Examples 3 in Standard Forms

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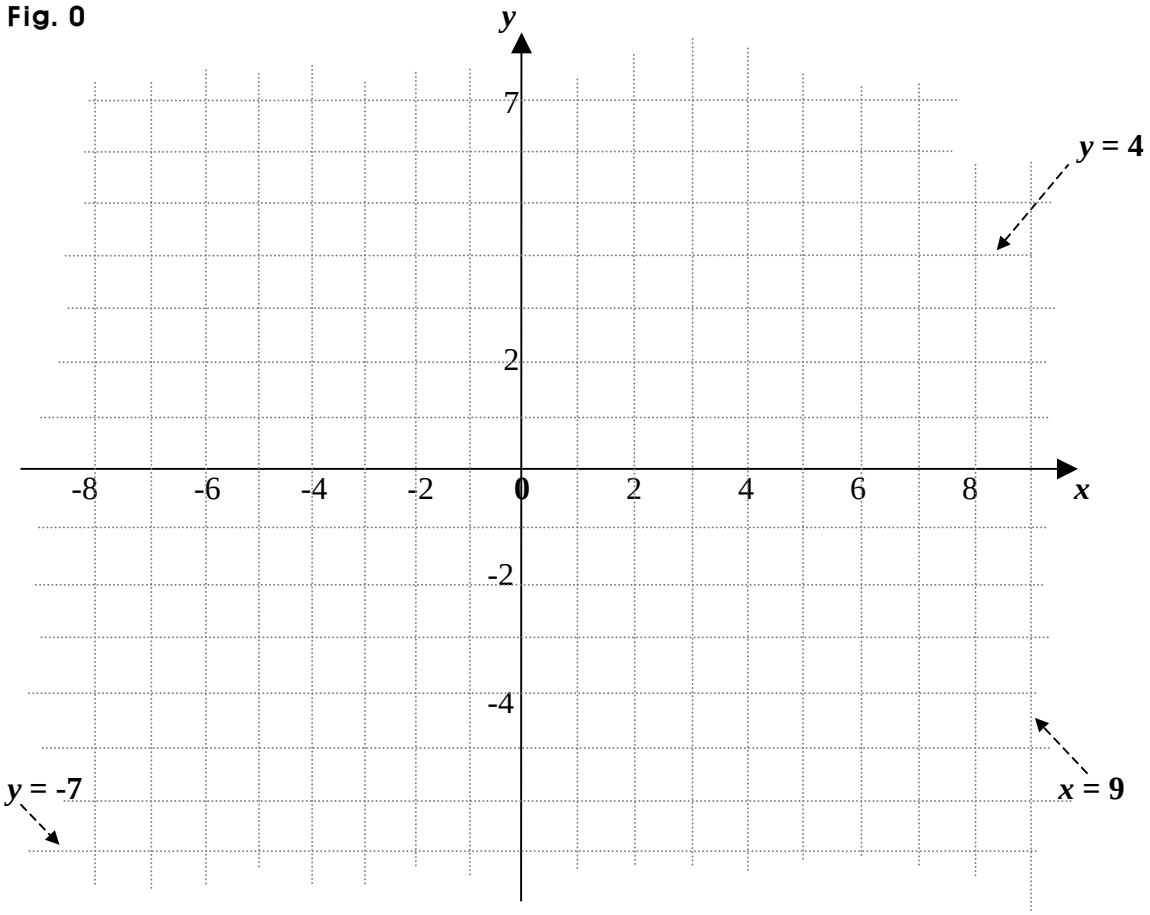
Specify the center and the two main axes of each hyperbola below, and then, graph it.

0. $\frac{(x+3)^2}{9}-(y-5)^2=1$ 1. $\frac{(x+1)^2}{9}-\frac{(y-5)^2}{4}=1$ 2. $(x-6)^2-4(y-\frac{7}{2})^2=9$

3. $4(x+5)^2-9y^2=36$ 4. $9x^2-16y^2=144$ 5. $4(x-3)^2-9(y+4)^2=36$

6. $9(x-7)^2-16(y+\frac{5}{2})^2=36$ 7. $(x+4)^2-4(y+\frac{11}{2})^2=9$

Fig. 0



Suggestions or Solutions
To the Problems in the Examples

Specify the center, the transverse axis, and the conjugate axis of each hyperbola below, and then, graph it.

0. $\frac{(x+3)^2}{9}-(y-5)^2=1$ 1. $\frac{(x+1)^2}{9}-\frac{(y-5)^2}{4}=1$ 2. $(x-6)^2-4(y-\frac{7}{2})^2=9$

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First, we have a fact as follows.

If a hyperbola centered at **(*u*, *v*)** is horizontal, the equation is $\frac{(x-u)^2}{a^2}-\frac{(y-v)^2}{b^2}=1$, where ***a*** is half the transverse axis, and ***b*** is half the conjugate axis, and if vertical, the equation is $\frac{(y-v)^2}{b^2}-\frac{(x-u)^2}{a^2}=1$, where ***b*** is half the transverse axis, and ***a*** is half the conjugate axis.

In either case though, the rectangle is ***a*** by ***b***, ***a*** is the width (horizontal length) of the rectangle, and ***b*** is the height (vertical length).

So beginning with 0, we can get $\frac{(x+3)^2}{9}-(y-5)^2=1\Rightarrow\frac{\{x-(-3)\}^2}{3^3}-\frac{(y-5)^2}{1^2}=1.$
So we can see that the hyperbola is horizontal, the center is at (-3, 5), the transverse axis is 6, and the conjugate axis is 2. And its rectangle is centered at (-3, 5), too, and is 6 by 2, that is, the horizontal length of the rectangle is 6, and the vertical length is 2.

1. $\frac{(x+1)^2}{9}-\frac{(y-5)^2}{4}=1\Rightarrow\frac{\{x-(-1)\}^2}{3^2}-\frac{(y-5)^2}{2^2}=1.$ So we can see that the hyperbola is horizontal, the center is (-1, 5), the transverse axis is 6, and the conjugate axis is 4. And the rectangle is 6 by 4, that is, the width is 6, and the height is 4.

2. $(x-6)^2-4(y-\frac{7}{2})^2=9 \Rightarrow \frac{(x-6)^2}{9}-\frac{4(y-\frac{7}{2})^2}{9}=1 \Rightarrow \frac{(x-6)^2}{3^2}-\frac{(y-\frac{7}{2})^2}{(\frac{3}{2})^2}=1.$

So the hyperbola is horizontal, the center is (-1, $\frac{7}{2}$), the transverse axis is 6, and the conjugate axis is 3. And the rectangle is 6 by 3, that is, the width is 6, and the height is 3.

3. $4(x+5)^2-9y^2=36 \Rightarrow \frac{4(x+5)^2}{36}-\frac{9y^2}{36}=1 \Rightarrow \frac{(x+5)^2}{9}-\frac{y^2}{4}=1 \Rightarrow \frac{(x+5)^2}{3^2}-\frac{y^2}{2^2}=1.$

So the hyperbola is horizontal, the center is (-5, 0), the transverse axis is 6, and the conjugate axis is 4. And the rectangle is 6 by 4, that is, the width is 6, and the height is 4.

4. $9x^2-16y^2=144 \Rightarrow \frac{9x^2}{144}-\frac{16y^2}{144}=\frac{x^2}{16}-\frac{y^2}{9}=\frac{x^2}{4^2}-\frac{y^2}{3^2}=1.$

So the hyperbola is horizontal, the center is (0, 0), the transverse axis is 8, and the conjugate axis is 6. And the rectangle is 8 by 6, that is, the width is 8, and the height is 6.

5. $4(x-3)^2-9(y+4)^2=36 \Rightarrow \frac{(x-3)^2}{9}-\frac{(y+4)^2}{4}=\frac{(x-3)^2}{3^2}-\frac{(y+4)^2}{2^2}=1.$

So the hyperbola is horizontal, the center is (3, -4), the transverse axis is 6, and the conjugate axis is 4. And the rectangle is 6 by 4, that is, the width is 6, and the height is 4.

6. $9(x-7)^2-16(y+\frac{5}{2})^2=36 \Rightarrow \frac{(x-7)^2}{4}-\frac{4(y+\frac{5}{2})^2}{9}=\frac{(x-7)^2}{2^2}-\frac{(y+\frac{5}{2})^2}{(\frac{3}{2})^2}=1.$

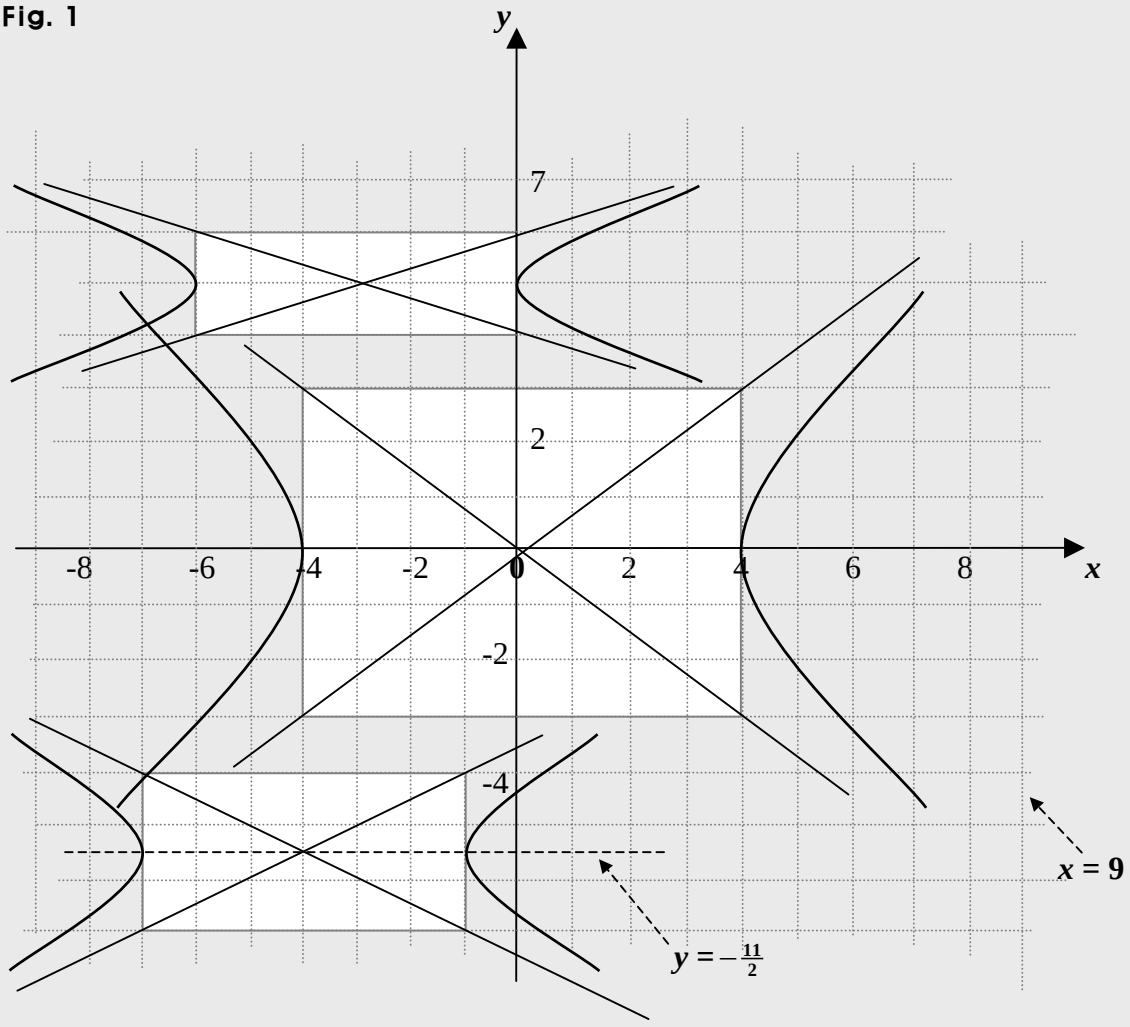
So the hyperbola is horizontal, the center is (7, $-\frac{5}{2}$), the transverse axis is 4, and the conjugate axis is 3. And the rectangle is 4 by 3, that is, the width is 4, and the height is 3.

7. $(x+4)^2-4(y+\frac{11}{2})^2=9 \Rightarrow \frac{(x+4)^2}{9}-\frac{4(y+\frac{11}{2})^2}{9}=\frac{(x+4)^2}{3^2}-\frac{(y+\frac{11}{2})^2}{(\frac{3}{2})^2}=1.$

So the hyperbola is horizontal, the center is (–4, $-\frac{11}{2}$), the transverse axis is 6, and the conjugate axis is 3. And the rectangle is 6 by 3, that is, the width is 6, and the height is 3.

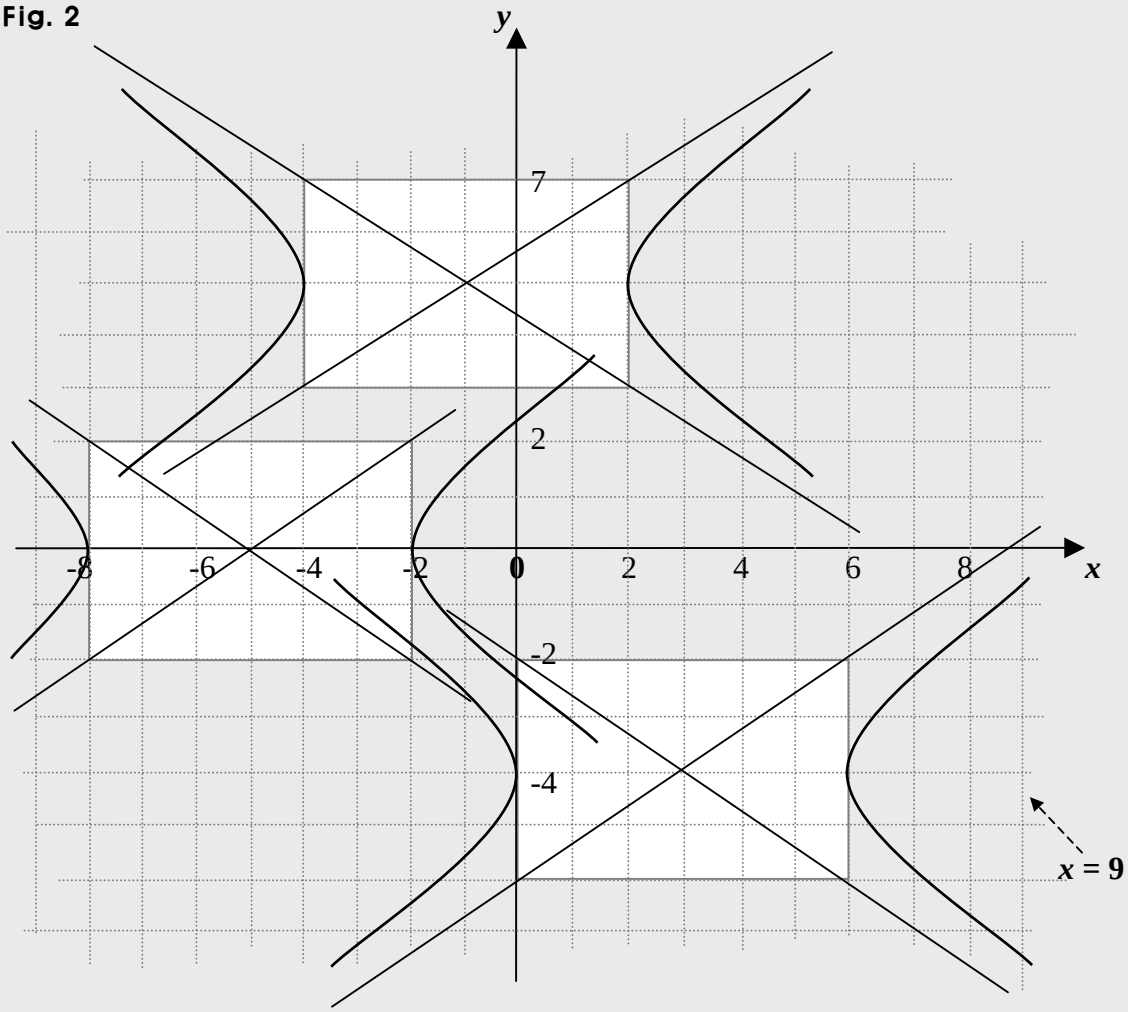
And graphing the equations, we get

Fig. 1



There are more hyperbolas in the next page.

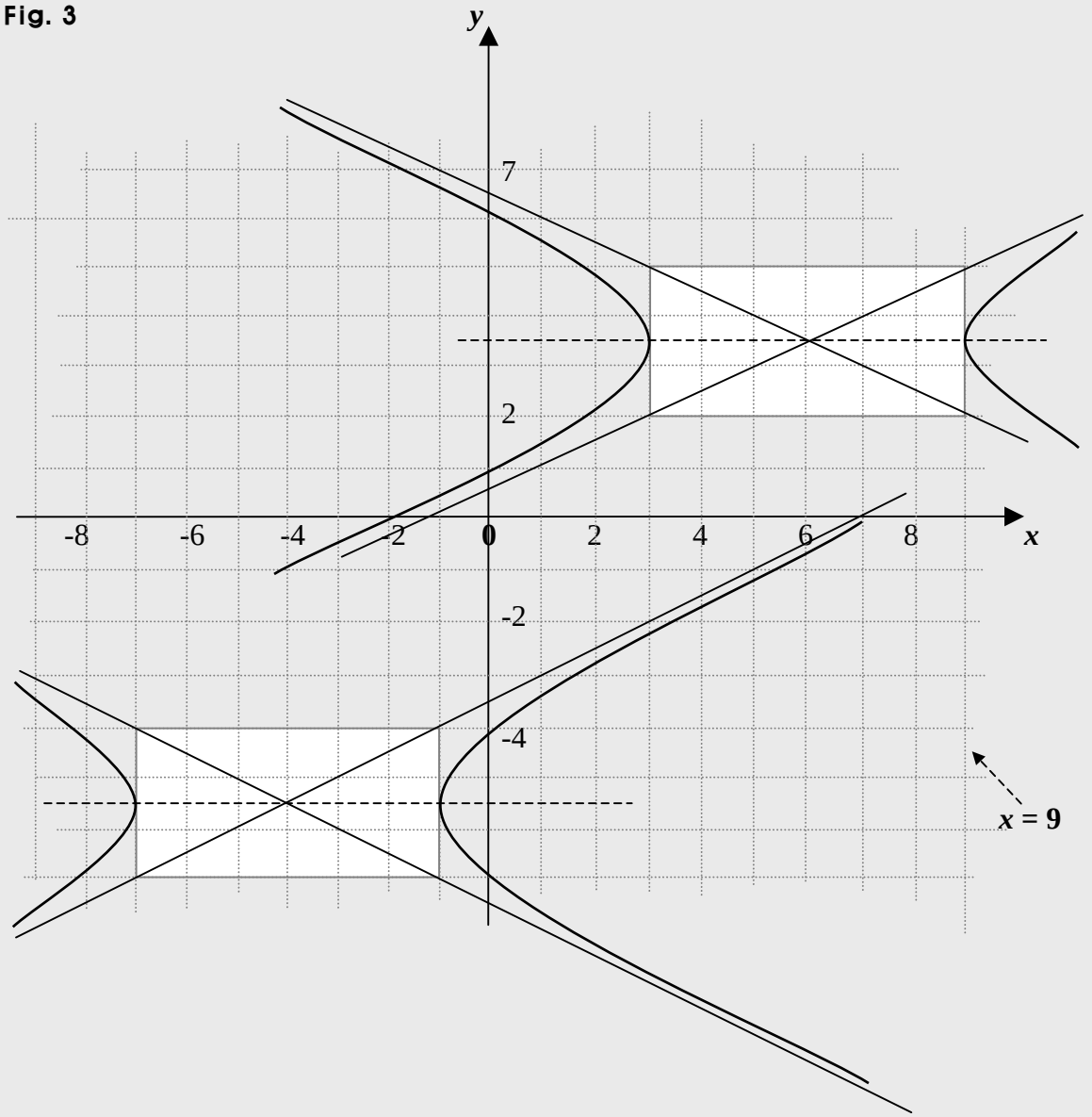
Fig. 2



All the hyperbolas themselves above are actually the same, but have different positions in the graph, so they are considered to be different hyperbolas, and thus in math, have different equations.

And the next page has the other hyperbolas.

Fig. 3



Hyperbolas are symmetric about its center, the conjugate axis, and the transverse axis, so each hyperbola above is shown just in part and not as a whole.

In fact, we cannot show a hyperbola entirely, because its length is infinite.

And although the hyperbola looks meeting the asymptotes as it gets longer, it does not meet them no matter how long it may get longer, and that’s what an asymptote means.