Examples 5 in Standard Forms

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Find the center and the main axes of each ellipse below.

0. $\boldsymbol{x^2 + 2x + 2y^2 + 2y - 1 = 0}$

1. $\boldsymbol{36x^2 + 36x + 100y^2 - 300y + 9 = 0}$

2. $\boldsymbol{x^2 + 12x + 9y^2 - 90y + 252 = 0}$

3. $\boldsymbol{2x - 2x^2 + 3y - y^2 + 1 = 0}$

4. $\boldsymbol{25x^2 + 250x + 16y^2 + 16y + 529 = 0}$

5. $\boldsymbol{9x^2 + 4y^2 - 24y = 0}$

Suggestions or Solutions
To the Problems in the Examples

0. $x^2 + 2x + 2y^2 + 2y - 1 = 0 \Rightarrow x^2 + 2x + 1 + 2(y^2 + y) - 1 = 0$

$$\Rightarrow (x+1)^2 + 2\{y^2 + y + (\tfrac{1}{2})^2 - (\tfrac{1}{2})^2\} - 1 = 0 \Rightarrow (x+1)^2 + 2\{y^2 + y + (\tfrac{1}{2})^2\} - \tfrac{1}{2} - 1 = 0$$

$$\Rightarrow (x+1)^2 + 2(y+\tfrac{1}{2})^2 - \tfrac{3}{2} = 0 \Rightarrow (x+1)^2 + 2(y+\tfrac{1}{2})^2 = \tfrac{3}{2} \Rightarrow \frac{(x+1)^2}{\frac{3}{2}} + \frac{2(y+\frac{1}{2})^2}{\frac{3}{2}} = 1$$

$$\Rightarrow \frac{(x+1)^2}{(\sqrt{\frac{3}{2}})^2} + \frac{(y+\frac{1}{2})^2}{\frac{3}{4}} = 1 \Rightarrow \frac{(x+1)^2}{(\sqrt{\frac{3}{2}})^2} + \frac{(y+\frac{1}{2})^2}{(\frac{\sqrt{3}}{2})^2} = 1.$$

So the ellipse is horizontal, the center is $(-1,-\tfrac{1}{2})$, the major radius is $\sqrt{\tfrac{3}{2}}$, and the minor radius is $\tfrac{\sqrt{3}}{2}$.

1. $36x^2 + 36x + 100y^2 - 300y + 9 = 0 \Rightarrow 36(x^2 + x) + 100(y^2 - 3y) + 9 = 0$

$$\Rightarrow 36(x^2 + x + \tfrac{1}{4} - \tfrac{1}{4}) + 100(y^2 - 3y + \tfrac{9}{4} - \tfrac{9}{4}) + 9 = 0$$

$$\Rightarrow 36(x^2 + x + \tfrac{1}{4}) - 9 + 100(y^2 - 3y + \tfrac{9}{4}) - 225 + 9 = 0$$

$$\Rightarrow 36(x+\tfrac{1}{2})^2 + 100(y-\tfrac{3}{2})^2 - 225 = 0 \Rightarrow 36(x+\tfrac{1}{2})^2 + 100(y-\tfrac{3}{2})^2 = 225$$

$$\Rightarrow \frac{36(x+\frac{1}{2})^2}{225} + \frac{100(y-\frac{3}{2})^2}{225} = 1 \Rightarrow \frac{4(x+\frac{1}{2})^2}{25} + \frac{4(y-\frac{3}{2})^2}{9} = 1 \Rightarrow \frac{(x+\frac{1}{2})^2}{\frac{25}{4}} + \frac{(y-\frac{3}{2})^2}{\frac{9}{4}} = 1$$

$$\Rightarrow \frac{(x+\frac{1}{2})^2}{(\frac{5}{2})^2} + \frac{(y-\frac{3}{2})^2}{(\frac{3}{2})^2} = 1.$$

So the ellipse is horizontal, the center is $(-\tfrac{1}{2},\tfrac{3}{2})$, the major radius is $\tfrac{5}{2}$, and the minor radius is $\tfrac{3}{2}$.

2. $x^2 + 12x + 9y^2 - 90y + 252 = 0$

$\Rightarrow x^2 + 12x + 9y^2 - 90y + 252 = x^2 + 12x + 9(y^2 - 10y) + 252$

$= x^2 + 12x + 36 - 36 + 9(y^2 - 10y + 25 - 25) + 252$

$= (x + 6)^2 - 36 + 9(y^2 - 10y + 25) - 225 + 252$

$= (x + 6)^2 + 9(y - 5)^2 - 9 = 0$

$$\Rightarrow \frac{(x+6)^2}{9} + (y-5)^2 = 1 \Rightarrow \frac{\{x-(-6)\}^2}{3^2} + \frac{(y-5)^2}{1^2} = 1.$$

So the ellipse is horizontal, the center is (-6, 5), the major radius is 3, and the minor radius is 1.

3. $2x - 2x^2 + 3y - y^2 + 1 = 0 \Rightarrow 2x^2 - 2x + y^2 - 3y - 1 = 0$

$$\Rightarrow 2x^2 - 2x + y^2 - 3y - 1 = 2(x^2 + x + \tfrac{1}{4} - \tfrac{1}{4}) + (y^2 - 3y + \tfrac{9}{4} - \tfrac{9}{4}) - 1$$

$$= 2(x + \tfrac{1}{2})^2 - \tfrac{1}{2} + (y - \tfrac{3}{2})^2 - \tfrac{9}{4} - 1 = 2(x + \tfrac{1}{2})^2 + (y - \tfrac{3}{2})^2 - \tfrac{15}{4} = 0$$

$$\Rightarrow 2(x + \tfrac{1}{2})^2 + (y - \tfrac{3}{2})^2 = \tfrac{15}{4} \Rightarrow \frac{2(x + \frac{1}{2})^2}{\frac{15}{4}} + \frac{(y - \frac{3}{2})^2}{\frac{15}{4}} = 1 \Rightarrow \frac{(x + \frac{1}{2})^2}{\frac{15}{8}} + \frac{(y - \frac{3}{2})^2}{\frac{15}{4}} = 1$$

$$\Rightarrow \frac{(x + \frac{1}{2})^2}{(\frac{\sqrt{15}}{2\sqrt{2}})^2} + \frac{(y - \frac{3}{2})^2}{(\frac{\sqrt{15}}{2})^2} = 1 \Rightarrow \frac{(x + \frac{1}{2})^2}{(\frac{\sqrt{30}}{4})^2} + \frac{(y - \frac{3}{2})^2}{(\frac{\sqrt{15}}{2})^2} = 1.$$

So the ellipse is vertical, the center is $(-\frac{1}{2}, \frac{3}{2})$, the major radius is $\frac{\sqrt{15}}{2}$, and the minor radius is $\frac{\sqrt{30}}{4}$.

4. $\mathbf{25x^2 + 250x + 16y^2 + 16y + 529 = 0}$

$\Rightarrow \mathbf{25x^2 + 250x + 16y^2 - 16y + 529 = 25(x^2 + 10x) + 16(y^2 - y) + 529}$

$$= 25(x^2 + 10x + 25 - 25) + 16(y^2 - y + \tfrac{1}{4} - \tfrac{1}{4}) + 529$$

$$= 25(x^2 + 10x + 25) - 625 + 16(y^2 - y + \tfrac{1}{4}) - 4 + 529$$

$$= 25(x+5)^2 + 16(y - \tfrac{1}{2})^2 - 100 = 0 \Rightarrow 25(x+5)^2 + 16(y - \tfrac{1}{2})^2 = 100$$

$$\Rightarrow \frac{25(x+5)^2}{100} + \frac{16(y-\frac{1}{2})^2}{100} = 1 \Rightarrow \frac{(x+5)^2}{4} + \frac{4(y-\frac{1}{2})^2}{25} = 1 \Rightarrow \frac{(x+5)^2}{4} + \frac{(y-\frac{1}{2})^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{\{x-(-5)\}^2}{2^2} + \frac{(y-\frac{1}{2})^2}{(\frac{5}{2})^2} = 1.$$

So the ellipse is vertical, the center is $\mathbf{(-5, \tfrac{1}{2})}$, the major radius is $\tfrac{5}{2}$, and the minor radius is 2.

5. $\mathbf{9x^2 + 4y^2 - 24y = 0}$

$\Rightarrow \mathbf{9x^2 + 4y^2 - 24y = 9x^2 + 4(y^2 - 6y) = 9x^2 + 4(y^2 - 6y + 9 - 9)}$

$\mathbf{= 9x^2 + 4(y^2 - 6y + 9) - 36 = 9x^2 + 4(y - 3)^2 - 36 = 0 \Rightarrow 9x^2 + 4(y - 3)^2 = 36}$

$$\Rightarrow \frac{9x^2}{36} + \frac{4(y-3)^2}{36} = 1 \Rightarrow \frac{x^2}{4} + \frac{(y-3)^2}{9} = 1 \Rightarrow \frac{x^2}{2^2} + \frac{(y-3)^2}{3^2} = 1.$$

So the ellipse is vertical, the center is (0, 3), the major radius is 3, and the minor radius is 2.