Examples 3 in Ellipses

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Examples 3 in Ellipses

Find if each ellipse below is horizontal or vertical, and the center, foci, vertices, major axis, minor axis, eccentricity, and directrices.

0. $\frac{x^2}{25}+\frac{y^2}{16}=1$

1. $\frac{x^2}{16}+\frac{y^2}{25}=1$

2. $\frac{(x-1)^2}{25}+\frac{(y-2)^2}{16}=1$

3. $25(x-1)^2 + 16(y-2)^2 = 400$

4. $\frac{(x-1)^2}{4}+y^2=1$

5. $4(x-1)^2 + (y-2)^2 = 4$

Suggestions or Solutions
To the Problem in the Example 0

Find all the elements of the ellipse as follows. $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$.

To begin with, the ellipse is horizontal, the center is (0, 0), the major axis is 10, and the minor axis is 8.

Next, assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $a = 5$, $b = 4$, and $c^2 = a^2 - b^2 \Rightarrow c^2 = 25 - 16 = 9 \Rightarrow c = 3$.

So the focal distance is 3, and the foci are (-3, 0) and (3, 0).

And the vertices are (-5, 0) and (5, 0).

Next, assuming e is the eccentricity, we get $e = c/a = 3/5$.

And next, the directrices are $x = \pm a/e = \pm a^2/c = \pm 25/3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we can put the ellipse given this way: $\dfrac{x^2}{5^2}+\dfrac{y^2}{4^2}=1$.

And the standard equation of an ellipse centered at the origin is $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$.

So the center of the ellipse given is (0, 0).

Next, if $a > b > 0$ in the equation above, the ellipse is horizontal, a is the major radius, and b is the minor radius.

So the ellipse given is horizontal, the major radius is 5, and the minor radius is 4.

And the major axis is twice the major radius, and thus, is 10, and the minor axis is twice the minor radius, and thus, is 8. What then, about the focal distance?

Assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $c^2 = a^2 - b^2$. So we get $c^2 = a^2 - b^2 = 5^2 - 4^2 = 25 - 16 = 9 \Rightarrow c = 3$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is horizontal, the center, foci, and vertices share the same ***y***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is c, which is 3.

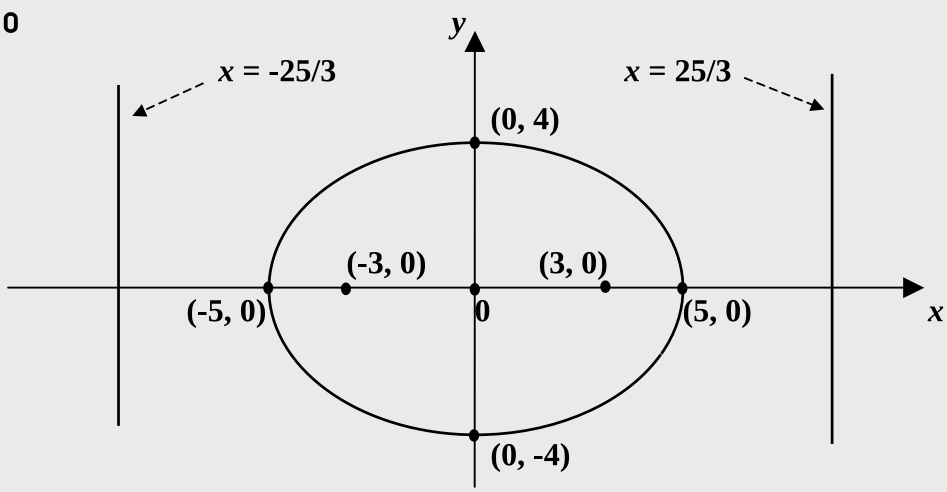
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is a, which is 5.

So since the center is (0, 0), and the ellipse is horizontal, the foci are (3, 0) and (-3, 0), and the vertices are (5, 0) and (-5, 0).

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming e is the eccentricity, we get $\boldsymbol{e = c/a = 3/5}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (0, 0), and the ellipse is horizontal, the directrices are $\boldsymbol{x = \pm a/e = \pm a^2/c = \pm 25/3}$.

Fig. 0



The ellipse is $\boldsymbol{\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1}$.

Suggestions or Solutions
To the Problem in the Example 1

Find all the elements of the ellipse as follows. $\frac{x^2}{\mathbf{16}} + \frac{y^2}{\mathbf{25}} = \mathbf{1}$.

To begin with, the ellipse is vertical, the center is (0, 0), the major axis is 10, and the minor axis is 8.

Next, assuming c is the focal distance, b is the major radius, and a is the minor radius, we get $b = 5$, $a = 4$, and $c^2 = b^2 - a^2 \Rightarrow c^2 = 25 - 16 = 9 \Rightarrow c = 3$.

So the focal distance is 3, and the foci are (0, 3) and (0, -3).

And the vertices are (0, 5) and (0, -5).

Next, assuming e is the eccentricity, we get $e = c/b = 3/5$.

And next, the directrices are $y = \pm b/e = \pm b^2/c = \pm 25/3$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we can put the ellipse given this way: $\frac{x^2}{4^2} + \frac{y^2}{5^2} = 1$.

And the standard equation of an ellipse centered at the origin is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

So the center of the ellipse given is (0, 0).

Next, if $b > a > 0$ in the equation above, the ellipse is vertical, b is the major radius, and a is the minor radius.

So the ellipse given is <u>vertical</u>, the major radius is 5, and the minor radius is 4.

And the major axis is twice the major radius, and thus, is 10, and the minor axis is twice the minor radius, and thus, is 8. What then, about the focal distance?

Assuming c is the focal distance, b is the major radius, and a is the minor radius, we get $c^2 = b^2 - a^2$. So we get $c^2 = b^2 - a^2 = 5^2 - 4^2 = 25 - 16 = 9 \Rightarrow c = 3$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is vertical, the center, foci, and vertices share the same ***x***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is c, which is 3.

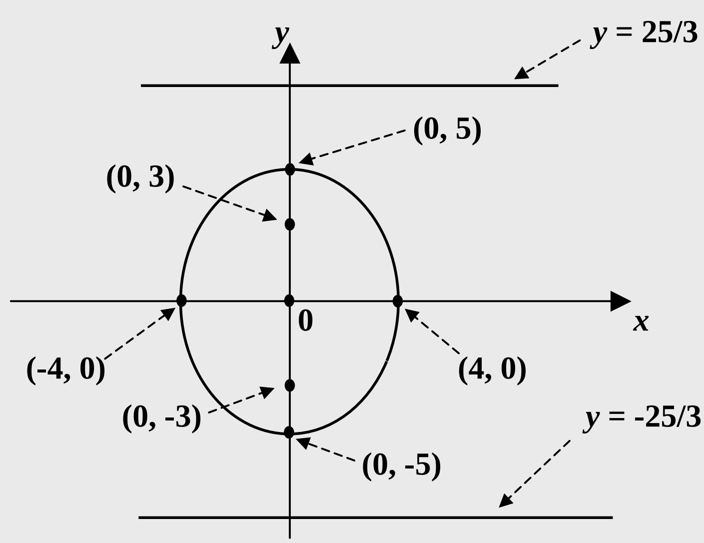
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is b, which is 5.

So since the center is (0, 0), and the ellipse is vertical, the foci are (0, 3) and (0, -3), and the vertices are (0, 5) and (0, -5).

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming e is the eccentricity, we get $\boldsymbol{e = c/b = 3/5}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (0, 0), and the ellipse is vertical, the directrices are $\boldsymbol{y = \pm b/e = \pm b^2/c = \pm 25/3}$.

Fig. 0



The ellipse is $\dfrac{x^2}{4^2} + \dfrac{y^2}{5^2} = \mathbf{1}$.

Suggestions or Solutions
To the Problem in the Example 2

Find all the elements of the ellipse as follows. $\dfrac{(x-1)^2}{25}+\dfrac{(y-2)^2}{16}=1.$

To begin with, the ellipse is horizontal, the center is (1, 2), the major axis is 10, and the minor axis is 8.

Next, assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $a = 5$, $b = 4$, and $c^2 = a^2 - b^2 \Rightarrow c^2 = 25 - 16 = 9 \Rightarrow c = 3$.

So the focal distance is 3, and the foci are (-2, 2) and (4, 2).

And the vertices are (-4, 2) and (6, 2).

Next, assuming e is the eccentricity, we get $e = c/a = 3/5$.

And next, the directrices are $x = \pm a/e + 1 = \pm a^2/c + 1 = \pm 25/3 + 1$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we can put the ellipse given this way: $\dfrac{(x-1)^2}{5^2}+\dfrac{(y-2)^2}{4^2}=1.$

And the standard equation of an ellipse centered at (u, v) is $\dfrac{(x-u)^2}{a^2}+\dfrac{(y-v)^2}{b^2}=1.$

So the center of the ellipse given is (1, 2).

Next, if $a > b > 0$ in the equation above, the ellipse is horizontal, a is the major radius, and b is the minor radius.

So the ellipse given is horizontal, the major radius is 5, and the minor radius is 4.

And the major axis is twice the major radius, and thus, is 10, and the minor axis is twice the minor radius, and thus, is 8. What then, about the focal distance?

Assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $c^2 = a^2 - b^2$. So we get $c^2 = a^2 - b^2 = 5^2 - 4^2 = 25 - 16 = 9 \Rightarrow c = 3$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is horizontal, the center, foci, and vertices share the same ***y***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is ***c***, which is 3.

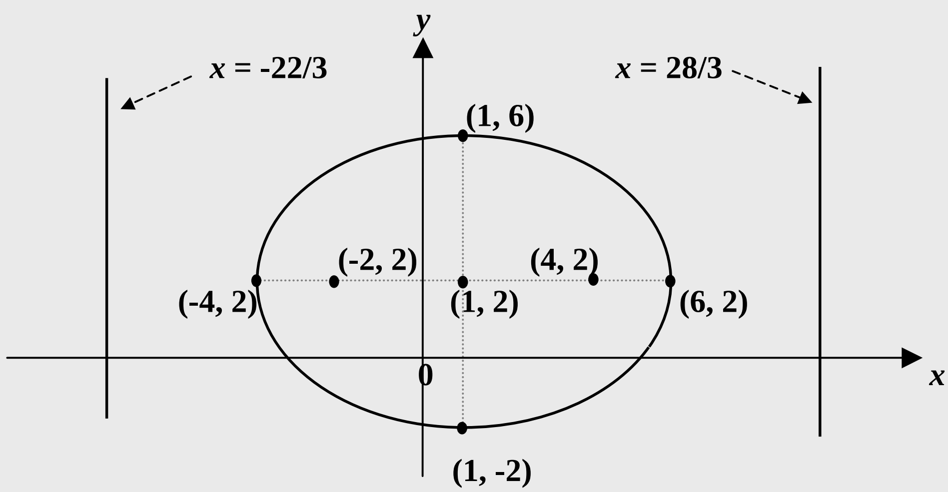
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is ***a***, which is 5.

So since the center is (1, 2), and the ellipse is horizontal, the two foci are (3 + 1, 0 + 2) and (-3 + 1, 0 + 2), that is, **(4, 2)** and **(-2, 2)**, and by the same token, the two vertices are (5 + 1, 0 + 2) and (-5 + 1, 0 + 2), that is, **(6, 2)** and **(-4, 2)**.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming ***e*** is the eccentricity, we get $\boldsymbol{e = c/a = 3/5}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (1, 2), and the ellipse is horizontal, the directrices are $\boldsymbol{x = \pm a/e + 1 = \pm a^2/c + 1 = \pm 25/3 + 1}$.

Fig. 0



The ellipse is $\boldsymbol{\dfrac{(x-1)^2}{5^2} + \dfrac{(y-2)^2}{4^2} = 1}$.

Suggestions or Solutions
To the Problem in the Example 3

Find all the elements of the ellipse as follows. $\mathbf{25(x-1)^2+16(y-2)^2=400}$.

To begin with, we can put the ellipse give this way: $\dfrac{(x-1)^2}{4^2}+\dfrac{(y-2)^2}{5^2}=1$.

So the ellipse is vertical, the center is (1, 2), the major axis is 10, and the minor axis is 8.

Next, assuming c is the focal distance, b is the major radius, and a is the minor radius, we get $b=5$, $a=4$, and $c^2=b^2-a^2 \Rightarrow c^2=25-16=9 \Rightarrow c=3$.

So the foci are (1, 5) and (1, -1). And the vertices are (1, 7) and (1, -3).

Next, assuming e is the eccentricity, we get $e=c/b=3/5$.

And next, the directrices are $y=\pm b/e+2=\pm b^2/c+2=\pm 25/3+2$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with we can put the ellipse given the way below.

$$25(x-1)^2+16(y-2)^2=400 \Rightarrow \frac{25}{400}(x-1)^2+\frac{16}{400}(y-2)^2=1$$

$$\Rightarrow \frac{5^2}{20^2}(x-1)^2+\frac{4^2}{20^2}(y-2)^2 \Rightarrow \frac{1}{16}(x-1)^2+\frac{1}{25}(y-2)^2=1 \Rightarrow \frac{(x-1)^2}{4^2}+\frac{(y-2)^2}{5^2}=1.$$

And the standard equation of an ellipse centered at (u, v) is $\dfrac{(x-u)^2}{a^2}+\dfrac{(y-v)^2}{b^2}=1$.

So the center of the ellipse given is (1, 2).

Next, if $b>a>0$ in the equation above, the ellipse is vertical, b is the major radius, and a is the minor radius. So the ellipse given is <u>vertical</u>, the major radius is 5, and the minor radius is 4. And the major axis is twice the major radius, and thus, is 10, and the minor axis is twice the minor radius, and thus, is 8. What then, about the focal distance?

Assuming c is the focal distance, b is the major radius, and a is the minor radius, we get $c^2=b^2-a^2$. So we get $c^2=b^2-a^2=5^2-4^2=25-16=9 \Rightarrow c=3$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is vertical, the center, foci, and vertices share the same ***x***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is ***c***, which is 3.

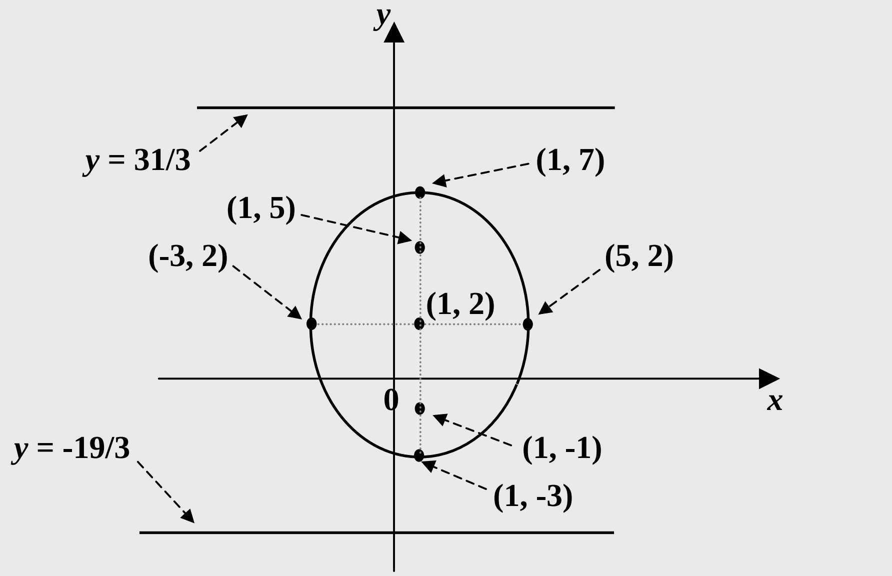
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is ***b***, which is 5.

So since the center is (1, 2), and the ellipse is vertical, the two foci are (0 + 1, 3 + 2) and (0 + 1, -3 + 2), that is, **(1, 5)** and **(1, -1)**, and next, the two vertices are (0 + 1, 5 + 2) and (0 + 1, -5 + 2), that is, **(1, 7)** and **(1, -3)**.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming ***e*** is the eccentricity, we get $e = c/b = 3/5$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (1, 2), and the ellipse is vertical, the directrices are $y = \pm b/e + 2 = \pm b^2/c + 2 = \pm 25/3 + 2$.

Fig. 0



The ellipse is $\dfrac{(x-1)^2}{4^2} + \dfrac{(y-2)^2}{5^2} = 1$.

Suggestions or Solutions
To the Problem in the Example 4

Find all the elements of the ellipse as follows. $\frac{(x-1)^2}{4}+y^2=1$.

To begin with, the ellipse is horizontal, the center is (1, 0), the major axis is 4, and the minor axis is 2.

Next, assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $a = 2$, $b = 1$, and $c^2 = a^2 - b^2 \Rightarrow c^2 = 4 - 1 = 3 \Rightarrow c = \sqrt{3}$.

So the focal distance is $\sqrt{3}$, and the foci are $(1-\sqrt{3}, 0)$ and $(1+\sqrt{3}, 0)$.
And the vertices are (-1, 0) and (3, 0).
Next, assuming e is the eccentricity, we get $e = c/a = \frac{\sqrt{3}}{2}$.
And next, the directrices are $x = \pm a/e + 1 = \pm a^2/c + 1 = \pm\frac{4\sqrt{3}}{3}+1$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, we can put the ellipse given this way: $\frac{(x-1)^2}{2^2}+\frac{y^2}{1^2}=1$.

And the standard equation of an ellipse centered at $(u, 0)$ is $\frac{(x-u)^2}{a^2}+\frac{y^2}{b^2}=1$.

So the center of the ellipse given is (1, 0).

Next, if $a > b > 0$ in the equation above, the ellipse is horizontal, a is the major radius, and b is the minor radius.
So the ellipse given is <u>horizontal</u>, the major radius is 2, and the minor radius is 1.

And the major axis is twice the major radius, and thus, is 4, and the minor axis is twice the minor radius, and thus, is 2. What then, about the focal distance?

Assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $c^2 = a^2 - b^2$. So we get $c^2 = a^2 - b^2 = 2^2 - 1^2 = 4 - 1 = 3 \Rightarrow c = \sqrt{3}$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is horizontal, the center, foci, and vertices share the same ***y***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is ***c***, which is $\sqrt{\mathbf{3}}$.

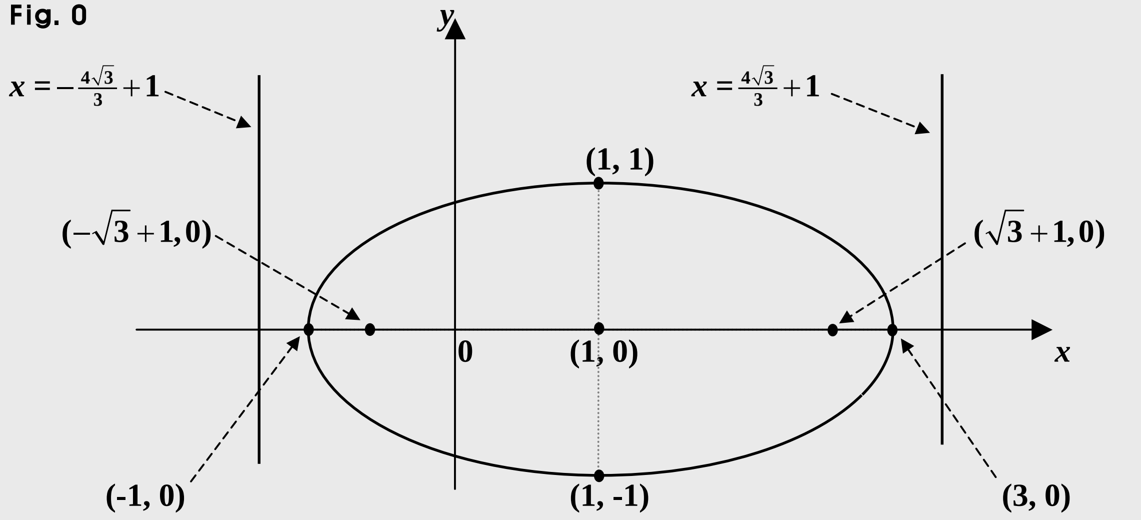
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is ***a***, which is 2.

So since the center is (1, 0), and the ellipse is horizontal, the two foci are $(\sqrt{3}+1,0)$ and $(-\sqrt{3}+1,0)$, and the vertices are (2 + 1, 0) and (-2 + 1, 0), that is, **(3, 0)** and **(-1, 0)**.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming ***e*** is the eccentricity, we get $e = c/a = \frac{\sqrt{3}}{2}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (1, 0), and it is horizontal, the directrices are $x = \pm a/e + 1 = \pm a^2/c + 1 = \pm\frac{4}{\sqrt{3}} + 1 = \pm\frac{4\sqrt{3}}{3} + 1$.

Fig. 0



The ellipse is $\frac{(x-1)^2}{4} + y^2 = 1$.

Suggestions or Solutions
To the Problem in the Example 5

Find all the elements of the ellipse as follows. $\mathbf{4(x-1)^2 + (y-2)^2 = 4}$.

To begin with, we can put the ellipse give this way: $\dfrac{(x-1)^2}{1^2} + \dfrac{(y-2)^2}{2^2} = 1$.
So the ellipse is vertical, the center is (1, 2), the major axis is 4, and the minor axis is 2.

Next, assuming c is the focal distance, a is the major radius, and b is the minor radius, we get $b = 2$, $a = 1$, and $c^2 = b^2 - a^2 \Rightarrow c^2 = 4 - 1 = 3 \Rightarrow c = \sqrt{3}$.

So the foci are $(1, 2-\sqrt{3})$ and $(1, 2+\sqrt{3})$. And the vertices are $(1, 4)$ and $(1, 0)$.
Next, assuming e is the eccentricity, we get $e = c/b = \frac{\sqrt{3}}{2}$.
And next, the directrices are $y = \pm b/e + 2 = \pm b^2/c + 2 = \pm\frac{4\sqrt{3}}{3} + 2$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with we can put the ellipse given the way below.

$$4(x-1)^2 + (y-2)^2 = 4 \Rightarrow (x-1)^2 + \frac{1}{4}(y-2)^2 = 1 \Rightarrow \frac{(x-1)^2}{1^2} + \frac{(y-2)^2}{2^2} = 1.$$

And the standard equation of an ellipse centered at (u, v) is $\dfrac{(x-u)^2}{a^2} + \dfrac{(y-v)^2}{b^2} = 1$.
So the center of the ellipse given is (1, 2).

Next, if $b > a > 0$ in the equation above, the ellipse is vertical, b is the major radius, and a is the minor radius.
So the ellipse given is vertical, the major radius is 2, and the minor radius is 1.

And the major axis is twice the major radius, and thus, is 4, and the minor axis is twice the minor radius, and thus, is 2. What then, about the focal distance?

Assuming c is the focal distance, b is the major radius, and a is the minor radius, we get $c^2 = b^2 - a^2$. So we get $c^2 = b^2 - a^2 = 2^2 - 1^2 = 4 - 1 = 3 \Rightarrow c = \sqrt{3}$.

Next, the center, foci, and vertices are all in the major axis.
So if the ellipse is vertical, the center, foci, and vertices share the same ***x***-coordinate.

The center is the midpoint between the foci, and the focal distance is the distance from the center to each focus, and is ***c***, which is $\sqrt{3}$.

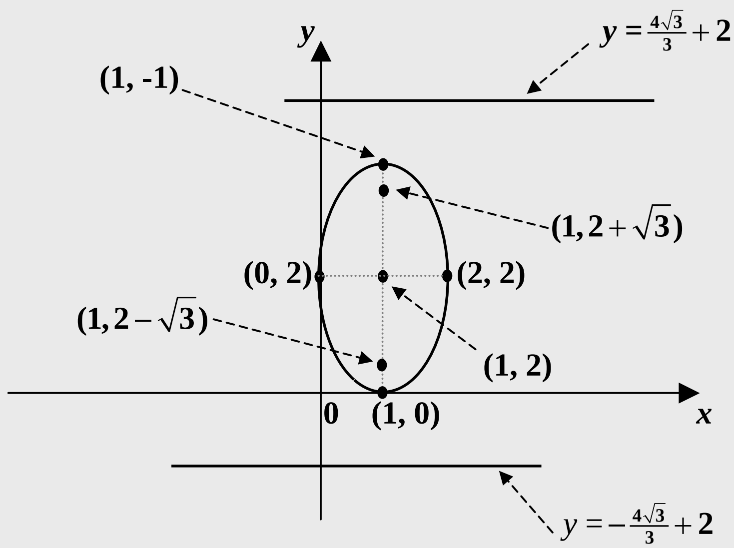
And also, the center is the midpoint between the vertices, too, which are the endpoints of the major axis, which is twice the major radius, which is the distance from the center to each vertex, and in this case, is ***b***, which is 2.

So since the center is (1, 2), and the ellipse is vertical, the two foci are $(0+1, 2+\sqrt{3})$ and $(0+1, 2-\sqrt{3})$, that is, $(1, 2+\sqrt{3})$ and $(1, 2-\sqrt{3})$, and the two vertices are (0 + 1, 2 + 2) and (0 + 1, -2 + 2), that is, **(1, 4)** and **(1, 0)**.

Next, the eccentricity of an ellipse is a ratio, the focal distance over the major radius.
So assuming e is the eccentricity, we get $e = c/b = \frac{\sqrt{3}}{2}$.

And next, an ellipse has two lines called the directrices, and the distance from each to the center is a ratio, the major radius over the eccentricity. So since the center is (1, 2), and it is vertical, the directrices are $y = \pm b/e + 2 = \pm b^2/c + 2 = \pm\frac{4}{\sqrt{3}} + 2 = \pm\frac{4\sqrt{3}}{3} + 2$.

Fig. 0



The ellipse is $\frac{(x-1)^2}{1^2} + \frac{(y-2)^2}{2^2} = 1$.