Examples 2 in Ellipses

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Assuming in each example below, ***C*** is the center of an ellipse, ***F*** is a focus, ***M*** is the major radius, and ***m*** is the minor radius, find the ellipse, the eccentricity, and the directrices, and put in a graph the ellipse, together with the foci, vertices, and directrices.

0. ***C*(2, 2)**, ***F*(-2, 2)**, and ***m* = 9**.

1. ***C*(3, 2)**, ***F*(3, -3)**, and ***M* = 9**.

2. ***C*(1, 2)**, ***F*(1, 5)**, and ***m* = 9**.

3. ***C*(3, 2)**, ***F*(5, 2)**, and ***m* = 9**.

Suggestions or Solutions
To the Problem in the Example 0

Assuming *C*(2, 2) is the center of an ellipse, *F*(-2, 2) is a focus, and 9 is the minor radius, find the ellipse, and its elements, and put them all in a graph.

To begin with, the ellipse is horizontal, the other focus is (6, 2), and assuming $\boldsymbol{c}$ is the focal distance, we get $\boldsymbol{c} = \mathbf{4}$.
So next, assuming $\boldsymbol{a}$ is the major radius, and $\boldsymbol{b}$ is the minor radius, we get

$\boldsymbol{a} = \mathbf{9}$, and $\boldsymbol{c}^2 = \boldsymbol{a}^2 - \boldsymbol{b}^2 \Rightarrow \mathbf{4}^2 = \mathbf{9}^2 - \boldsymbol{b}^2 \Rightarrow \boldsymbol{b}^2 = \mathbf{65}$.

So the major axis is **18**, the minor axis is $\mathbf{2\sqrt{65}}$, and the ellipse is $\frac{(x-2)^2}{81} + \frac{(y-2)^2}{65} = 1$.

And the vertices are (-7, 2) and (11, 2).

Next, assuming $\boldsymbol{e}$ is the eccentricity, we get $\boldsymbol{e} = \boldsymbol{c}/\boldsymbol{a} = \mathbf{4/9}$.

And next, the directrices are $\boldsymbol{x} = \pm \boldsymbol{a}/\boldsymbol{e} + \mathbf{2} = \pm \boldsymbol{a}^2/\boldsymbol{c} + \mathbf{2} = \pm \mathbf{81/4} + \mathbf{2}$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we know that the ellipse we want to find is centered at (2, 2).

And the standard equation of an ellipse centered at (2, 2) is $\frac{(x-2)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$.

So if finding the values of $\boldsymbol{a}$ and $\boldsymbol{b}$, we find the ellipse. How then, can we get them?

To begin with, if the ellipse is horizontal, we get $\boldsymbol{a} > \boldsymbol{b} > \mathbf{0}$.
Then, we call $\boldsymbol{a}$ the major radius, and call $\boldsymbol{b}$ the minor radius.

If it is vertical however, we get $\boldsymbol{b} > \boldsymbol{a} > \mathbf{0}$.
Then, we call $\boldsymbol{a}$ the minor radius, and call $\boldsymbol{b}$ the major radius.

Next, the center is (2, 2), and one of the foci is (-2, 2).
So we can notice that <u>the center and the foci share the same ***y***-coordinate</u>, which is 2.

We can see thus, the ellipse is <u>horizontal</u>. So first, assuming the other focus is ($\boldsymbol{p}$, 2), since the center is (2, 2), and is the midpoint between the foci, we get $2 = \{\boldsymbol{p} + (-2)\}/2$.

The other focus is thus, (6, 2). And next, assuming c is the focal distance, we get $c = 4$, because the focal distance is the distance from the center to a focus.

Next, we can say that a is the major radius, and thus, is 9. What then, about b?

We have $c^2 = a^2 - b^2$, where a is the major radius, and b is the minor radius.

Thus, we get $c^2 = a^2 - b^2 \Rightarrow 4^2 = 9^2 - b^2 \Rightarrow b^2 = 81 - 16 = 65$.

So the ellipse is $\frac{(x-2)^2}{81} + \frac{(y-2)^2}{65} = 1$, often put this way: $\frac{(x-2)^2}{9^2} + \frac{(y-2)^2}{(\sqrt{65})^2} = 1$.

Next, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, that is, $2a$, and is 18, since $a = 9$. And since the ellipse is horizontal, the center and vertices share the same y-coordinate, too, which is 2. So since the center is (2, 2), the vertices are (-9 + 2, 2) and (9 + 2, 2).

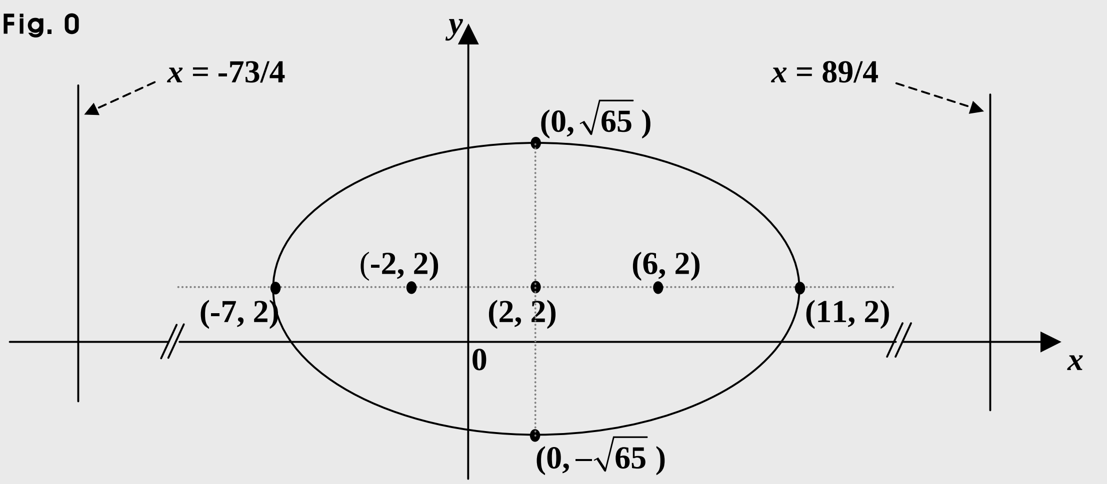
Next, the minor axis is twice the minor radius, that is, $2b$, and thus, is $2\sqrt{65}$.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.

So assuming e is the eccentricity, we get $e = c/a = 4/9$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (2, 2), and the ellipse is horizontal, the directrices are $x = \pm a/e + 2 = \pm a^2/c + 2 = \pm 81/4 + 2$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Assuming $C(3, 2)$ is the center of an ellipse, $F(3, -3)$ is a focus, and 9 is the major radius, find the ellipse, and its elements, and put them all in a graph.

To begin with, the ellipse is vertical, the other focus is (3, 7), and assuming c is the focal distance, we get $c = 5$.
So next, assuming b is the major radius, and a is the minor radius, we get

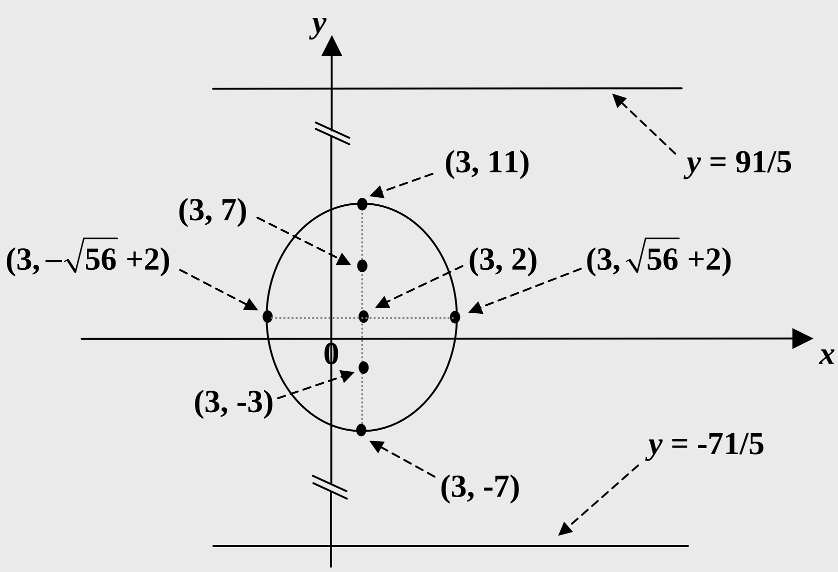
$b = 9$, and $c^2 = b^2 - a^2 \Rightarrow 5^2 = 9^2 - a^2 \Rightarrow a^2 = 56$.

So the major axis is **18**, the minor axis is $2\sqrt{56}$, and the ellipse is $\dfrac{x^2}{56} + \dfrac{y^2}{81} = 1$.

And the vertices are **(3, 11)** and **(3, -7)**.
Next, assuming e is the eccentricity, we get $e = c/b = 5/9$.

And next, the directrices are $y = b/e + 2 = \pm b^2/c + 2 = \pm 81/5 + 2$.

Fig. 0



If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we know that the ellipse we want to find is centered at (3, 2).

And the standard equation of an ellipse centered at (3, 2) is $\dfrac{(x-3)^2}{a^2} + \dfrac{(y-2)^2}{b^2} = 1$.

So if finding the values of a and b, we find the ellipse. How then, can we get them?

To begin with, if the ellipse is horizontal, we get $a > b > 0$.
Then, we call a the major radius, and call b the minor radius.

If it is vertical however, we get $b > a > 0$.
Then, we call a the minor radius, and call b the major radius.

Next, the center is (3, 2), and one of the foci is (3, -3).
So we can notice that the center and the foci share the same x-coordinate, which is 3.

We can see thus, the ellipse is vertical. So first, assuming the other focus is (3, q), since the center is (3, 2), and is the midpoint between the foci, we get 2 = {q + (-3)}/2. The other focus is thus, (3, 7). And next, assuming c is the focal distance, we get $c = 5$, because the focal distance is the distance from the center to a focus.

Next, we can say that b is the major radius, and thus, is 9. What then, about a?

We have $c^2 = b^2 - a^2$, where b is the major radius, and a is the minor radius.

Thus, we get $c^2 = b^2 - a^2 \Rightarrow 5^2 = 9^2 - a^2 \Rightarrow a^2 = 81 - 25 = 56$.

So the ellipse is $\frac{x^2}{56} + \frac{y^2}{81} = 1$, which is often put this way, of course: $\frac{x^2}{(\sqrt{56})^2} + \frac{y^2}{9^2} = 1$.

Next, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, that is, $2b$, and is 18, since $b = 9$. And since the ellipse is vertical, the center and vertices share the same x-coordinate, too, which is 0. So since the center is (3, 2), the vertices are **(3, 9 + 2)** and **(3, -9 + 2)**.

Next, the minor axis is twice the minor radius, that is, $2a$, and thus, is $2\sqrt{56}$.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.

So assuming e is the eccentricity, we get $e = c/b = 5/9$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, which is the major radius over the eccentricity. So since the center is (3, 2), and the ellipse is vertical, the directrices are $y = \pm b/e + 2 = \pm b^2/c + 2 = \pm 81/5 + 2$.

Suggestions or Solutions
To the Problem in the Example 2

Assuming $C(2, 1)$ is the center of an ellipse, $F(2, 6)$ is a focus, and 9 is the minor radius, find the ellipse, and its elements, and put them all in a graph.

To begin with, the ellipse is vertical, the other focus is (2, -4), and assuming c is the focal distance, we get $c = 5$.
So next, assuming b is the major radius, and a is the minor radius, we get

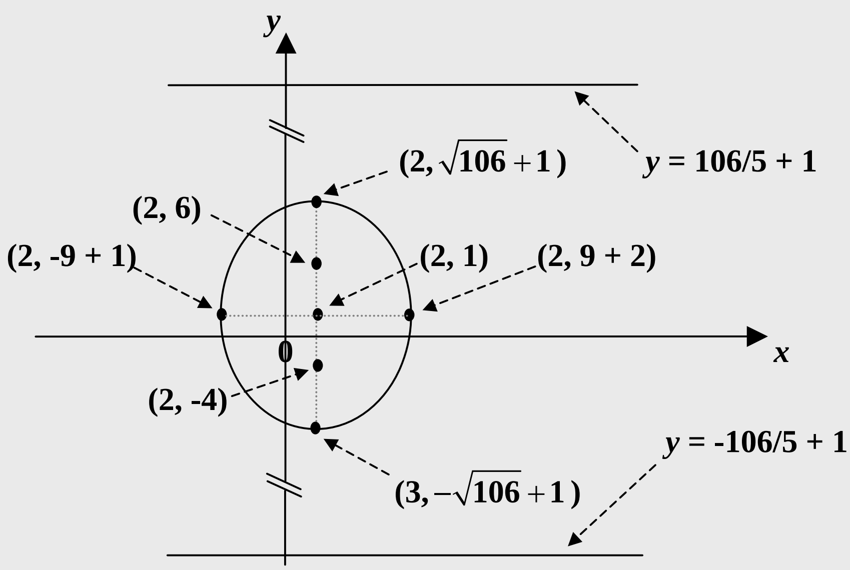
$a = 9$, and $c^2 = b^2 - a^2 \Rightarrow 5^2 = b^2 - 9^2 \Rightarrow b^2 = 106$.

So the major axis is $2\sqrt{106}$, the minor axis is 18, and the ellipse is $\frac{x^2}{81} + \frac{y^2}{106} = 1$.

And the vertices are $(2, \sqrt{106} + 1)$ and $(2, -\sqrt{106} + 1)$.
Next, assuming e is the eccentricity, we get $e = c/b = \frac{5}{\sqrt{106}}$.

And next, the directrices are $y = b/e + 1 = \pm b^2/c + 1 = \pm 106/5 + 1$.

Fig. 0



If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we know that the ellipse we want to find is centered at (2, 1).

And the standard equation of an ellipse centered at (2, 1) is $\frac{(x-2)^2}{a^2} + \frac{(y-1)^2}{b^2} = 1$.

So if finding the values of $\boldsymbol{a}$ and $\boldsymbol{b}$, we find the ellipse. How then, can we get them?

To begin with, if the ellipse is horizontal, we get $\boldsymbol{a} > \boldsymbol{b} > \mathbf{0}$.
Then, we call $\boldsymbol{a}$ the major radius, and call $\boldsymbol{b}$ the minor radius.

If it is vertical however, we get $\boldsymbol{b} > \boldsymbol{a} > \mathbf{0}$.
Then, we call $\boldsymbol{a}$ the minor radius, and call $\boldsymbol{b}$ the major radius.

Next, the center is (2, 1), and one of the foci is (2, 6).
So we can notice that the center and the foci share the same $\boldsymbol{x}$-coordinate, which is 2.

We can see thus, the ellipse is vertical. So first, assuming the other focus is (2, $\boldsymbol{q}$), since the center is (2, 1), and is the midpoint between the foci, we get $1 = (\boldsymbol{q} + 6)/2$.
The other focus is thus, (2, -4). And next, assuming $\boldsymbol{c}$ is the focal distance, we get $\boldsymbol{c} = \mathbf{5}$, because the focal distance is the distance from the center to a focus.

Next, we can say that $\boldsymbol{a}$ is the minor radius, and thus, is 9. What then, about $\boldsymbol{b}$?

We have $\boldsymbol{c}^2 = \boldsymbol{b}^2 - \boldsymbol{a}^2$, where $\boldsymbol{b}$ is the major radius, and $\boldsymbol{a}$ is the minor radius.

Thus, we get $\boldsymbol{c}^2 = \boldsymbol{b}^2 - \boldsymbol{a}^2 \Rightarrow \mathbf{5}^2 = \boldsymbol{b}^2 - \mathbf{9}^2 \Rightarrow \boldsymbol{b}^2 = \mathbf{81 + 25 = 106}$.

So the ellipse is $\dfrac{\boldsymbol{x}^2}{\mathbf{81}} + \dfrac{\boldsymbol{y}^2}{\mathbf{106}} = \mathbf{1}$, which is often put this way, of course: $\dfrac{\boldsymbol{x}^2}{\mathbf{9}^2} + \dfrac{\boldsymbol{y}^2}{(\sqrt{\mathbf{106}})^2} = \mathbf{1}$.

Next, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, that is, $\mathbf{2}\boldsymbol{b}$, and is $\mathbf{2}\sqrt{\mathbf{106}}$. And since the ellipse is vertical, the center and vertices share the same $\boldsymbol{x}$-coordinate, too, which is 2.
So since the center is (2, 1), the vertices are $(\mathbf{2}, \sqrt{\mathbf{106}} + \mathbf{1})$ and $(\mathbf{2}, -\sqrt{\mathbf{106}} + \mathbf{1})$.

Next, the minor axis is twice the minor radius, that is, $\mathbf{2}\boldsymbol{a}$, and thus, is 18.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.

So assuming $\boldsymbol{e}$ is the eccentricity, we get $\boldsymbol{e} = \boldsymbol{c}/\boldsymbol{b} = \frac{5}{\sqrt{106}}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (2, 1), and the ellipse is vertical, the directrices are $\boldsymbol{y} = \pm\boldsymbol{b}/\boldsymbol{e} + \mathbf{1} = \pm\boldsymbol{b}^2/\boldsymbol{c} + \mathbf{1} = \pm\mathbf{106/5 + 1}$.

Suggestions or Solutions
To the Problem in the Example 3

Assuming $C(3, 2)$ is the center of an ellipse, $F(5, 2)$ is a focus, and 9 is the minor radius, find the ellipse, and its elements, and put them all in a graph.

To begin with, the ellipse is horizontal, the other focus is (1, 2), and assuming c is the focal distance, we get $c = 2$.
So next, assuming a is the major radius, and b is the minor radius, we get

$b = 9$, and $c^2 = a^2 - b^2 \Rightarrow 2^2 = a^2 - 9^2 \Rightarrow a^2 = 85$.

So the major axis is $2\sqrt{85}$, the minor axis is 18, and the ellipse is $\frac{(x-3)^2}{85} + \frac{(y-2)^2}{81} = 1$.

And the vertices are $(3-\sqrt{85}, 2)$ and $(3+\sqrt{85}, 2)$.
Next, assuming e is the eccentricity, we get $e = c/a = \frac{2}{\sqrt{85}}$.

And next, the directrices are $x = \pm a/e + 3 = \pm a^2/c + 3 = \pm 85/2 + 3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we know that the ellipse we want to find is centered at (3, 2).
And the standard equation of an ellipse centered at (3, 2) is $\frac{(x-3)^2}{a^2} + \frac{(y-2)^2}{b^2} = 1$.
So if finding the values of a and b, we find the ellipse. How then, can we get them?

To begin with, if the ellipse is horizontal, we get $a > b > 0$.
Then, we call a the major radius, and call b the minor radius.

If it is vertical however, we get $b > a > 0$.
Then, we call a the minor radius, and call b the major radius.

Next, the center is (3, 2), and one of the foci is (5, 2).
So we can notice that the center and the foci share the same y-coordinate, which is 2.

We can see thus, the ellipse is horizontal. So first, assuming the other focus is $(p, 2)$, since the center is (3, 2), and is the midpoint between the foci, we get $3 = (p + 5)/2$.

The other focus is thus, (1, 2). And next, assuming c is the focal distance, we get $c = 2$, because the focal distance is the distance from the center to a focus.

Next, we can say that b is the minor radius, and thus, is 9. What then, about a?

We have $c^2 = a^2 - b^2$, where a is the major radius, and b is the minor radius.

Thus, we get $c^2 = a^2 - b^2 \Rightarrow 2^2 = a^2 - 9^2 \Rightarrow a^2 = 81 + 4 = 85$.

So the ellipse is $\frac{(x-3)^2}{85} + \frac{(y-2)^2}{81} = 1$, often put this way: $\frac{(x-3)^2}{(\sqrt{85})^2} + \frac{(y-2)^2}{9^2} = 1$.

Next, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, that is, $2a$, and is $2\sqrt{85}$. And since the ellipse is horizontal, the center and vertices share the same y-coordinate, too, which is 2. So since the center is (3, 2), the vertices are $(3-\sqrt{85}, 2)$ and $(3+\sqrt{85}, 2)$.

Next, the minor axis is twice the minor radius, that is, $2b$, and thus, is 18.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius. So assuming e is the eccentricity, we get $e = c/a = \frac{2}{\sqrt{85}}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (3, 2), and the ellipse is horizontal, the directrices are $x = \pm a/e + 3 = \pm a^2/c + 3 = \pm 85/2 + 3$.

Fig. 0