Examples 2 in Standard Forms

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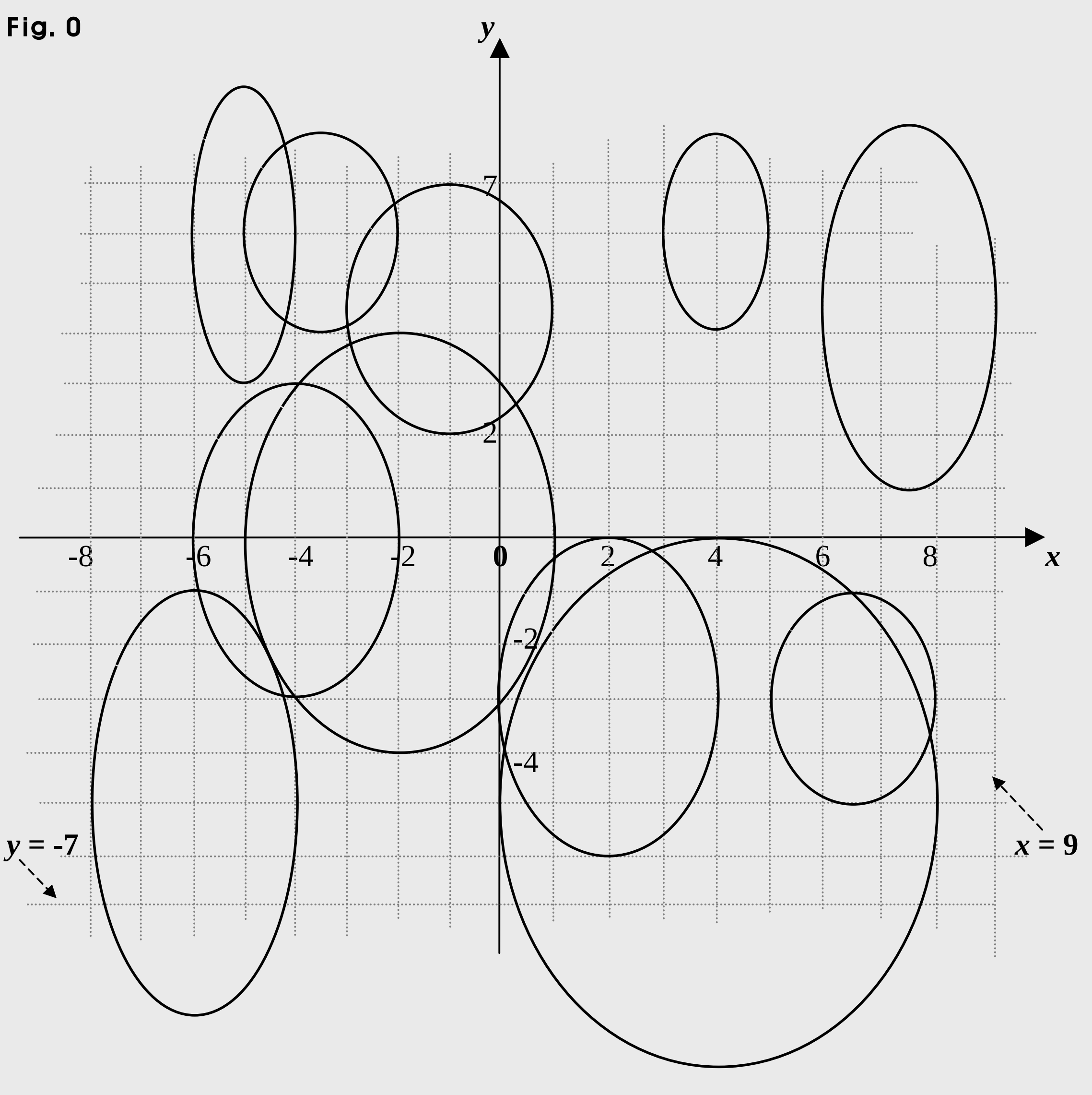
Label each ellipse below, and then find the equation of each of the ellipses.

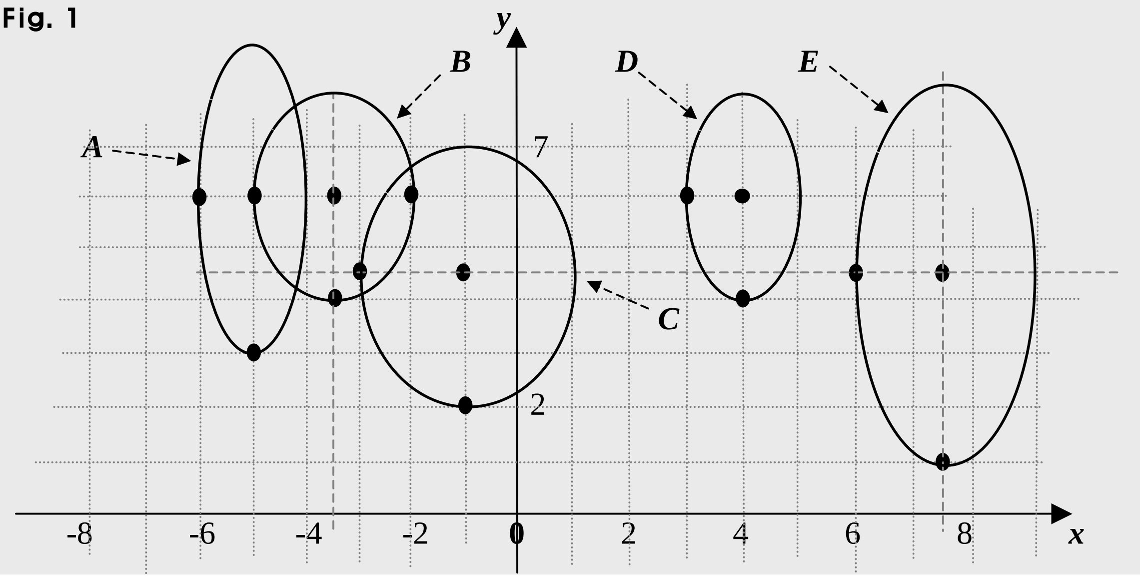
Fig. 0



Suggestions or Solutions
To the Problems in the Examples

The ellipses are all vertical, and beginning with the ellipses below, we can see first, the ellipse ***A*** is centered at (-5, 6), the major radius is 3, and the minor radius is 1.

Fig. 1



And in the graph, we can find the center and the radii each of all the other ellipses has.

Next, if an ellipse is **vertical**, knowing the center and the major and minor radii, we can get the equation of the ellipse using the standard form, $\dfrac{(x-u)^2}{a^2}+\dfrac{(y-v)^2}{b^2}=1$, where (u, v) is the center, and $b > a$, so b is the major radius, and a is the minor radius.

And thus, beginning with the ellipse ***A***, we can see it has the center at (-5, 6), the major radius is 3, and the minor radius is 1. So we get $u = -5$, $v = 6$, $a = 1$, and $b = 3$.

Thus, we get $\dfrac{\{x-(-5)\}^2}{1^2}+\dfrac{(y-6)^2}{3^2}=1 \Rightarrow \dfrac{(x+5)^2}{1^2}+\dfrac{(y-6)^2}{3^2}=1$, which is the equation of the ellipse ***A***. And we can put it this way, too, of course: $(x+5)^2+\dfrac{(y-6)^2}{9}+=1$.

And this way, as well: $9(x + 5)^2 + (y - 6)^2 = 9$.

Next, looking at the ellipse ***B***, we can see it has the center at $(-\frac{7}{2}, 6)$, the major radius is 2, and the minor radius is $\frac{3}{2}$. So we get $u = -\frac{7}{2}$, $v = 6$, $a = \frac{3}{2}$, and $b = 2$.

Thus, we get $\frac{\{x-(-\frac{7}{2})\}^2}{(\frac{3}{2})^2} + \frac{(y-6)^2}{2^2} = 1 \Rightarrow \frac{(x+\frac{7}{2})^2}{(\frac{3}{2})^2} + \frac{(y-6)^2}{2^2} = 1$, which is the equation of the ellipse ***B***, and can be put this way, too: $\frac{(x+\frac{7}{2})^2}{\frac{9}{4}} + \frac{(y-6)^2}{4} = 1$.

And this way, as well: $\frac{4(x+\frac{7}{2})^2}{9} + \frac{(y-6)^2}{4} = 1$, or $16(x+\frac{7}{2})^2 + 9(y-6)^2 = 36$.

Looking at next, the ellipse ***C***, we can see the center is $(-1, \frac{9}{2})$, the major radius is $\frac{5}{2}$, and the minor radius is 2. So we get $u = -1$, $v = \frac{9}{2}$, $a = 2$, and $b = \frac{5}{2}$.

Thus, we get $\frac{(x+1)^2}{2^2} + \frac{(y-\frac{9}{2})^2}{(\frac{5}{2})^2} = 1$, which is the equation of ***C***, and can be put this way, too: $\frac{(x+1)^2}{4} + \frac{(y-\frac{9}{2})^2}{\frac{25}{4}} = 1$, $\frac{(x+1)^2}{4} + \frac{4(y-\frac{9}{2})^2}{25} = 1$, or $25(x+1)^2 + 16(y-\frac{9}{2})^2 = 100$.

Next, examining the ellipse ***D***, we can see the center is (4, 6), the major radius is 2, and the minor radius is 1. So we get $u = 4$, $v = 6$, $a = 1$, and $b = 2$.

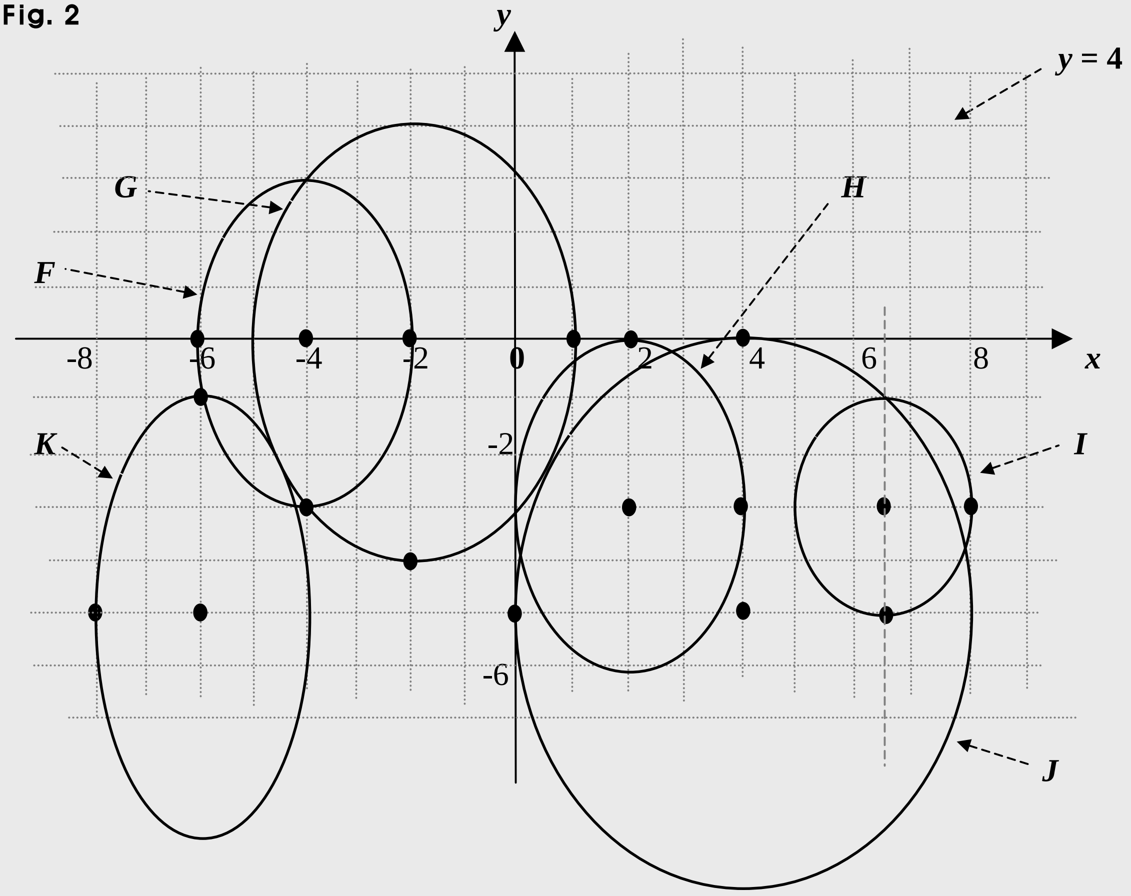
Thus, we get $\frac{(x-4)^2}{1^2} + \frac{(y-6)^2}{2^2} = 1$, which is the equation of the ellipse ***D***, and can be put this way, too: $(x-4)^2 + \frac{(y-6)^2}{4} = 1$. And this way, as well: $4(x-4)^2 + (y-6)^2 = 4$.

And next, looking at the ellipse ***E***, we can see the center is $(\frac{15}{2}, \frac{9}{2})$, the major radius is $\frac{7}{2}$, and the minor radius is $\frac{3}{2}$. So we get $u = \frac{15}{2}$, $v = \frac{9}{2}$, $a = \frac{3}{2}$, and $b = \frac{7}{2}$.

Thus, we get $\dfrac{(x-\frac{15}{2})^2}{(\frac{3}{2})^2}+\dfrac{(y-\frac{9}{2})^2}{(\frac{7}{2})^2}=1$, which is the equation of ***E***, and can be put this way, too: $\dfrac{(x-\frac{15}{2})^2}{\frac{9}{4}}+\dfrac{(y-\frac{9}{2})^2}{\frac{49}{4}}=1$, $\dfrac{4(x-\frac{15}{2})^2}{9}+\dfrac{4(y-\frac{9}{2})^2}{49}=1$ or $\dfrac{(x-\frac{15}{2})^2}{9}+\dfrac{(y-\frac{9}{2})^2}{49}=\dfrac{1}{4}$.

Next, moving on to the ellipses below, we can see that the ellipse ***F*** is centered at (-4, 0), and a point at (-7, 4) or (-3, 4).

Fig. 2



And if an ellipse is **horizontal**, knowing the center and the major and minor radii, we can get the equation of the ellipse using the standard form, $\dfrac{(x-u)^2}{a^2}+\dfrac{(y-v)^2}{b^2}=1$, where ***(u, v)*** is the center, and $\boldsymbol{a > b}$, so ***a*** is the major radius, and ***b*** is the minor radius.

And thus, beginning with the ellipse ***F***, we can see the center is (-4, 0), the major radius is 3, and the minor radius is 2. So we get ***u* = -4**, ***v* = 0**, ***a* = 2**, and ***b* = 3**.

Thus, we get $\frac{\{x-(-4)\}^2}{2^2}+\frac{(y-0)^2}{3^2}=1 \Rightarrow \frac{(x+4)^2}{2^2}+\frac{y^2}{3^2}=1$, which is the equation of ***F***, and can be put this way below, too: $\frac{(x+4)^2}{4}+\frac{y^2}{9}=1$, or this way: $9(x + 4)^2 + 4y^2 = 36$.

Next, looking at the ellipse ***G***, we can see the center is (-2, 0), the major radius is 4, and the minor radius is 3. So we get ***u* = -2**, ***v* = 0**, ***a* = 3**, and ***b* = 4**.

Thus, we get $\frac{\{x-(-2)\}^2}{3^2}+\frac{(y-0)^2}{4^2}=1 \Rightarrow \frac{(x+2)^2}{3^2}+\frac{y^2}{4^2}=1$, which is the equation of ***G***, and can be put this way, too: $\frac{(x+2)^2}{9}+\frac{y^2}{16}=1$, or this way: $16(x + 2)^2 + 9(y - 6)^2 = 144$.

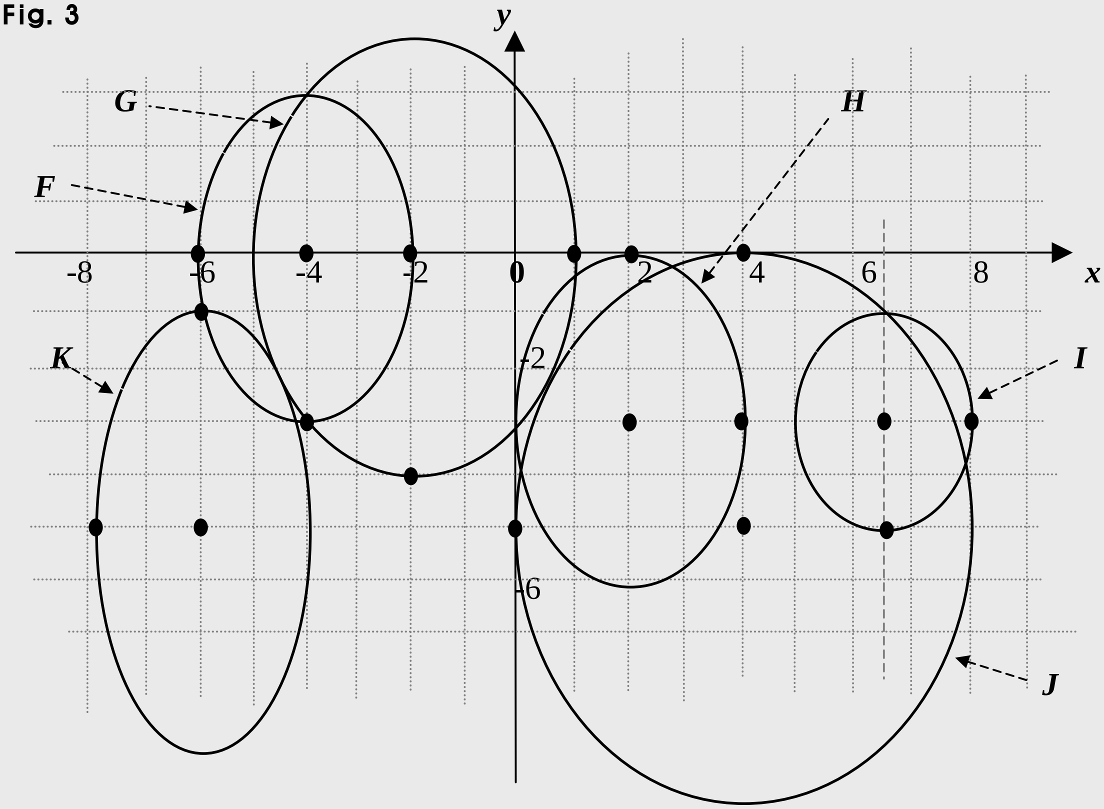
Next, examining the ellipse ***H***, we can see the center is (2, -3), the major radius is 3, and the minor radius is 2. So we get ***u* = 2**, ***v* = -3**, ***a* = 2**, and ***b* = 3**.

Thus, we get $\frac{(x-2)^2}{2^2}+\frac{\{y-(-3)\}^2}{3^2}=1 \Rightarrow \frac{(x-2)^2}{2^2}+\frac{(y+3)^2}{3^2}=1$, which is the equation of ***H***, and can be put this way, too: $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{9}=1$ or $9(x - 2)^2 + 4(y + 3)^2 = 36$.

Next, examining the ellipse ***I***, we can see the center is $(\frac{13}{2}, -3)$, the major radius is 2, and the minor radius is $\frac{3}{2}$. So we get $u = \frac{13}{2}$, ***v* = -3**, $a = \frac{3}{2}$, and ***b* = 2**.

Thus, we get $\frac{(x-\frac{13}{2})^2}{(\frac{3}{2})^2}+\frac{(y+3)^2}{2^2}=1$, which is the equation of ***E***, and can be put this way, too: $\frac{(x-\frac{13}{2})^2}{\frac{9}{4}}+\frac{(y+3)^2}{4}=1$, $\frac{4(x-\frac{13}{2})^2}{9}+\frac{(y+3)^2}{4}=1$, or $16(x - \frac{13}{2})^2 + 9(y + 3)^2 = 36$.

Fig. 3



Next, examining the ellipse ***J***, we can see the center is (4, -5), the major radius is 5, and the minor radius is 4. So we get ***u* = 4**, ***v* = -5**, ***a* = 4**, and ***b* = 5**.

Thus, we get $\dfrac{(x-4)^2}{4^2}+\dfrac{\{y-(-5)\}^2}{5^2}=1 \Rightarrow \dfrac{(x-4)^2}{4^2}+\dfrac{(y+5)^2}{5^2}=1$, which is the ellipse ***J***,

and can be put this way, too: $\dfrac{(x-4)^2}{16}+\dfrac{(y+5)^2}{25}=1$ or $25(x-4)^2+16(y+5)^2=400$.

Next, examining the ellipse ***K***, we can see the center is (-6, -5), the major radius is 4, and the minor radius is 2. So we get ***u* = -6**, ***v* = -5**, ***a* = 2**, and ***b* = 4**.

Thus, we get $\dfrac{\{x-(-6)\}^2}{2^2}+\dfrac{\{y-(-5)\}^2}{4^2}=1 \Rightarrow \dfrac{(x+6)^2}{2^2}+\dfrac{(y+5)^2}{4^2}=1$, which is the ellipse ***K***, and can be put this way, too: $\dfrac{(x+6)^2}{4}+\dfrac{(y+5)^2}{16}=1$ or $16(x+6)^2+4(y+5)^2=64$.