Examples 2 in Standard Forms

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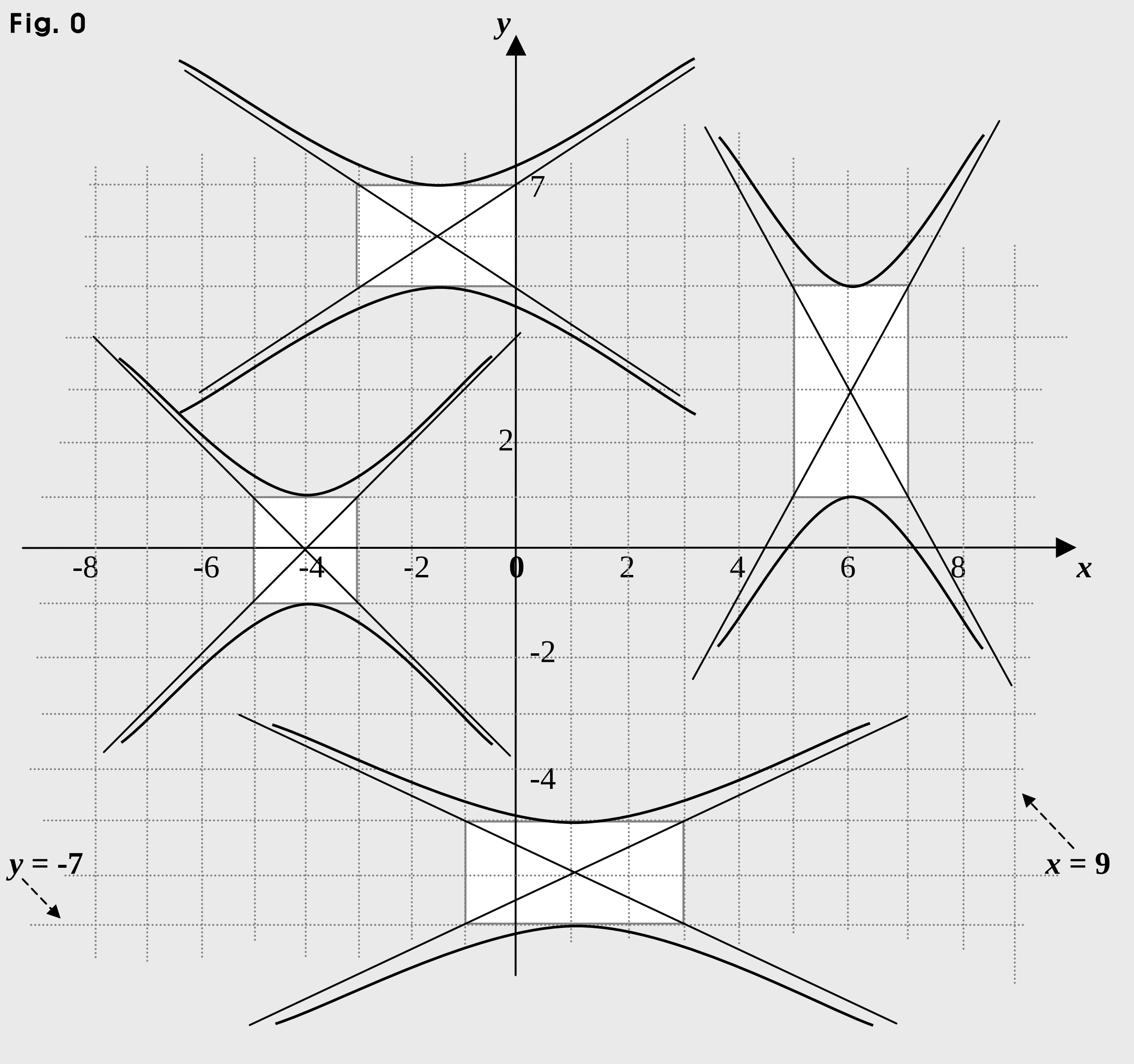
Label each hyperbola below, and then find the equation of each hyperbola.

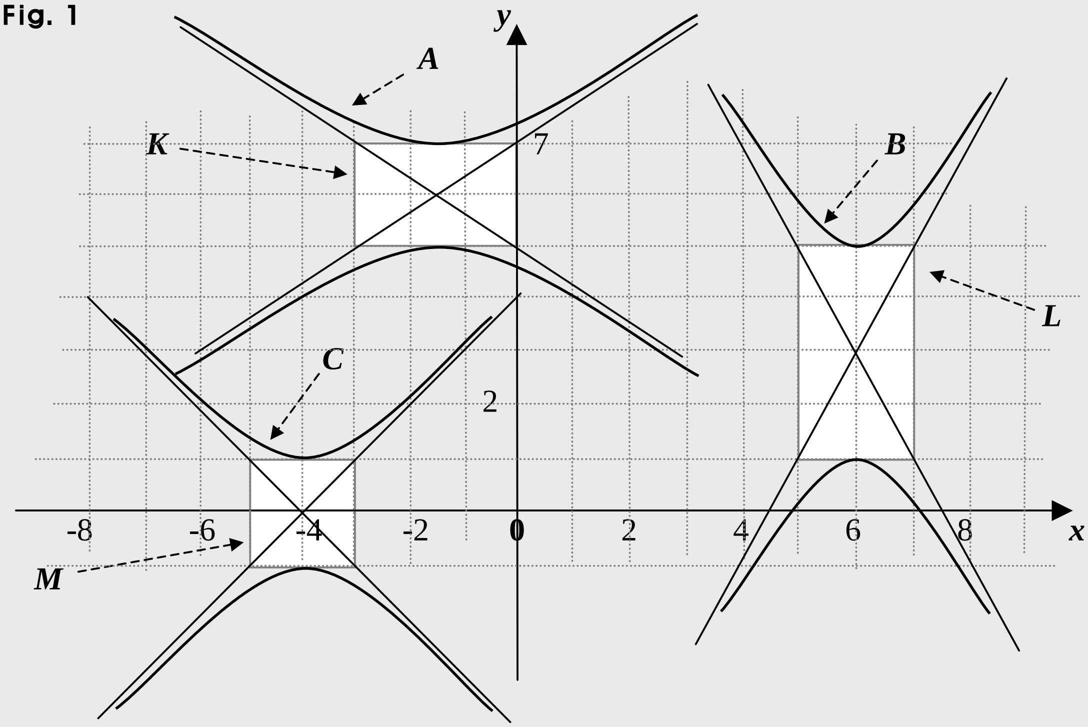
Fig. 0



Suggestions or Solutions
To the Problems in the Examples

The hyperbolas are all vertical, and beginning with the hyperbola ***A*** below, we can see first, the center is $(-\frac{3}{2}, 6)$, and the rectangle ***K*** is 3 by 2.

Fig. 1



The rectangle is tangent to the hyperbola at the vertices, and the diagonals overlap the two asymptotes. And finding the equation of the hyperbola, we can use <u>half the width and half the height</u> of the rectangle. It's because of the fact below.

If a hyperbola is horizontal, the equation is $\dfrac{(x-u)^2}{a^2}-\dfrac{(y-v)^2}{b^2}=1$, and if vertical, the equation is $\dfrac{(y-v)^2}{b^2}-\dfrac{(x-u)^2}{a^2}=1$.

Then (u, v) is the center of the hyperbola, ***a*** is half the width (horizontal length) of the rectangle, and ***b*** is the half the height (vertical length).

And thus, beginning with the hyperbola ***A***, we can see the center is $(-\frac{3}{2}, 6)$, and of the rectangle ***K***, the width is 3, and the height is 2. So we get $\boldsymbol{u = -\frac{3}{2}}$, $\boldsymbol{v = 6}$, $\boldsymbol{a = \frac{3}{2}}$, and $\boldsymbol{b = 1}$.

Thus, we get $\dfrac{(y-6)^2}{1^2} - \dfrac{\{x-(-\frac{3}{2})\}^2}{(\frac{3}{2})^2} = 1 \Rightarrow \dfrac{(y-6)^2}{1^2} - \dfrac{(x+\frac{3}{2})^2}{(\frac{3}{2})^2} = 1$, which is the equation of the hyperbola ***A***. And we can put it this way, too, of course: $(y-6)^2 - \dfrac{(x+\frac{3}{2})^2}{\frac{9}{4}} = 1$.

And this way, as well: $(y-6)^2 - \dfrac{4(x+\frac{3}{2})^2}{9} = 1$ or $9(y-6)^2 - 4(x+\frac{3}{2})^2 = 9$.

Next, moving on to the hyperbola ***B***, we can see the center is (6, 3), and of the rectangle ***L***, the width is 2, and the height is 4. So we get $\boldsymbol{u = 6}$, $\boldsymbol{v = 3}$, $\boldsymbol{a = 1}$, and $\boldsymbol{b = 2}$.

Thus, we get $\dfrac{(y-3)^2}{2^2} - \dfrac{(x-6)^2}{1^2} = 1$, which is the equation of the hyperbola ***B***.

And we can put it this way, too: $\dfrac{(y-3)^2}{4} - (x-6)^2 = 1$ or $(y-3)^2 - 4(x-6)^2 = 4$.

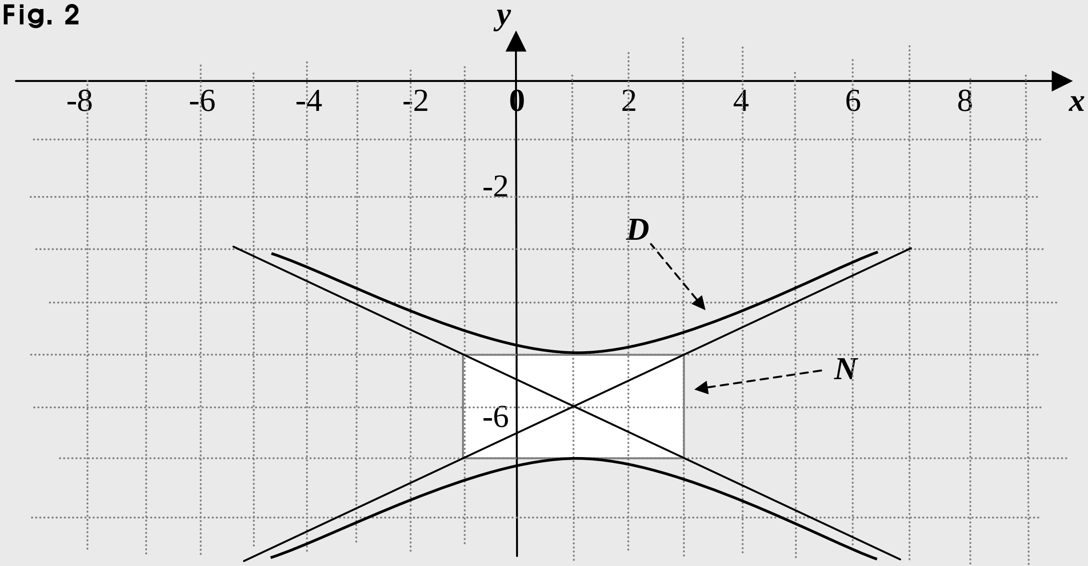
Moving next, on to the hyperbola ***C***, we can see the center is (-4, 0), and of the rectangle ***M***, the width is 2, and the height is 2. So we get $\boldsymbol{u = -4}$, $\boldsymbol{v = 0}$, $\boldsymbol{a = 1}$, and $\boldsymbol{b = 1}$.

Thus, we get $\dfrac{(y-0)^2}{1^2} - \dfrac{\{x-(-4)\}^2}{1^2} = 1 \Rightarrow \dfrac{y^2}{1^2} - \dfrac{(x+4)^2}{1^2} = 1$, which is the equation of the hyperbola ***C***.

And of course, we can put it this way, too: $y^2 - (x+4)^2 = 4$.

And next, moving on to the other hyperbola, we have

Fig. 2



Then, we can see the center of hyperbola ***D*** is (1, -6), and of the rectangle ***N***, the width is 4, and the height is 2. So we get $\boldsymbol{u = 1}$, $\boldsymbol{v = -6}$, $\boldsymbol{a = 2}$, and $\boldsymbol{b = 1}$.

Thus, we get $\frac{(y+6)^2}{1^2} - \frac{(x-1)^2}{2^2} = 1$, which is the equation of the hyperbola ***D***.

And we can put it this way, too: $(y+6)^2 - \frac{(x-1)^2}{4} = 1$ or $4(y + 6)^2 - (x - 1)^2 = 4$.