# Examples 2 in Hyperbolas



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# Examples 2 in Hyperbolas

Assuming in each example below, ***C*** is the center of a hyperbola, ***F*** is a focus, ***V*** is a vertex, ***T*** is half the transverse axis, and ***t*** is half the conjugate axis, find the hyperbola, the eccentricity, the asymptotes, and the directrices, and put in a graph the hyperbola, together with the foci, vertices, asymptotes, and directrices.

0. ***C*****(0, 0),** ***F*****(0, 3)**, and ***t*** **= 2**.

1. ***C*****(0, 1)**, ***F*****(-2, 1)**, and ***T*** **= 1**.

2. ***C*****(1, 1)**, ***F*****(1, 6)**, and ***t*** **= 4**.

## Suggestions or Solutions
## To the Problem in the Example 0

**Assuming $C(0, 0)$ is the center of a hyperbola, $F(0, 3)$ is a focus, and 2 is half the conjugate axis, find the hyperbola, and its elements, and put them all in a graph.**

To begin with, the center is (0, 0), one focus is (0, 3), and the $x$-coordinates are the same.
So the hyperbola is vertical, the other focus is (0, -3), and assuming $c$ is the focal distance, we get $c = 3$. Then, assuming $a$ is half the conjugate axis, and $b$ is half the transverse axis, we get

$a = 2$, and $c^2 = a^2 + b^2 \Rightarrow 3^2 = 2^2 + b^2 \Rightarrow b^2 = 5$.

So the transverse axis is $2\sqrt{5}$, the conjugate axis is $4$, and the hyperbola is $\dfrac{y^2}{5} - \dfrac{x^2}{4} = 1$.

So the vertices are $(0, \sqrt{5})$ and $(0, -\sqrt{5})$, and the asymptotes are $y = \pm(b/a)x = \pm\frac{\sqrt{5}}{2}x$.

Next, assuming $e$ is the eccentricity, we get $e = c/b = \frac{3\sqrt{5}}{5}$.

And next, the directrices are $y = \pm b/e = \pm b^2/c = \pm 5/3$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

To begin with, the hyperbola we want to find is centered at the origin. And if it is horizontal, the equation is $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. If it is vertical, the equation is $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$.
So if finding if the hyperbola is horizontal or vertical, and the values of $a$ and $b$, we find the hyperbola. How then, can we get them?

To begin with, if the hyperbola is <u>horizontal</u>, <u>the coefficient of $x^2$ is positive</u>.
Then, <u>$a$ is half the transverse axis</u>, and $b$ is half the conjugate axis.

If it is <u>vertical</u> however, <u>the coefficient of $y^2$ is positive.</u>
Then, $a$ is half the conjugate axis, and <u>$b$ is half the transverse axis</u>.

Next, the center is (0, 0), and one of the foci is (0, 3).
So we can notice that <u>the center and the foci share the same $x$-coordinate</u>, which is 0.
We can see thus, the hyperbola is <u>vertical</u>.

So first, assuming the other focus is (0, $\boldsymbol{q}$), since the center is (0, 0), and is the midpoint between the foci, we get 0 = ($\boldsymbol{q}$ + 3)/2. The other focus is thus, (0, -3).

Next, assuming $\boldsymbol{c}$ is the focal distance, we get $\boldsymbol{c = 3}$, because the focal distance is the distance from the center to a focus.

Next, we can say that $\boldsymbol{a}$ is half the conjugate axis, and thus, is 2.
So the conjugate axis is 4. What then, about $\boldsymbol{b}$?

We have $\boldsymbol{c^2 = a^2 + b^2}$. So we get $\mathbf{3^2 = 2^2} + \boldsymbol{b^2} \Rightarrow \boldsymbol{b^2} = \mathbf{9 - 4 = 5}$.

And $\boldsymbol{b}$ is half the transverse axis, so the transverse axis is $\mathbf{2\sqrt{5}}$.

So the hyperbola is $\dfrac{\boldsymbol{y}^2}{\mathbf{5}} - \dfrac{\boldsymbol{x}^2}{\mathbf{4}} = \mathbf{1}$, which is often put this way, of course: $\dfrac{\boldsymbol{y}^2}{(\sqrt{\mathbf{5}})^2} - \dfrac{\boldsymbol{x}^2}{\mathbf{2}^2} = \mathbf{1}$.

Next, if the hyperbola is <u>vertical</u>, the center and vertices share the <u>same $\boldsymbol{x}$-coordinate</u>, which is 0 in this case.
And the center is the midpoint between the <u>vertices</u>, too, which are the <u>endpoints</u> of the <u>transverse axis</u>, which is $\mathbf{2}\boldsymbol{b}$, and is $\mathbf{2\sqrt{5}}$.
And $\boldsymbol{b}$ is half the transverse axis, which is the distance from a vertex to the center.
So since the center is (0, 0), the two vertices are $\mathbf{(0, \sqrt{5})}$ and $\mathbf{(0, -\sqrt{5})}$.

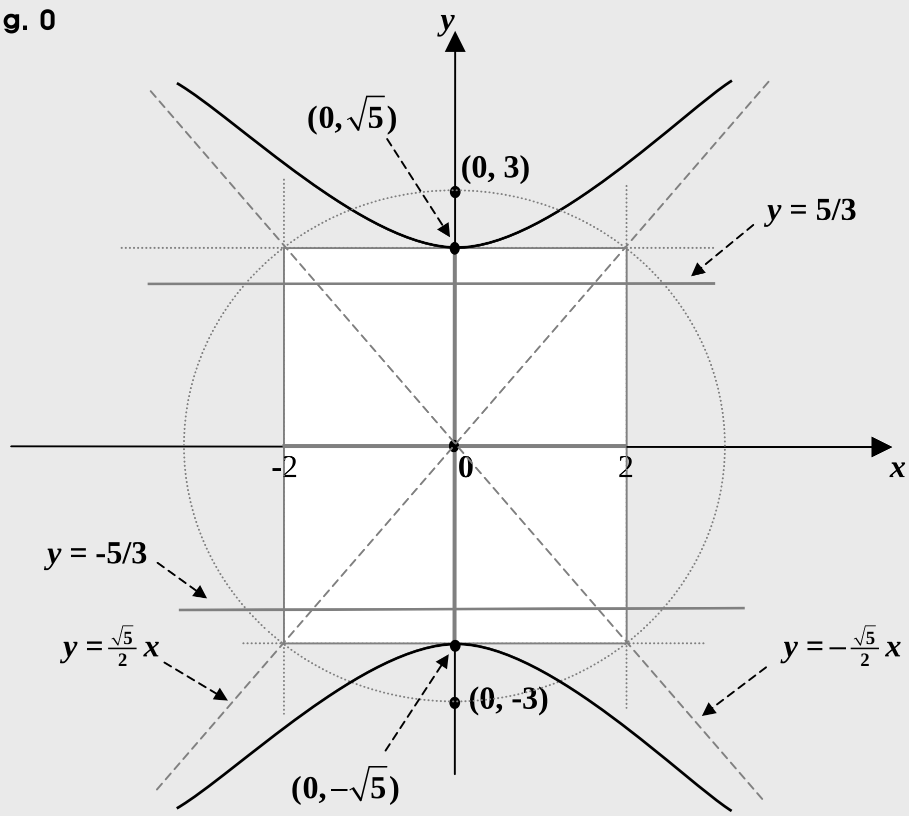
And next, a hyperbola has two lines called the <u>asymptotes</u>, the slope of one is $\boldsymbol{b/a}$, and the other is $\boldsymbol{-b/a}$. And the asymptotes pass through the center, which is (0, 0) in this case.
So the asymptotes are $\boldsymbol{y} = \pm(\boldsymbol{b/a})\boldsymbol{x} = \pm\frac{\sqrt{5}}{2}\boldsymbol{x}$.

Next, the <u>eccentricity</u> of a hyperbola is a ratio, <u>the focal distance over half the transverse</u>.
So assuming $\boldsymbol{e}$ is the eccentricity, we get $\boldsymbol{e = c/b} = \frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$.

Next, a hyperbola has two lines called the <u>directrices</u>, and the distance from each to the center is a ratio, which is <u>half the transverse over the eccentricity</u>.

So since the center is (0, 0), and the hyperbola is vertical, the directrices are
$\boldsymbol{y = \pm b/e = \pm b^2/c} = \mathbf{\pm 5/3}$.

Fig. 0




The hyperbola is $\frac{y^2}{5} - \frac{x^2}{4} = 1$, often put this way, too, of course: $\frac{y^2}{(\sqrt{5})^2} - \frac{x^2}{2^2} = 1$.

Note that half the transverse axis is called the semi major axis, too, and half the conjugate axis is called the semi minor axis.

## Suggestions or Solutions
## To the Problem in the Example 1

**Assuming *C*(0, 1) is the center of a hyperbola, *F*(-2, 1) is a focus, and 1 is half the transverse axis, find the hyperbola, and its elements, and put them all in a graph.**

To begin with, the center is (0, 1), a focus is (-2, 1), and the $y$-coordinates are the same. So the hyperbola is horizontal, the other focus is (2, 1), and assuming $c$ is the focal distance, we get $c = 2$. Then, assuming $a$ is half the transverse axis, and $b$ is half the conjugate axis, we get

$a = 1$, and $c^2 = a^2 + b^2 \Rightarrow 2^2 = 1^2 + b^2 \Rightarrow b^2 = 3$. So the transverse axis is **2**, the conjugate axis is $2\sqrt{3}$, and the hyperbola is $x^2 - \frac{(y-1)^2}{3} = 1$.

So the vertices are (-1, 1) and (1, 1), and the asymptotes are $y = \pm\sqrt{3}x + 1$.

Next, assuming $e$ is the eccentricity, we get $e = c/a = 2$.

And next, the directrices are $x = \pm a/e = \pm a^2/c = \pm 1/2$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

To begin with, the hyperbola we want to find is centered at (0, 1). And if it is horizontal, the equation is $\frac{x^2}{a^2} - \frac{(y-1)^2}{b^2} = 1$. If it is vertical, the equation is $\frac{(y-1)^2}{b^2} - \frac{x^2}{a^2} = 1$.

So if finding if the hyperbola is horizontal or vertical, and the values of $a$ and $b$, we find the hyperbola. How then, can we get them?

To begin with, if the hyperbola is horizontal, the coefficient of $x^2$ is positive.
Then, $a$ is half the transverse axis, and $b$ is half the conjugate axis.

If it is vertical however, the coefficient of $y^2$ is positive.
Then, $a$ is half the conjugate axis, and $b$ is half the transverse axis.

Next, the center is (0, 1), and one of the foci is (-2, 1).

So we can notice that the center and the foci share the same $y$-coordinate, which is 1.

We can see thus, the hyperbola is horizontal.

So first, assuming the other focus is (***p***, 1), since the center is (0, 1), and is the midpoint between the foci, we get 0 = {***p*** + (-2)}/2. The other focus is thus, (2, 1).

Next, assuming ***c*** is the focal distance, we get ***c* = 2**, because the focal distance is the distance from the center to a focus.

Next, we can say that ***a*** is half the transverse axis, and thus, is 1.
So the transverse axis is 2. What then, about ***b***?

We have $c^2 = a^2 + b^2$. So we get $2^2 = 1^2 + b^2 \Rightarrow b^2 = 4 - 1 = 3$.

And ***b*** is half the conjugate axis, so the conjugate axis is $2\sqrt{3}$.

So the hyperbola is $x^2 - \frac{y^2}{3} = 1$, which is often put this way, of course: $\frac{x^2}{1^2} - \frac{y^2}{(\sqrt{3})^2} = 1$.

Next, if the hyperbola is horizontal, the center and vertices share the same ***y***-coordinate, which is 1 in this case.
And the center is the midpoint between the vertices, too, which are the endpoints of the transverse axis, which is **2*a***, and is 2.
And ***a*** is half the transverse axis, which is the distance from a vertex to the center.
So since the center is (0, 1), the two vertices are (-1, 1) and (1, 1).

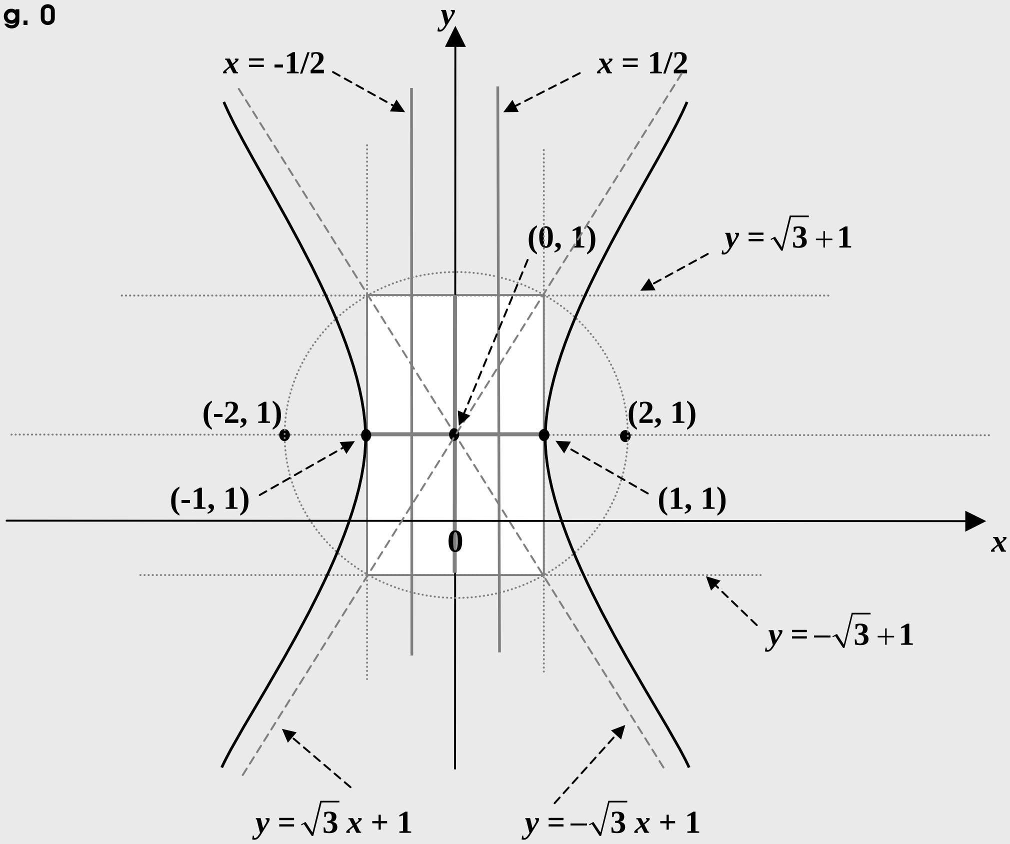
And next, a hyperbola has two lines called the asymptotes, the slope of one is ***b*/*a***, and the other is **–*b*/*a***. And the asymptotes pass through the center, which is (0, 1) in this case.
And a line of slope ***m*** passing through **(*s*, *t*)** is $y - t = m(x - s)$.
So getting the asymptotes, we get $y - 1 = \pm(b/a)x = \pm\frac{\sqrt{3}}{1}x = \pm\sqrt{3}x \Rightarrow y = \pm\sqrt{3}x + 1$.

Next, the eccentricity of a hyperbola is a ratio, the focal distance over half the transverse.
So assuming ***e*** is the eccentricity, we get $e = c/a = 2/1 = 2$.

Next, a hyperbola has two lines called the directrices, and the distance from each to the center is the value of a ratio, which is half the transverse over the eccentricity.

So since the center is (0, 1), and the hyperbola is horizontal, the directrices are
$x = \pm a/e = \pm a^2/c = \pm 1/2$.

Fig. 0




The hyperbola is $x^2 - \frac{(y-1)^2}{3} = 1$, often put this way, too, of course: $\frac{x^2}{1^2} - \frac{(y-1)^2}{(\sqrt{3})^2} = 1$.

Note that half the transverse axis is called the semi major axis, too, and half the conjugate axis is called the semi minor axis.

## Suggestions or Solutions
## To the Problem in the Example 2

**Assuming *C*(1, 1) is the center of a hyperbola, *F*(1, 6) is a focus, and 4 is half the conjugate axis, find the hyperbola, and its elements, and put them all in a graph.**

To begin with, the center is (1, 1), one focus is (1, 6), and the ***x***-coordinates are the same. So the hyperbola is vertical, the other focus is (1, -4), and assuming ***c*** is the focal distance, we get $c = 5$. Then, assuming ***a*** is half the conjugate axis, and ***b*** is half the transverse axis, we get

$a = 4$, and $c^2 = a^2 + b^2 \Rightarrow 5^2 = 4^2 + b^2 \Rightarrow b^2 = 3^2$. So the transverse axis is **6**, the conjugate axis is **8**, and the hyperbola is $\dfrac{(y-1)^2}{9} - \dfrac{(x-1)^2}{16} = 1$.

So the vertices are (1, 4) and (1, -2), and the asymptotes are $y - 1 = \pm(3/4)(x - 1)$.

Next, assuming ***e*** is the eccentricity, we get $e = c/b = 5/4$.

And next, the directrices are $y = \pm b/e + 1 = \pm b^2/c + 1 = \pm 9/5 + 1$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

To begin with, the hyperbola we want to find is centered at (1, 1). And if it is horizontal, the equation is $\dfrac{(x-1)^2}{a^2} - \dfrac{(y-1)^2}{b^2} = 1$. If vertical, the equation is $\dfrac{(y-1)^2}{b^2} - \dfrac{(x-1)^2}{a^2} = 1$.
So if finding if the hyperbola is horizontal or vertical, and the values of ***a*** and ***b***, we find the hyperbola. How then, can we get them?

To begin with, if the hyperbola is horizontal, the coefficient of $x^2$ is positive.
Then, ***a*** is half the transverse axis, and ***b*** is half the conjugate axis.

If it is vertical however, the coefficient of $y^2$ is positive.
Then, ***a*** is half the conjugate axis, and ***b*** is half the transverse axis.

Next, the center is (1, 1), and one of the foci is (1, 6).
So we can notice that the center and the foci share the same ***x***-coordinate, which is 1.
We can see thus, the hyperbola is vertical.

So first, assuming the other focus is (1, $\boldsymbol{q}$), since the center is (1, 1), and is the midpoint between the foci, we get 1 = ($\boldsymbol{q}$ + 6)/2.  The other focus is thus, (1, -4).

Next, assuming $\boldsymbol{c}$ is the focal distance, we get $\boldsymbol{c = 5}$, because the focal distance is the distance from the center to a focus.

Next, we can say that $\boldsymbol{a}$ is half the conjugate axis, and thus, is 4.
So the conjugate axis is 8.  What then, about $\boldsymbol{b}$?

We have $\boldsymbol{c^2 = a^2 + b^2}$.  So we get $\mathbf{5^2 = 4^2} + \boldsymbol{b^2} \Rightarrow \boldsymbol{b^2} \mathbf{= 25 - 16 = 9}$.

And $\boldsymbol{b}$ is half the transverse axis, so the transverse axis is 6.

So the hyperbola is $\dfrac{(y-1)^2}{9} - \dfrac{(x-1)^2}{16} = 1$, often put this way: $\dfrac{(y-1)^2}{3^2} - \dfrac{(x-1)^2}{4^2} = 1$.

Next, if the hyperbola is <u>vertical</u>, the center and vertices share the <u>same ***x***-coordinate</u>, which is 1 in this case.
And the center is the midpoint between the <u>vertices</u>, too, which are the <u>endpoints</u> of the <u>transverse axis</u>, which is $\mathbf{2}\boldsymbol{b}$, and is 6.
And $\boldsymbol{b}$ is half the transverse axis, which is the distance from a vertex to the center.
So since the center is (1, 1), the two vertices are (1, 4) and (1, -2)

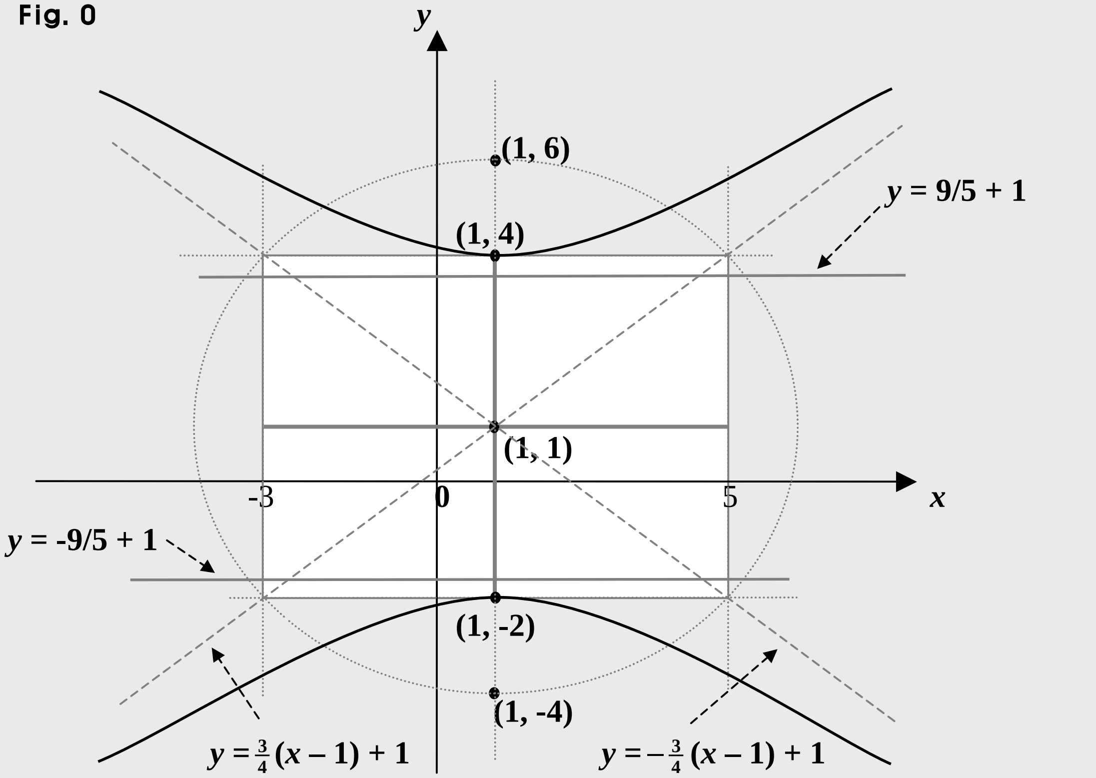
And next, a hyperbola has two lines called the <u>asymptotes</u>, the slope of one is $\boldsymbol{b/a}$, and the other is $\boldsymbol{-b/a}$. And the asymptotes pass through the center, which is (1, 1) in this case.
And a line of slope $\boldsymbol{m}$ passing through $\boldsymbol{(s, t)}$ is $\boldsymbol{y - t = m(x - s)}$.
So getting the asymptotes, we get $\boldsymbol{y - 1 = \pm(b/a)(x - 1) \Rightarrow y - 1 = \pm(3/4)(x - 1)}$.

Next, the <u>eccentricity</u> of a hyperbola is a ratio, <u>the focal distance over half the transverse</u>.
So assuming $\boldsymbol{e}$ is the eccentricity, we get $\boldsymbol{e = c/b = 5/3}$.

Next, a hyperbola has two lines called the <u>directrices</u>, and the distance from each to the center is a ratio, which is <u>half the transverse over the eccentricity</u>.

So since the center is (1, 1), and the hyperbola is vertical, the directrices are
$\boldsymbol{y = \pm b/e + 1 = \pm b^2/c + 1 = \pm 5/3 + 1}$.

Fig. 0




The hyperbola is $\frac{y^2}{5} - \frac{x^2}{4} = \mathbf{1}$, often put this way, too, of course: $\frac{y^2}{(\sqrt{5})^2} - \frac{x^2}{2^2} = \mathbf{1}$.

Note that half the transverse axis is called the semi major axis, too, and half the conjugate axis is called the semi minor axis.