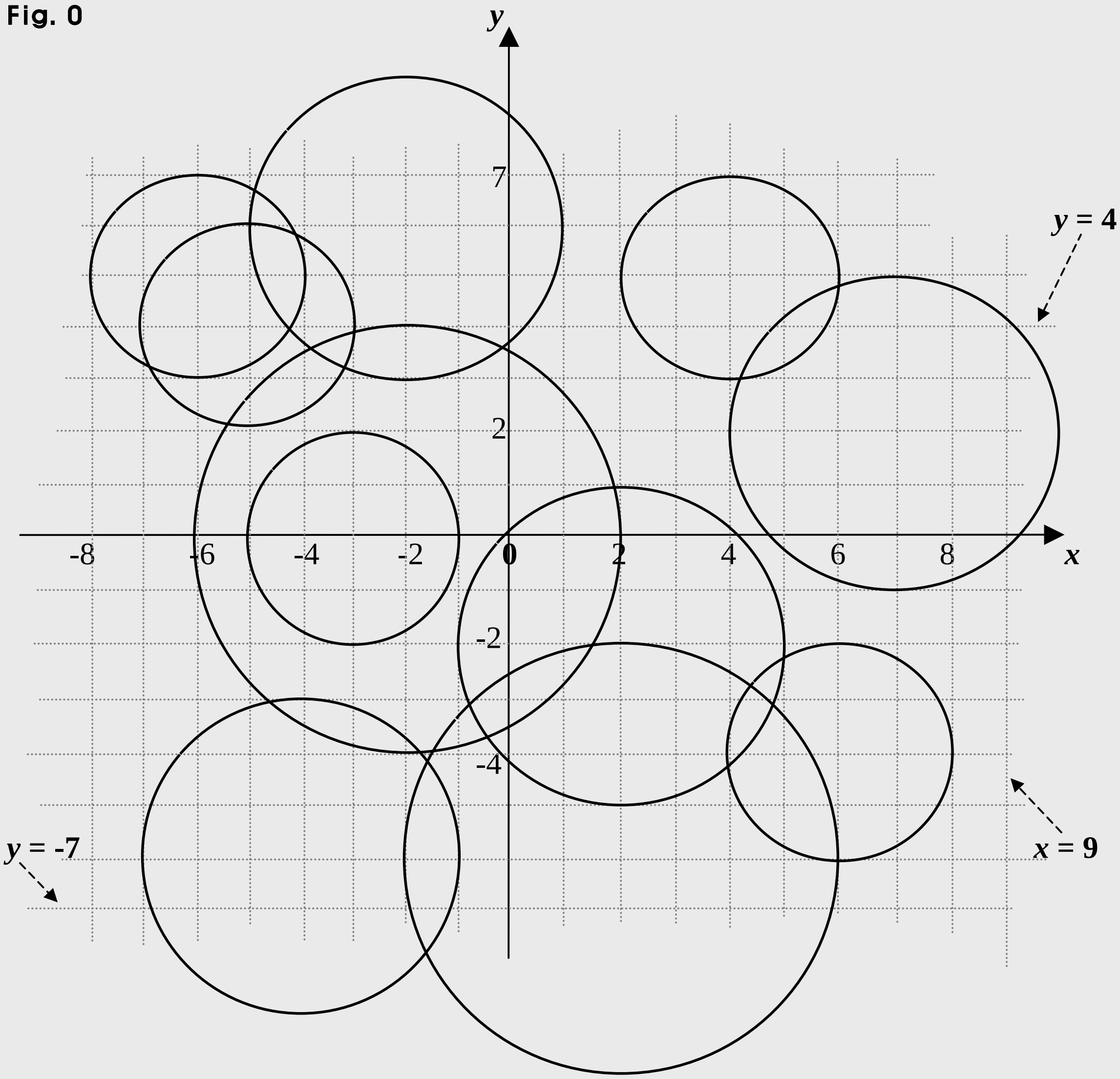
Examples 1 in Standard Forms

Click this to start.

Examples 1 in Standard Forms

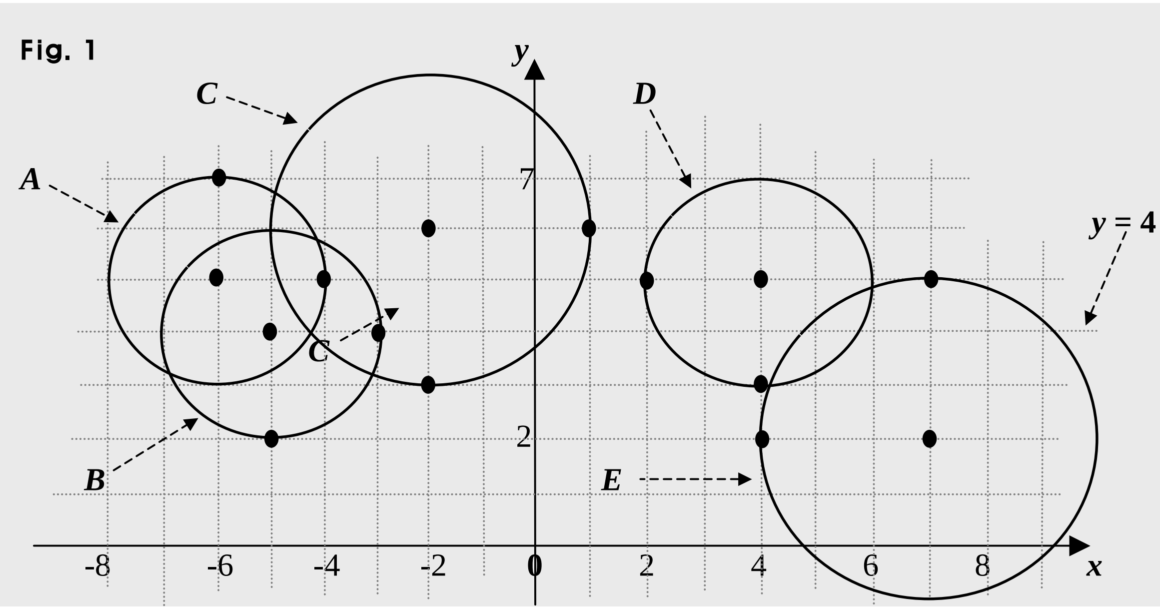
Label each circle below, and then find the equation of each of the circles.

Fig. 0



Suggestions or Solutions
To the Problems in the Examples

Beginning with the circles below, we can see first, that the circle ***A*** is centered at (-6, 5), and has a point (-6, 7) or (-4, 5).



And we can specify the center and a point each of all the other circles has.

Next, if we know the center and a point in the circle, we can get the radius, and then, can get the equation of the circle using the standard form as follows.

$\boldsymbol{(x - a)^2 + (y - b)^2 = r^2}$**,** where $\boldsymbol{(a, b)}$ is the center, and $\boldsymbol{r}$ is the radius.

And thus, beginning with the circle ***A***, we can see the circle has the center at (-6, 5) and a point at (-6, 7), so the radius is 7 – 5 = 2, and of course, we can get the radius using the distance formula the way as follows. $\boldsymbol{\{-6 - (-6)\}^2 + (5 - 7)^2 = 2^2 \Rightarrow r = 2}$.

So next, since the circle ***A*** is centered at (-6, 5), and the radius is 2, we get

$\boldsymbol{\{x - (-6)\}^2 + (y - 5)^2 = 2^2 \Rightarrow (x + 6)^2 + (y - 5)^2 = 4}$, which is the equation of the circle ***A***.

What if we use the other point (-4, 5) instead of the point (-6, 7)?

The point (-4, 5) is in the circle ***A***, too, so we will get the same equation. Since the center is (-6, 5), the radius is –4 – (-6) = –4 + 6 = 2, so we can get it the way as follows.

$\{x - (-6)\}^2 + (y - 5)^2 = 2^2 \Rightarrow (x + 6)^2 + (y - 5)^2 = 4$.

Note that in math, the same circles have the same center and the same radius.
So even if the same radius, if the center is different, it's a different circle.

Next, moving on to the circle ***B***, we can see that the center is (-5, 4) and it has a point at (-5, 2), so the radius is 4 – 2 = 2, and of course, we can get the radius using the distance formula the way as follows. $\{-5 - (-5)\}^2 + (4 - 2)^2 = 2^2 \Rightarrow r = 2$.

So next, since the circle ***B*** is centered at (-5, 4), and the radius is 2, we get
$\{x - (-5)\}^2 + (y - 4)^2 = 2^2 \Rightarrow (x + 5)^2 + (y - 4)^2 = 4$, which is the equation of the circle ***B***.

Next, looking at the circle ***C***, we can see it has the center at (-2, 6) and a point at (-2, 3), so the radius is 6 – 3 = 3, and thus, we get

$\{x - (-2)\}^2 + (y - 6)^2 = 3^2 \Rightarrow (x + 2)^2 + (y - 6)^2 = 9$, which is the equation of the circle ***C***.

Next, examining the circle ***D***, we can see it is centered at (4, 5) and has a point at (2, 5), so the radius is 4 – 2 = 2, and thus, we get

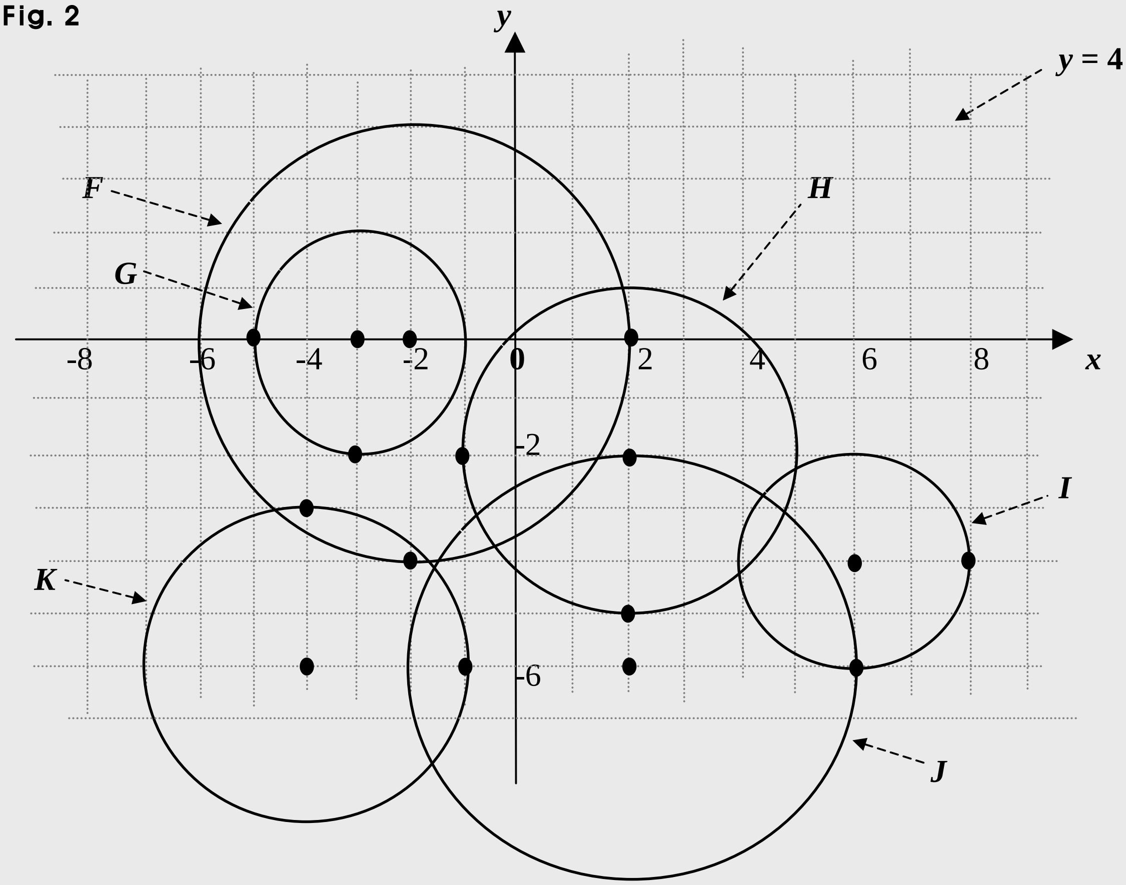
$(x - 4)^2 + (y - 5)^2 = 2^2$, which is the equation of the circle ***D***.

Next, examining the circle ***E***, we can see it is centered at (7, 2) and has a point at (4, 2), so the radius is 7 – 4 = 3, and thus, we get

$(x - 7)^2 + (y - 2)^2 = 3^2$, which is the equation of the circle ***E***.

Next, moving on to the circles below, we can see that the circle ***F*** is centered at (-5, 6), and a point at (-7, 4) or (-3, 4).

Fig. 2



And knowing the center and a point in the circle, we can get the radius, and then, can get the equation of the circle using the standard form as follows.

$(x - a)^2 + (y - b)^2 = r^2$, where (a, b) is the center, and r is the radius.

And thus, beginning with the circle ***F***, we can see the circle has the center at (-2, 0) and a point at (2, 0), so the radius is 2 – (-2) = 4.

So next, since the circle ***F*** is centered at (-2, 0), and the radius is 2, we get

$\{x - (-2)\}^2 + (y - 0)^2 = 4^2 \Rightarrow (x + 2)^2 + y^2 = 16$, which is the equation of the circle ***F***.

Next, looking at the circle ***G***, we can see it has the center at (-3, 0) and a point at (-5, 0), so the radius is –3 – (–5) = –3 + 5 = 2, and thus, we get

$\{x - (-3)\}^2 + (y - 0)^2 = 2^2 \Rightarrow (x + 3)^2 + y^2 = 4$, which is the equation of the circle ***G***.

Next, examining the circle ***H***, we can see it is centered at (2, -2) and has a point at (2, -5), the radius is |-5 – (-2)| = 3, so we get

$\mathbf{(x - 2)^2 + \{y - (-2)\}^2 = 2^2 \Rightarrow (x - 2)^2 + (y + 2)^2 = 4}$, which is the equation of the circle ***H***.

Next, examining the circle ***I***, we can see it is centered at (6, -4) and has a point at (8, -4), so the radius is |6 – 8| = 2, and thus, we get

$\mathbf{(x - 6)^2 + \{y - (-4)\}^2 = 2^2 \Rightarrow (x - 6)^2 + (y + 4)^2 = 4}$, which is the equation of the circle ***I***.

Next, examining the circle ***J***, we can see it is centered at (2, -6) and has a point at (2, -2), so the radius is |-6 – (-2)| = 4, and thus, we get

$\mathbf{(x - 2)^2 + \{y - (-6)\}^2 = 4^2 \Rightarrow (x - 2)^2 + (y + 6)^2 = 16}$, which is the equation of the circle ***J***.

Next, examining the circle ***K***, since it is centered at (-4, -6) and has a point at (-1, -6), the radius is |-4 – (-1)| = 3, so we get

$\mathbf{\{x - (-4)\}^2 + \{y - (-6)\}^2 = 3^2 \Rightarrow (x + 4)^2 + (y + 6)^2 = 9}$, which is the equation of ***K***.