})$ and $(s, 2s+1-2\sqrt{5})$.

By means of the equation above, we can determine twice as many circles as the number of the values of the constant $\boldsymbol{s}$, which is the $\boldsymbol{x}$-coordinate of the center, which is a point in either of the two lines $y = 2x+1+2\sqrt{5}$ and $y = 2x+1-2\sqrt{5}$, each of which can be the line $\boldsymbol{B}$, of course.

Fig. 2



Suggestions or Solutions
To the Problem in the Example 2

Find all the circles that have the same radii, and are tangent to a line $y = 2x + 1$.

Suppose ***A*** is the line $\boldsymbol{y = 2x + 1}$, ***B*** is a line $\boldsymbol{y = 2x + c}$ where ***c*** is constant and $\boldsymbol{c \neq 1}$, ***r*** is the distance between the two lines ***A*** and ***B***, and ***C*** is the circle to be constructed.

Suppose also, ***s*** and ***t*** are constant, and a point $\boldsymbol{(s, t)}$ is in ***B***, and is the center of a circle tangent to ***A***.

Then, we get $\boldsymbol{(s, t) = (s, 2s + c)}$, which is the center of ***C***, and ***r*** is the radius of ***C***.

The distance from a point $\boldsymbol{(u, v)}$ to a line $\boldsymbol{px + qy + r = 0}$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

So the distance from $\boldsymbol{(s, 2s + c)}$ to ***A*** is $\frac{|2s+(-1)(2s+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2s-2s-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

Thus, we get $\frac{|1-c|}{\sqrt{5}} = r \Rightarrow \frac{(1-c)^2}{5} = r^2 \Rightarrow (1-c)^2 = 5r^2 \Rightarrow 1-c = \pm r\sqrt{5} \Rightarrow c = 1 \pm r\sqrt{5}$.

Therefore, the circle ***C*** is $(x-s)^2 + (y-2s-1 \pm r\sqrt{5})^2 = r^2$ where ***r*** and ***s*** are constant.

If not quite sure of the idea behind the processes above, follow the steps below.

Finding a circle, we need the center and radius. This time however, both the radius and center are not specified. So they are probably hidden. Where then, can they possibly be?

They must be somewhere in the problem description.   So let's break apart the problem.

To begin with, many, or rather infinitely many circles can be tangent to a line. So we are after not one but infinitely many circles. Thus, we can expect the radius and center to be expressed in terms of some constants.

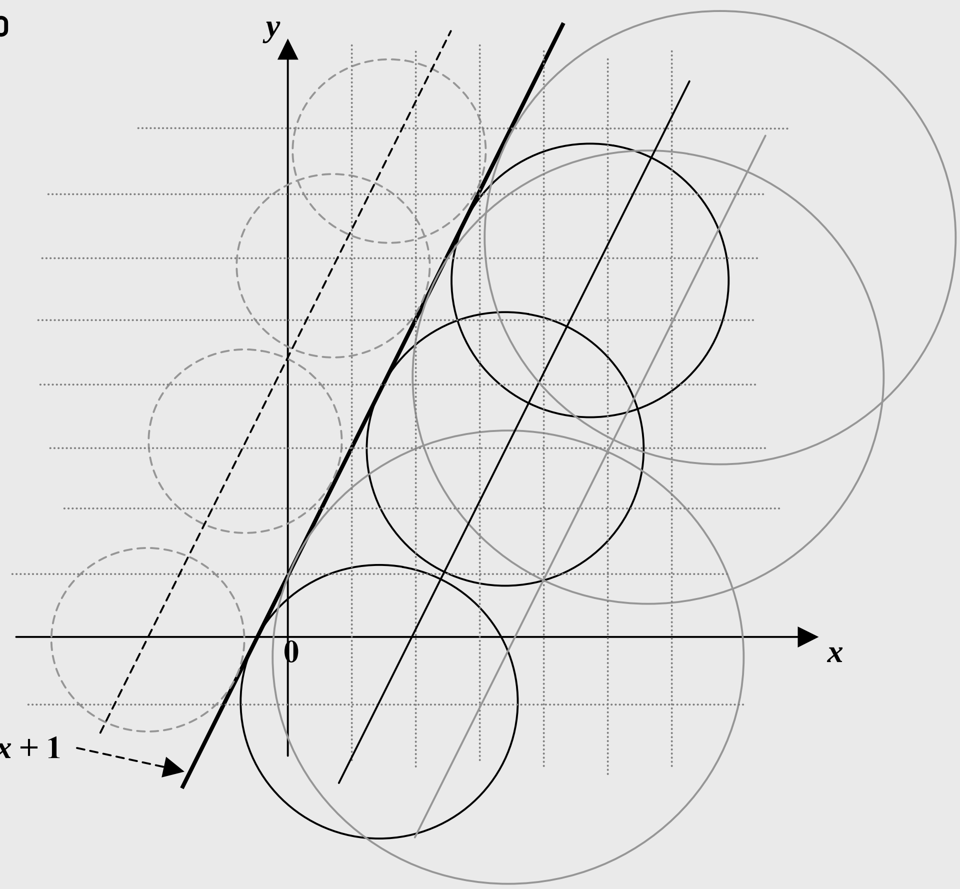
Next, no matter what the radii may be, they all have to be the same.

The information on the centers has to be given somehow, too.

So this time, too, we may want to begin with the centers' whereabouts.

So putting in a graph the line given and some probable circles we can get

Fig. 0



The center in a circle has much to do with the radius and a line tangent to the circle.

So suppose ***A*** is the line $\boldsymbol{y = 2x + 1}$, which the tangent line. Then, every circle tangent to the line ***A*** and centered at a point in a line parallel to the line ***A*** has the same radius.

That is, the line parallel to the line ***A*** is a particular distance away from the line ***A***, and the particular distance is the same radius.

So suppose now, ***B*** is a line parallel to the line ***A***.

In fact, not just one but infinitely many lines can be parallel to the line ***A***.
Anyway, we can put the line ***B*** this way: $\boldsymbol{y = 2x + c}$, where ***c*** is a constant, which is $\neq \mathbf{1}$.

For each value of ***c***, the equation $\boldsymbol{y = 2x + c}$ indicates a line parallel to the line ***A***.

Thus, the line ***B*** can represent all lines parallel to the line ***A***, and those circles that have their centers in the line ***B*** and are tangent to the line ***A*** are the solution to this problem.

In this problem however, the solution is not a line or lines as ***B*** but all circles of the same radii tangent to the line ***A***. So we want to assume that ***B*** is an arbitrary line parallel to ***A***.

Let's now, find the equation that can represent all those circles.

To begin with, we have the fact as follows.

• If we choose a point in the line ***B***, and the point is the center of a circle tangent to the line ***A***, the distance between the two lines ***A*** and ***B*** is the radius of the circle.

Then in fact, we can have two lines, each of which can be the line ***B***, because each of the two can be the same distance away from the line ***A*** as shown in the figure below.

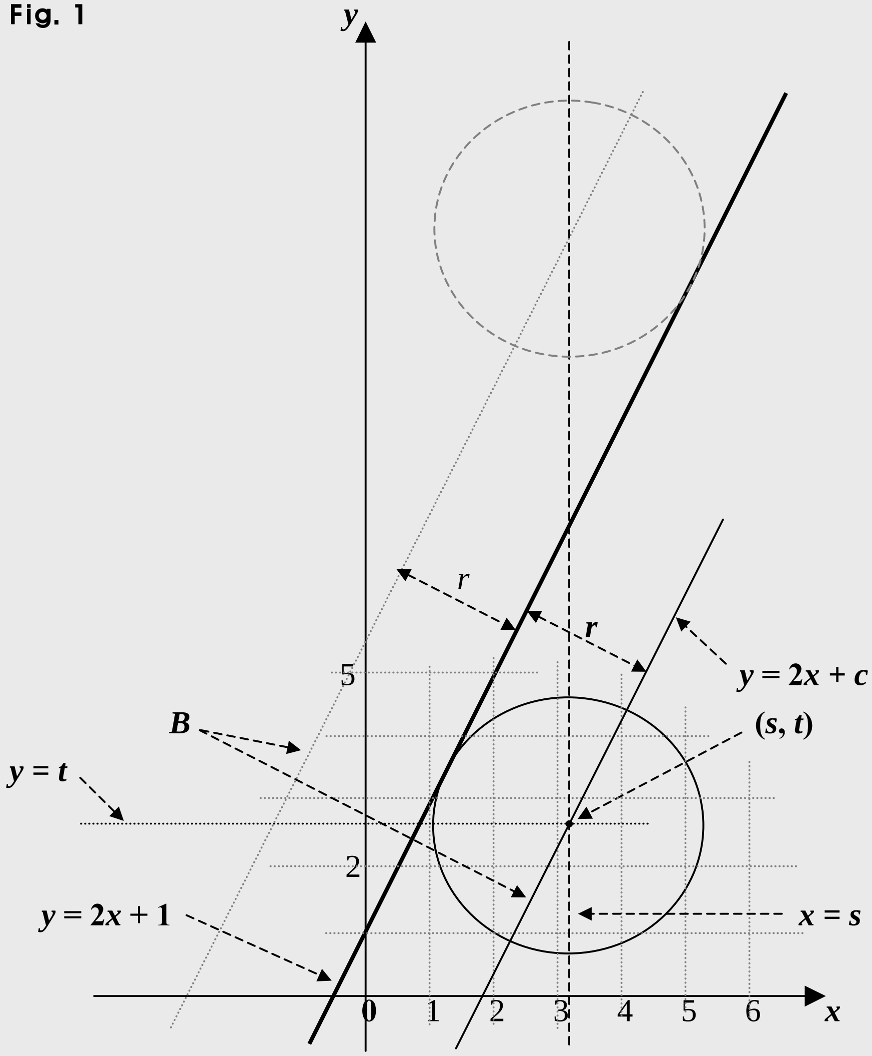
• So we can construct the solution the way as follows.

To begin with, we specify the value of the radius. We can set it to be ***r***, for instance.

Then, we will get two values of ***c***, because we can have two lines, each of which can be the line ***B*** since two lines can be the same distance away from the line ***A***. One is above the line ***A***, and the other is below the line ***A***.

Next, specifying only the ***x***-coordinate of a point in the line ***B***, we can get two circles tangent to the line ***A***. One is above the line ***A***, and the other is below the line ***A***.

Fig. 1



Suppose now, that ***s*** and ***t*** are constant, that a point **(*s*, *t*)** is in ***B***, which is parallel to the line ***A***, and that the point **(*s*, *t*)** is the center of a circle of radius ***r***, which is tangent to ***A***.

Then, since **(*s*, *t*)** is in the line ***B***, which is ***y* = 2*x* + *c***, we get ***t* = 2*s* + *c***.

So the center can be put in **(*s*, 2*s* + *c*)**, for now. And we will take care of ***c*** shortly.

The distance between the lines ***A*** and ***B*** is the distance from the center **(*s*, 2s + *c*)** to the line ***A***, so the distance is the radius ***r***. Thus, we can put ***c*** in terms of the radius ***r***.

Suppose now, $\boldsymbol{C}$ is the circle tangent to $\boldsymbol{A}$ and centered at $(\boldsymbol{s}, \boldsymbol{t})$, and let's find $\boldsymbol{c}$.
How then, can we find $\boldsymbol{c}$?

We have a formula for such a distance as above, and the formula goes as follows.

• The distance from a point (u, v) to a line $px + qy + r = 0$ is $\frac{|pu+qv+r|}{\sqrt{p^2+q^2}}$.

The line $\boldsymbol{A}$ is $y = 2x + 1$, equivalent to $2x - y + 1 = 0$. And the center is $(s, 2s + c)$.

So the distance, that is, the radius is as follows. $\frac{|2s+(-1)(2s+c)+1|}{\sqrt{2^2+(-1)^2}} = \frac{|2s-2s-c+1|}{\sqrt{4+1}} = \frac{|1-c|}{\sqrt{5}}$.

Therefore, the circle $\boldsymbol{C}$ can be put for now, this way: $(x - s)^2 + \{y - (2s + c)\}^2 = \frac{(1-c)^2}{5}$.

So the center of the circle $\boldsymbol{C}$ at $(s, 2s + c)$, and the radius is $\frac{|1-c|}{\sqrt{5}}$. And we know $\boldsymbol{r}$ is the radius of the circle $\boldsymbol{C}$. So we get $\frac{|1-c|}{\sqrt{5}} = r$.

Thus, we get $\frac{|1-c|}{\sqrt{5}} = r \Rightarrow \frac{(1-c)^2}{5} = r^2 \Rightarrow (1-c)^2 = 5r^2 \Rightarrow 1-c = \pm r\sqrt{5} \Rightarrow c = 1 \pm r\sqrt{5}$.

Now that we've found $\boldsymbol{c}$, let's get back to the line $\boldsymbol{B}$.

The line $\boldsymbol{B}$ is $y = 2x + c$, and we have $c = 1 \pm r\sqrt{5}$. So we get two lines, each of which is a particular distance away from the line $\boldsymbol{A}$, and the particular distance is $\boldsymbol{r}$.
One of the two is $y = 2x + 1 + r\sqrt{5}$, and the other is $y = 2x + 1 - r\sqrt{5}$.

Now, putting threads together, we have $(x - s)^2 + \{y - (2s + c)\}^2 = r^2$, and $c = 1 \pm r\sqrt{5}$.

So the circle $\boldsymbol{C}$ is $(x-s)^2 + (y-2s-1 \pm r\sqrt{5})^2 = r^2$, where $\boldsymbol{s}$ and $\boldsymbol{r}$ are constant.

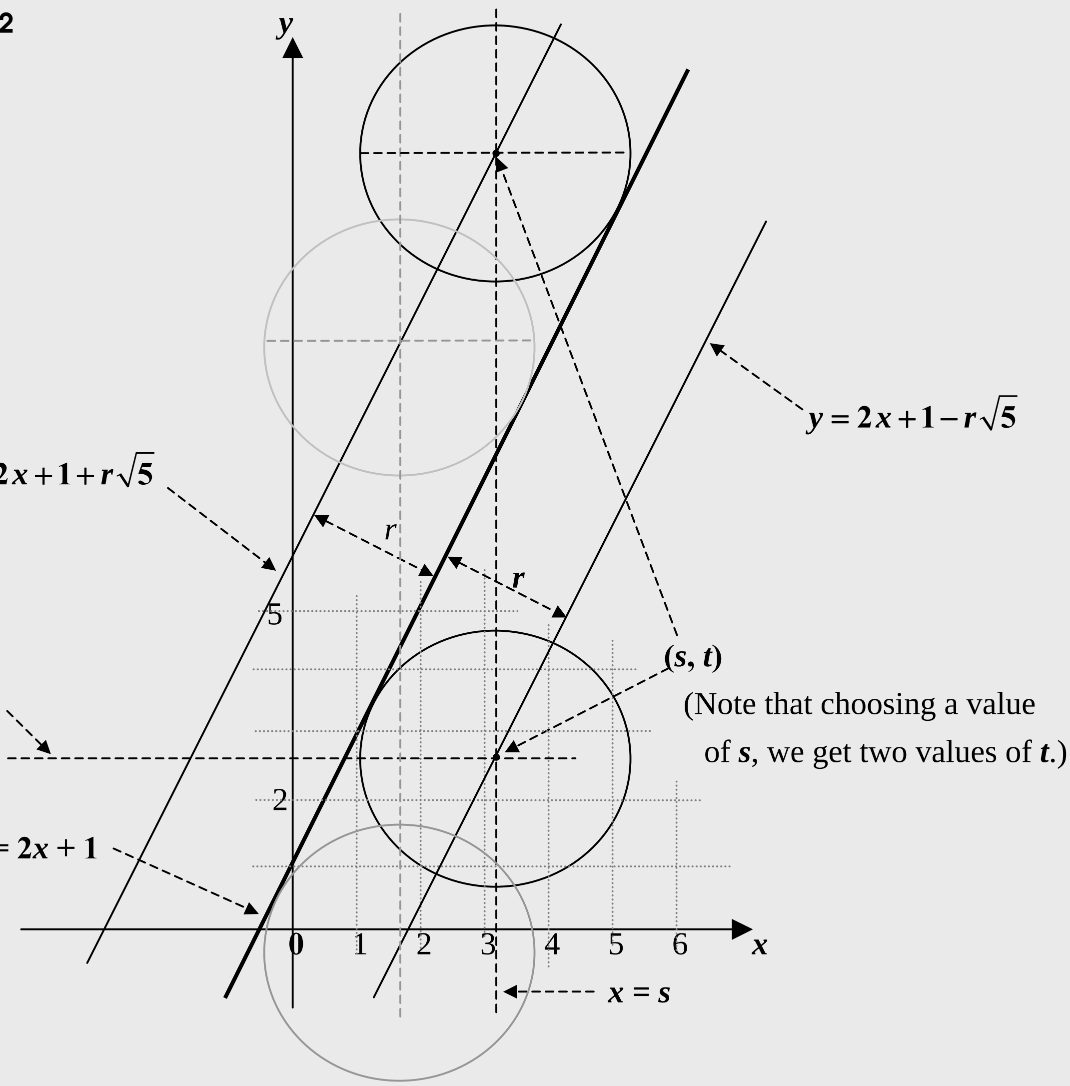
The equation above represents two circles for each value of $\boldsymbol{s}$, which is the $\boldsymbol{x}$-coordinate of the center. One circle is $(x-s)^2 + (y-2s-1+r\sqrt{5})^2 = r^2$, and the other circle is $(x-s)^2 + (y-2s-1-r\sqrt{5})^2 = r^2$.

Now, choosing a value of the radius $\boldsymbol{r}$, we get two parallel lines, each of which is $\boldsymbol{r}$ away from the line $\boldsymbol{A}$. One of the two is $y = 2x + 1 + r\sqrt{5}$, and the other is $y = 2x + 1 - r\sqrt{5}$.

Then, choosing a point in either of the two lines, we can get two circles tangent to the line $\boldsymbol{A}$. Each of the two has a radius of $\boldsymbol{r}$, of course.

So choosing a point in either line, we choose the centers of the two circles.
Choosing the centers, we have only to specify the ***x***-coordinate at one of the two centers since the two centers have the same ***x***-coordinates.

Fig. 2



Assuming for instance, $r = \sqrt{5}$, we can get two lines, each of which is $\sqrt{5}$ away from the line ***A***, $y = 2x + 1$. One of the two is $y = 2x + 6$, and the other is $y = 2x - 4$. Then, choosing a point in either of the two lines, we can get two circles tangent to the line ***A***. In each of the two, the radius is $\sqrt{5}$, of course. So choosing a point in either line, we choose the centers of the two circles. Choosing the centers, we have only to specify the ***x***-coordinate of one of the two centers since the two centers have the same ***x***-coordinates.

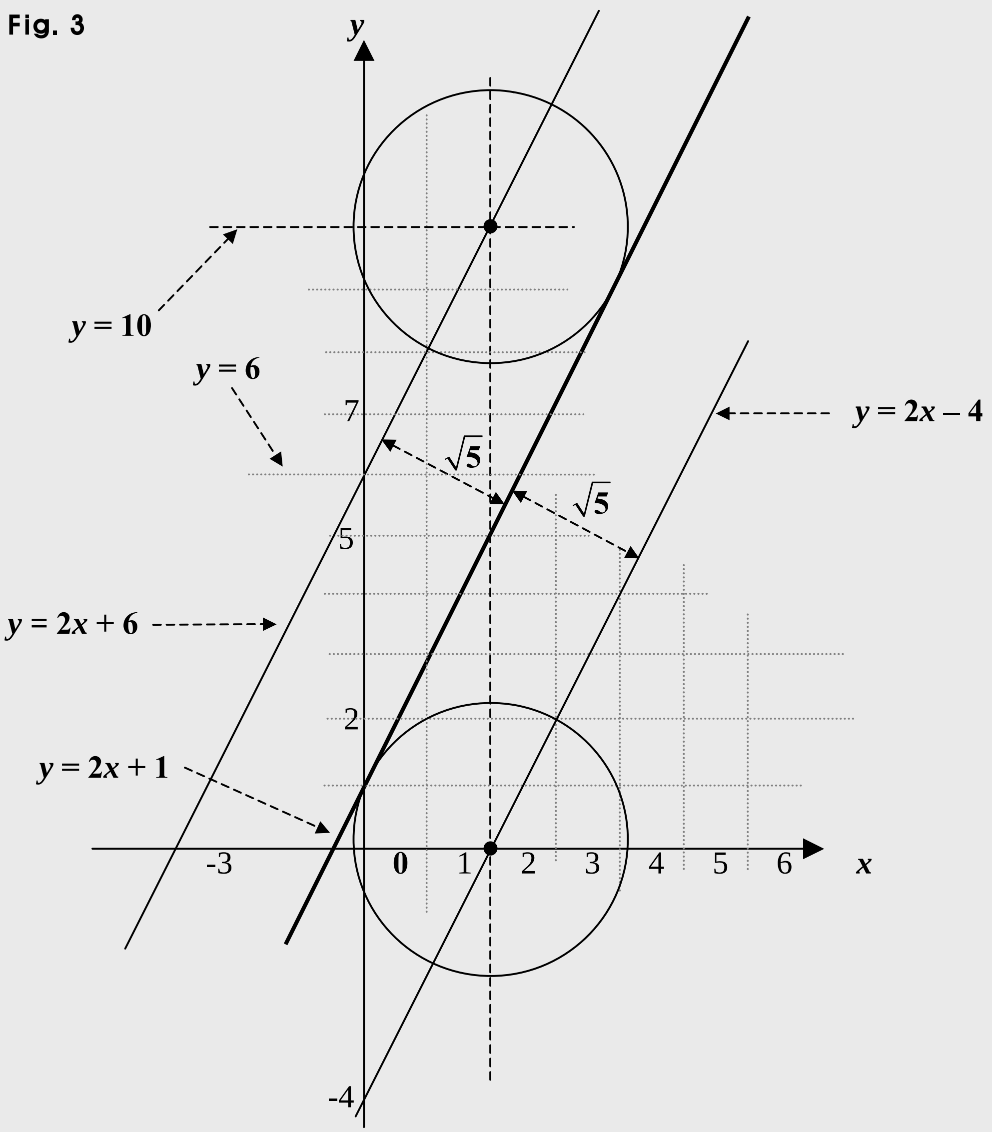
Suppose for instance, we choose 2 for the ***x***-coordinate of the center. Then, we get

$$(x-s)^2+(y-2s-1\pm r\sqrt{5})^2=r^2 \text{ where } s=2, \text{ and } r=\sqrt{5}.$$

$$\Rightarrow (x-2)^2+(y-4-1\pm 5)^2=5 \Rightarrow (x-2)^2+(y-5\pm 5)^2=5.$$

So one circle is $(x-2)^2+y^2=5$, and the other circle is $(x-2)^2+(y-10)^2=5$.

Fig. 3



Specifying another value of ***s***, we get another pair of circles, one of which is centered at a points in the line ***y* = 2*x* – 4**, and the other is centered at a point in the line ***y* = 2*x* + 6**.