Examples 4 in Standard Forms

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Specify the center and the two main axes of each ellipse below, and then, graph it.

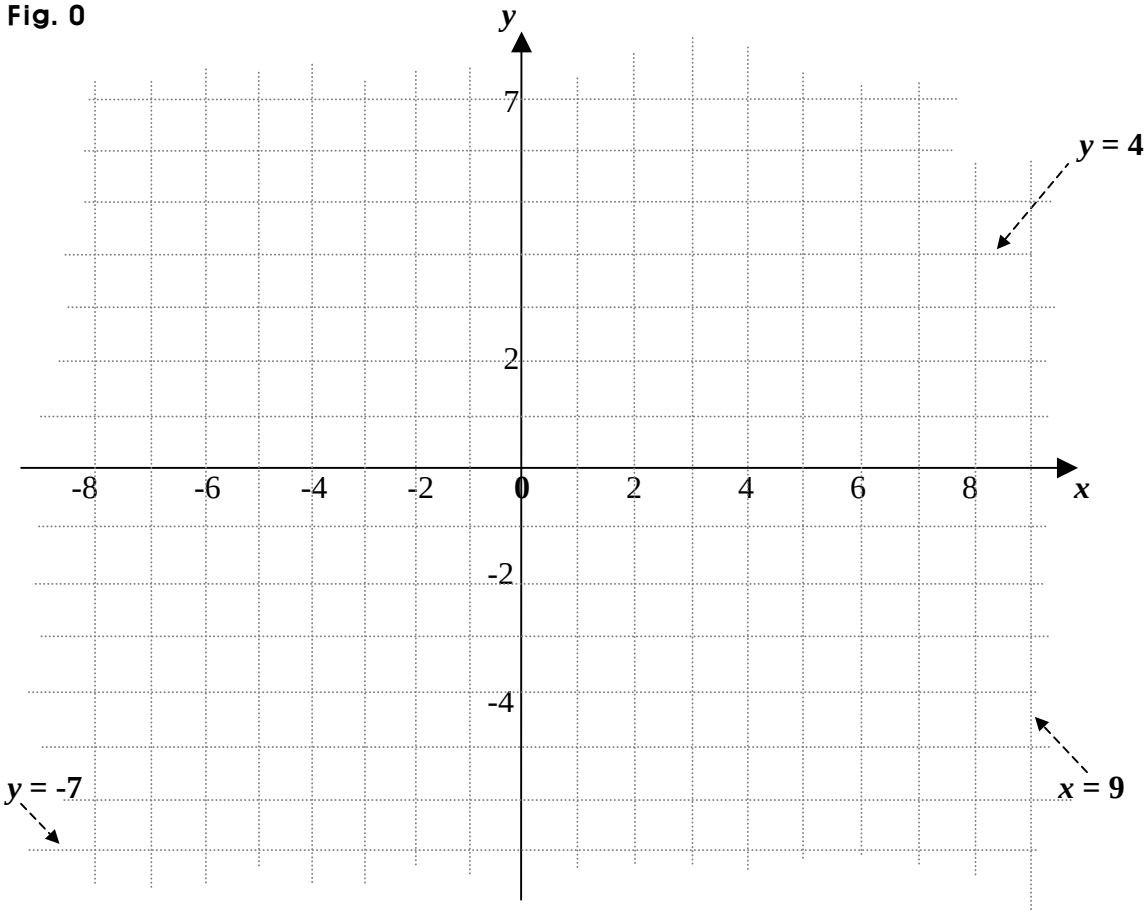
0. $\dfrac{(x+\frac{13}{2})^2}{\frac{9}{4}}+\dfrac{(y-\frac{13}{2})^2}{\frac{25}{4}}=1$ 1. $\dfrac{(x+5)^2}{4}+\dfrac{(y-\frac{11}{2})^2}{\frac{25}{4}}=1$ 2. $\dfrac{x^2}{4}+\dfrac{(y-5)^2}{9}=1$

3. $4(x-\frac{9}{2})^2+(y-7)^2=9$ 4. $196(x-7)^2+100(y-\frac{7}{2})^2=1225$

5. $\dfrac{(x+\frac{9}{2})^2}{25}+\dfrac{(y-\frac{1}{2})^2}{49}=\dfrac{1}{4}$ 6. $25x^2+9y^2=225$ 7. $49(x-3)^2+36(y-\frac{5}{2})^2=441$

8. $\dfrac{(x-5)^2}{9}+\dfrac{4(y+\frac{9}{2})^2}{81}=1.$ 9. $\dfrac{(x-7)^2}{16}+\dfrac{(y+\frac{5}{2})^2}{25}=\dfrac{1}{4}$ A. $\dfrac{(x+4)^2}{36}+\dfrac{(y+\frac{11}{2})^2}{49}=\dfrac{1}{4}$

Fig. 0



Suggestions or Solutions
To the Problems in the Examples

Specify the center and the major and minor radii of each ellipse below, and then, graph it.

0. $\frac{(x+\frac{13}{2})^2}{\frac{9}{4}}+\frac{(y-\frac{13}{2})^2}{\frac{25}{4}}=1$ 1. $\frac{(x+5)^2}{4}+\frac{(y-\frac{11}{2})^2}{\frac{25}{4}}=1$ 2. $\frac{x^2}{4}+\frac{(y-5)^2}{9}=1$

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First, if an ellipse is **vertical**, knowing the center and the major and minor radii, we can get the equation of the ellipse using the standard form, $\frac{(x-u)^2}{a^2}+\frac{(y-v)^2}{b^2}=1,$ where (u, v) is the center, and $b > a$, so b is the major radius, and a is the minor radius.

So beginning with 0, we can get: $\frac{(x+\frac{13}{2})^2}{\frac{9}{4}}+\frac{(y-\frac{13}{2})^2}{\frac{25}{4}}=1\Rightarrow\frac{(x+\frac{13}{2})^2}{(\frac{3}{2})^2}+\frac{(y-\frac{13}{2})^2}{(\frac{5}{2})^2}=1.$

So we can see that the ellipse is vertical, the center is at $(-\frac{13}{2}, \frac{13}{2})$, the major radius is $\frac{5}{2}$, and the minor radius is $\frac{3}{2}$.

1. $\frac{(x+5)^2}{4}+\frac{(y-\frac{11}{2})^2}{\frac{25}{4}}=1\Rightarrow\frac{\{x-(-5)\}^2}{2^2}+\frac{(y-\frac{11}{2})^2}{(\frac{5}{2})^2}=1.$ So we can see that the ellipse is vertical, the center is (-5, $\frac{11}{2}$), the major radius is $\frac{5}{2}$, and the minor radius is 2.

2. $\frac{x^2}{4}+\frac{(y-5)^2}{9}=1.\Rightarrow\frac{(x-0)^2}{2^2}+\frac{(y-5)^2}{3^2}=1.$ So we can see that the ellipse is vertical, the center is (0, 5), the major radius is 3, and the minor radius is 2.

3. $4(x-\frac{9}{2})^2+(y-7)^2=9 \Rightarrow \frac{4(x-\frac{9}{2})^2}{9}+\frac{(y-7)^2}{9}=1 \Rightarrow \frac{(x-\frac{9}{2})^2}{(\frac{3}{2})^2}+\frac{(y-7)^2}{3^2}=1$. So the ellipse is vertical, the center is ($\frac{9}{2}$, 7), the major radius is 3, and the minor radius is $\frac{3}{2}$.

4. $196(x-7)^2+100(y-\frac{7}{2})^2=1225 \Rightarrow \frac{4(x-7)^2}{25}+\frac{4(y-\frac{7}{2})^2}{49}=1 \Rightarrow \frac{(x-7)^2}{(\frac{5}{2})^2}+\frac{(y-\frac{7}{2})^2}{(\frac{7}{2})^2}=1$.

So the ellipse is vertical, the center is (7, $\frac{7}{2}$), the major radius is $\frac{7}{2}$, and the minor radius is $\frac{5}{2}$.

5. $\frac{(x+\frac{9}{2})^2}{25}+\frac{(y-\frac{1}{2})^2}{49}=\frac{1}{4} \Rightarrow \frac{4(x+\frac{9}{2})^2}{25}+\frac{4(y-\frac{1}{2})^2}{49}=1 \Rightarrow \frac{(x+\frac{9}{2})^2}{(\frac{5}{2})^2}+\frac{(y-\frac{1}{2})^2}{(\frac{7}{2})^2}=1$.

So the ellipse is vertical, the center is $(-\frac{9}{2}, \frac{1}{2})$, the major radius is $\frac{7}{2}$, and the minor radius is $\frac{5}{2}$.

6. $25x^2+9y^2=225 \Rightarrow \frac{25x^2}{225}+\frac{9y^2}{225}=\frac{x^2}{9}+\frac{y^2}{25}=\frac{x^2}{3^2}+\frac{y^2}{5^2}=1$.

So the ellipse is vertical, the center is (0, 0), the major radius is 5, and the minor radius is 3.

7. $4(x-3)^2+9(y+4)^2=36 \Rightarrow \frac{(x-3)^2}{9}+\frac{(y+4)^2}{4}=\frac{(x-3)^2}{3^2}+\frac{(y+4)^2}{2^2}=1$. So the ellipse is vertical, the center is (3, -4), the major radius is 3, and the minor radius is 2.

8. $\frac{(x-5)^2}{9}+\frac{4(y+\frac{9}{2})^2}{81}=1 \Rightarrow \frac{(x-5)^2}{9}+\frac{(y+\frac{9}{2})^2}{\frac{81}{4}}=\frac{(x-5)^2}{3^2}+\frac{(y+\frac{9}{2})^2}{(\frac{9}{2})^2}=1$. So the ellipse is vertical, the center is (5, $-\frac{9}{2}$), the major radius is $\frac{9}{2}$, and the minor radius is 3.

9. $\frac{(x-7)^2}{16}+\frac{(y+\frac{5}{2})^2}{25}=\frac{1}{4} \Rightarrow \frac{4(x-7)^2}{16}+\frac{4(y+\frac{5}{2})^2}{25}=1 \Rightarrow \frac{(x-7)^2}{2^2}+\frac{(y+\frac{5}{2})^2}{(\frac{5}{2})^2}=1$.

So the ellipse is vertical, the center is (7, $-\frac{5}{2}$), the major radius is $\frac{5}{2}$, and the minor radius is 2.

A. $\frac{(x+4)^2}{36}+\frac{(y+\frac{11}{2})^2}{49}=\frac{1}{4}\Rightarrow\frac{4(x+4)^2}{36}+\frac{4(y+\frac{11}{2})^2}{49}=1\Rightarrow\frac{(x+4)^2}{3^2}+\frac{(y+\frac{11}{2})^2}{(\frac{7}{2})^2}=1.$

So the ellipse is vertical, the center is $(-4,-\frac{11}{2})$, the major radius is $\frac{7}{2}$, and the minor radius is 3.

And graphing the equations, we get

Fig. 1