Examples 2 in Standard Forms

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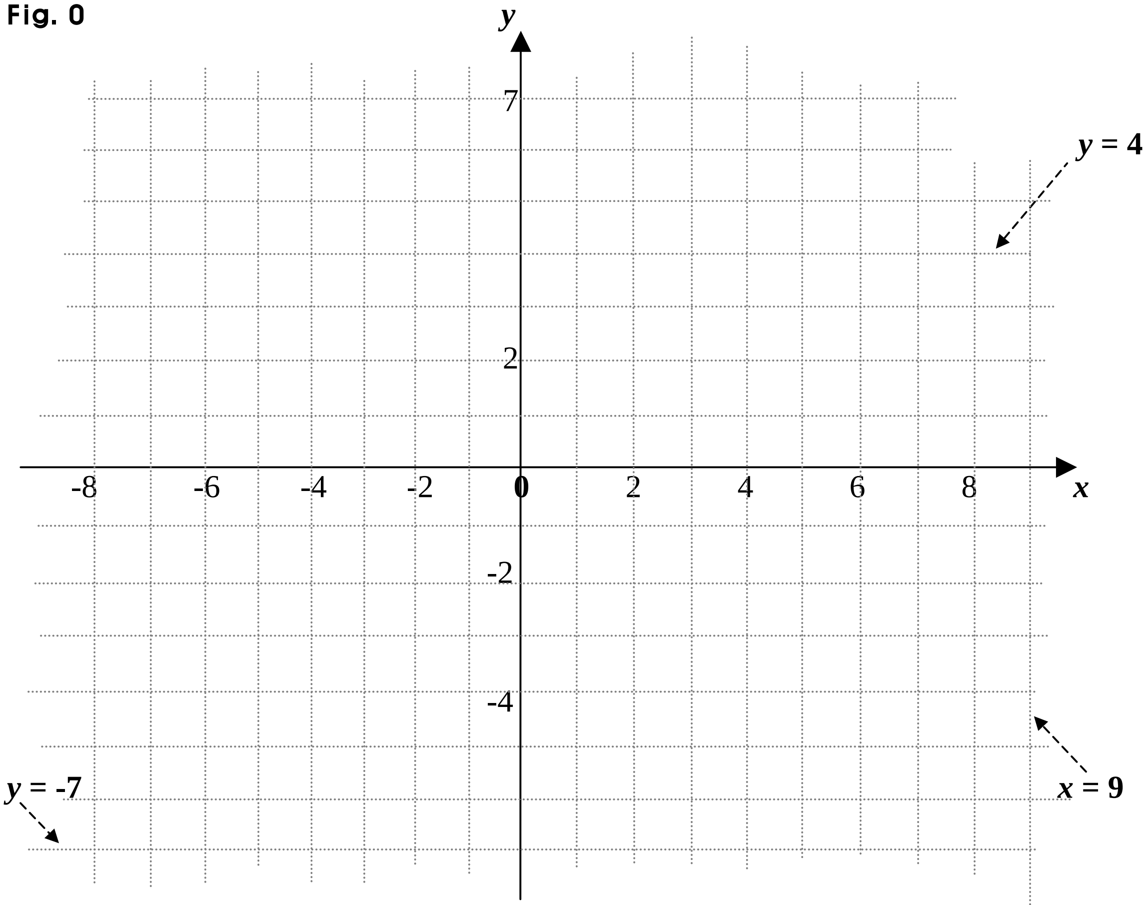
0. Specify the center and the radius of each circle below, and then, graph it.

0. $(x + 5)^2 + (y - 6)^2 = 4$ 1. $x^2 + (y - 2)^2 = 9$ 2. $(x - 5)^2 + (y - 3)^2 = 16$
3. $(x + 4)^2 + (y - 5)^2 = 12$ 4. $(x + 4)^2 + y^2 = 7$ 5. $(x + 4)^2 + (y + 3)^2 = 15$
6. $(x - \frac{9}{2})^2 + (y + 3)^2 = \frac{49}{4}$

Fig. 0



1. Find the center and the radius of each circle below.

0. $x^2 + 2x + y^2 + 2y + 1 = 0$
1. $x^2 - 3x + y^2 - 5y + 1 = 0$
2. $2x - x^2 + 3y - y^2 + 1 = 0$
3. $x^2 - 7x + y^2 + 2y + 1 = 0$

Suggestions or Solutions
To the Problems in the Example 0

Specify the center and the radius of each circle below, and then, graph it.

0. $\mathbf{(x + 5)^2 + (y - 6)^2 = 4}$ 1. $\mathbf{x^2 + (y - 2)^2 = 9}$ 2. $\mathbf{(x - 5)^2 + (y - 3)^2 = 16}$

3. $\mathbf{(x + 4)^2 + (y - 5)^2 = 12}$ 4. $\mathbf{(x + 4)^2 + y^2 = 7}$ 5. $\mathbf{(x + 4)^2 + (y + 3)^2 = 15}$

6. $\mathbf{(x - \frac{9}{2})^2 + (y + 3)^2 = \frac{49}{4}}$

First, if we know the center and a point in the circle, we can get the radius, and then, can get the equation of the circle using the standard form as follows.

$\mathbf{(x - a)^2 + (y - b)^2 = r^2}$**,** where $\mathbf{(a, b)}$ is the center, and $\mathbf{r}$ is the radius.

So to begin with, examining the circle $\mathbf{(x + 5)^2 + (y - 6)^2 = 4}$, since $\mathbf{4 = 2^2}$, we can see that the center is at (-6, 5), and the radius is 2.

Next, we can put the circle $\mathbf{x^2 + (y - 2)^2 = 9}$ this way: $\mathbf{(x - 0)^2 + (y - 2)^2 = 3^2}$, so we can see that the center is at (0, 2), and the radius is 3.

Next, examining the circle $\mathbf{(x - 5)^2 + (y - 3)^2 = 16}$, since $\mathbf{16 = 4^2}$, we can see that the center is at (5, 3), and the radius is 4.

Next, moving on to $\mathbf{(x + 4)^2 + (y - 5)^2 = 12}$, we can put it the way below.
$\mathbf{\{x - (-4)\}^2 + (y - 5)^2 = (\sqrt{12})^2 = (2\sqrt{3})^2}$. So the center is at (-4, 5), and the radius is $\mathbf{2\sqrt{3}}$.

Next, moving on to $\mathbf{(x + 4)^2 + y^2 = 7}$, we can put it this way: $\mathbf{\{x - (-4)\}^2 + (y - 0)^2 = 7}$. So the center is at (-4, 0), and the radius is a square root of 7.

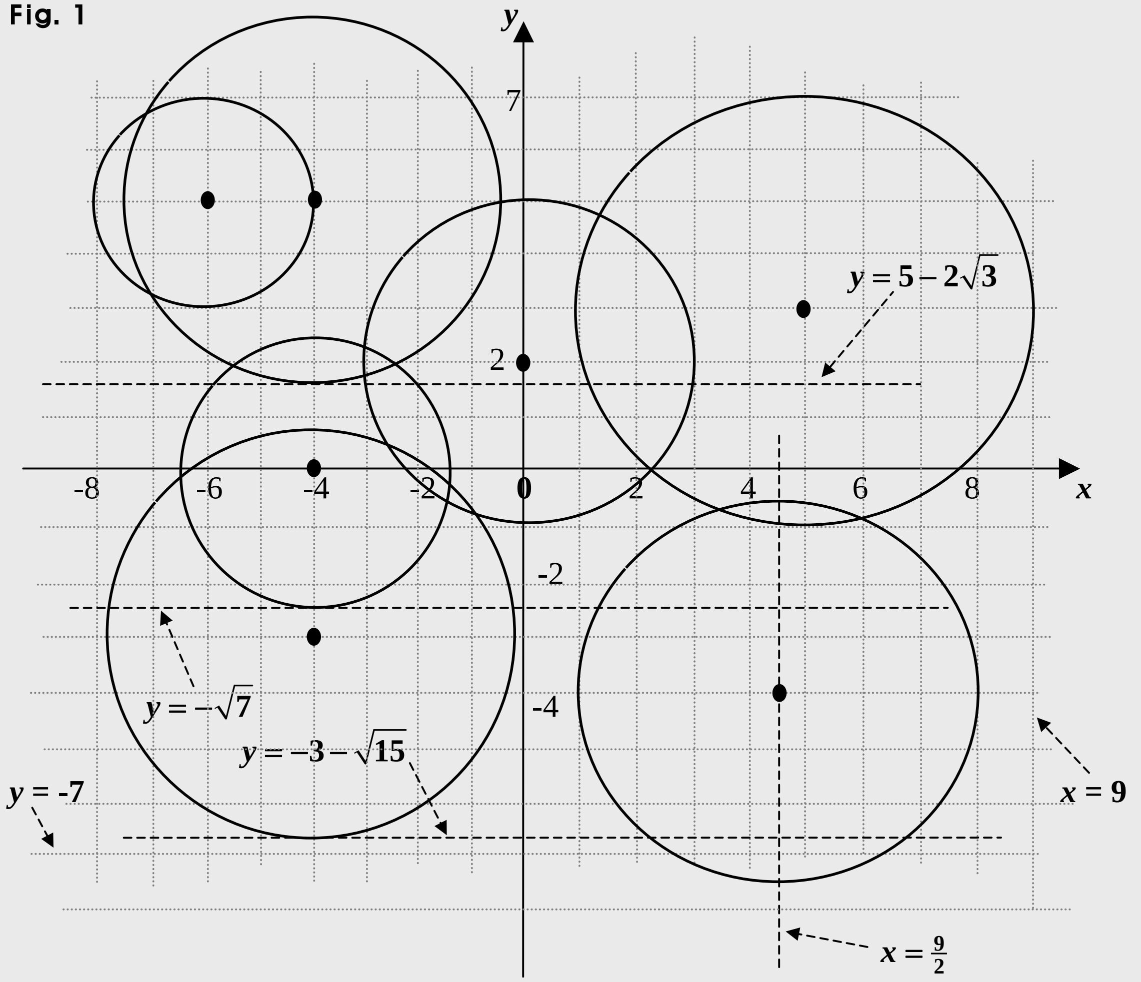
Moving next, on to $\mathbf{(x + 4)^2 + (y + 3)^2 = 15}$, we can see that the center is at (-4, -3), and the radius is a square root of 15.

Next, moving on to $(x - \frac{9}{2})^2 + (y + 4)^2 = \frac{49}{4}$, we can put it the way below.

$(x - \frac{9}{2})^2 + \{y - (-4)\}^2 = \frac{49}{4} = (\frac{7}{2})^2$. So the center is at ($\frac{9}{2}$, -4), and the radius is $\frac{7}{2}$.

And graphing the equations, we get

Fig. 1



Suggestions or Solutions
To the Problems in the Example 1

0. $\mathbf{x^2 + 2x + y^2 + 2y + 1 = 0 \Rightarrow x^2 + 2x + 1 - 1 + y^2 + 2y + 1 = 0}$

$\mathbf{\Rightarrow (x + 1)^2 + (y + 1)^2 - 1 = 0 \Rightarrow (x + 1)^2 + (y + 1)^2 = 1}$.

So the center is (-1, -1) and the radius is 1.

1. $\mathbf{x^2 - 3x + y^2 - 5y + 1 = 0 \Rightarrow \; x^2 - 3x + (\frac{3}{2})^2 - (\frac{3}{2})^2 + y^2 - 5y + (\frac{5}{2})^2 - (\frac{5}{2})^2 + 1}$

$\mathbf{\Rightarrow \; (x - \frac{3}{2})^2 + (y - \frac{5}{2})^2 - \frac{9}{4} - \frac{25}{4} + 1 = (x - \frac{3}{2})^2 + (y - \frac{5}{2})^2 - \frac{15}{2} = 0}$

$\mathbf{\Rightarrow \; (x - \frac{3}{2})^2 + (y - \frac{5}{2})^2 = \frac{15}{2} = (\sqrt{\frac{15}{2}})^2}$. So the center is $\mathbf{(\frac{3}{2}, \frac{5}{2})}$, and the radius is $\sqrt{\frac{15}{2}}$.

2. $\mathbf{2x - x^2 + 3y - y^2 + 1 = 0 \Rightarrow x^2 - 2x + y^2 - 3y - 1 = 0}$

$\mathbf{\Rightarrow x^2 - 2x + 1 - 1 + y^2 - 3y + (\frac{3}{2})^2 - (\frac{3}{2})^2 - 1 = 0 \; \Rightarrow (x - 1)^2 + (y - \frac{3}{2})^2 - \frac{9}{4} - 1 = 0}$

$\mathbf{\Rightarrow (x - 1)^2 + (y - \frac{3}{2})^2 - \frac{9}{4} - 1 = 0 \; \Rightarrow (x - 1)^2 + (y - \frac{3}{2})^2 = \frac{13}{4} = (\frac{\sqrt{13}}{2})^2}$.

So the center is $\mathbf{(1, \frac{3}{2})}$, and the radius is $\frac{\sqrt{13}}{2}$.

3. $\mathbf{x^2 - 7x + y^2 + 2y + 1 = 0 \Rightarrow x^2 - 7x + (\frac{7}{2})^2 - (\frac{7}{2})^2 + (y + 1)^2 = 0}$

$\mathbf{\Rightarrow \; (x - \frac{7}{2})^2 - (\frac{7}{2})^2 + (y + 1)^2 = 0 \Rightarrow \; (x - \frac{7}{2})^2 + (y + 1)^2 = (\frac{7}{2})^2}$.

So the center is $\mathbf{(\frac{7}{2}, -1)}$, and the radius is $\frac{7}{2}$.