Examples 4 in Standard Forms

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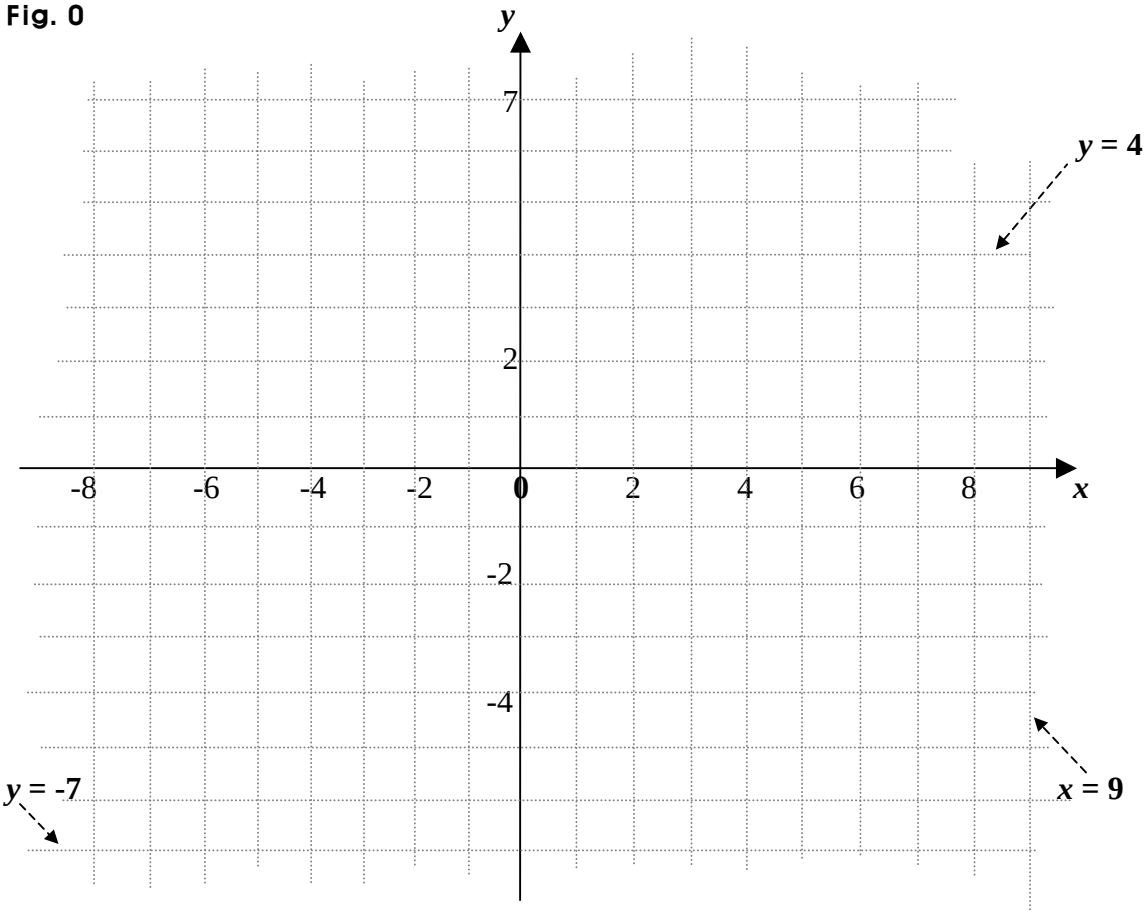
Specify the center and the two main axes of each hyperbola below, and then, graph it.

0. $\mathbf{4(y-5)^2-(x+4)^2=4}$ 1. $\mathbf{4y^2-x^2=4}$ 2. $\mathbf{4(y+\frac{11}{2})^2-9(x+2)^2=9}$

3. $\mathbf{9(y-5)^2-4(x+1)^2=36}$ 4. $\mathbf{(y+3)^2-(x+6)^2=1}$ 5. $\mathbf{(y+4)^2-(x-4)^2=4}$

6. $\mathbf{400(y+\frac{11}{2})^2-36(x+\frac{9}{2})^2=225}$ 7. $\mathbf{144(y-\frac{5}{2})^2-100(x-\frac{9}{2})^2=225}$

Fig. 0



Suggestions or Solutions
To the Problems in the Examples

Specify the center, the transverse axis, and the conjugate axis of each hyperbola below, and then, graph it.

0. $\mathbf{4(y-5)^2-(x+4)^2=4}$ 1. $\mathbf{4y^2-x^2=4}$ 2. $\mathbf{4(y+\frac{11}{2})^2-9(x+2)^2=9}$

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6. $\mathbf{400(y+\frac{11}{2})^2-36(x+\frac{9}{2})^2=225}$ 7. $\mathbf{144(y-\frac{5}{2})^2-100(x-\frac{9}{2})^2=225}$

First, we have a fact as follows.

If a hyperbola centered at **(*u*, *v*)** is horizontal, the equation is $\frac{(x-u)^2}{a^2}-\frac{(y-v)^2}{b^2}=1$, where ***a*** is half the transverse axis, and ***b*** is half the conjugate axis, and if vertical, the equation is $\frac{(y-v)^2}{b^2}-\frac{(x-u)^2}{a^2}=1$, where ***b*** is half the transverse axis, and ***a*** is half the conjugate axis.

In either case though, the rectangle is ***a*** by ***b***, ***a*** is the width (horizontal length) of the rectangle, and ***b*** is the height (vertical length).

So beginning with 0, we can get

$$\mathbf{4(y-5)^2-(x+4)^2=4}\Rightarrow(y-5)^2-\frac{(x+4)^2}{4}=1\Rightarrow\frac{(y-5)^2}{1^2}-\frac{\{x-(-4)\}^2}{2^2}=1.$$

So we can see that the hyperbola is vertical, the center is at (-4, 5), the transverse axis is 2, and the conjugate axis is 4. And its rectangle is centered at (-4, 6), too, and is 4 by 2, that is, the horizontal length of the rectangle is 4, and the vertical length is 2.

1. $\mathbf{4y^2-x^2=4}\Rightarrow y^2-\frac{x^2}{4^2}=1\Rightarrow\frac{(y-0)^2}{1^2}-\frac{(x-0)^2}{4^2}=1$. So we can see that the hyperbola is vertical, the center is (0, 0), the transverse axis is 2, and the conjugate axis is 8. And the rectangle is 8 by 2, that is, the width is 8, and the height is 2.

2. $4(y+\frac{11}{2})^2-9(x+2)^2=9 \Rightarrow \frac{4(y+\frac{11}{2})^2}{9}-(x+2)^2=1$

$\Rightarrow \frac{4(y+\frac{11}{2})^2}{9}-(x+2)^2=\frac{(y+\frac{11}{2})^2}{\frac{9}{4}}-(x+2)^2=\frac{(y+\frac{11}{2})^2}{(\frac{3}{2})^2}-\frac{(x+2)^2}{1^2}=1.$

So the hyperbola is vertical, the center is $(-2,-\frac{11}{2})$, the transverse axis is 3, and the conjugate axis is 2. And the rectangle is 2 by 3, that is, the width is 2, and the height is 3.

3. $9(y-5)^2-4(x+1)^2=36 \Rightarrow \frac{(y-5)^2}{4}-\frac{(x+1)^2}{9}=1$

$\Rightarrow \frac{(y-5)^2}{4}-\frac{(x+1)^2}{9}=\frac{(y-5)^2}{2^2}-\frac{(x+1)^2}{3^2}=1.$

So the hyperbola is vertical, the center is (-1, 5), the transverse axis is 4, and the conjugate axis is 6. And the rectangle is 6 by 4, that is, the width is 6, and the height is 4.

4. $(y+3)^2-(x+6)^2=1 \Rightarrow \frac{(y+3)^2}{1^2}-\frac{(x+6)^2}{1^2}=1.$

So the hyperbola is vertical, the center is (-6, -3), the transverse axis is 2, and the conjugate axis is 2. And the rectangle is a 2 by 2 square.

5. $(y+4)^2-(x-4)^2=4 \Rightarrow \frac{(y+4)^2}{2^2}-\frac{(x-4)^2}{2^2}=1.$

So the hyperbola is vertical, the center is (4, -4), the transverse axis is 4, and the conjugate axis is 4. And the rectangle is a 4 by 4 square.

6. $400(y+\frac{11}{2})^2-36(x+\frac{9}{2})^2=225 \Rightarrow \frac{16(y+\frac{11}{2})^2}{9}-\frac{4(x+\frac{9}{2})^2}{25}=1$

$\Rightarrow \frac{16(y+\frac{11}{2})^2}{9}-\frac{4(x+\frac{9}{2})^2}{25}=\frac{4(y+\frac{11}{2})^2}{\frac{9}{4}}-\frac{(x+\frac{9}{2})^2}{\frac{25}{4}}=\frac{4(y+\frac{11}{2})^2}{(\frac{3}{2})^2}-\frac{(x+\frac{9}{2})^2}{(\frac{5}{2})^2}=1$

So the hyperbola is vertical, the center is $(-\frac{9}{2}, -\frac{11}{2})$, the transverse axis is 3, and the conjugate axis is 5. And the rectangle is 5 by 3, that is, the width is 5, and the height is 3.

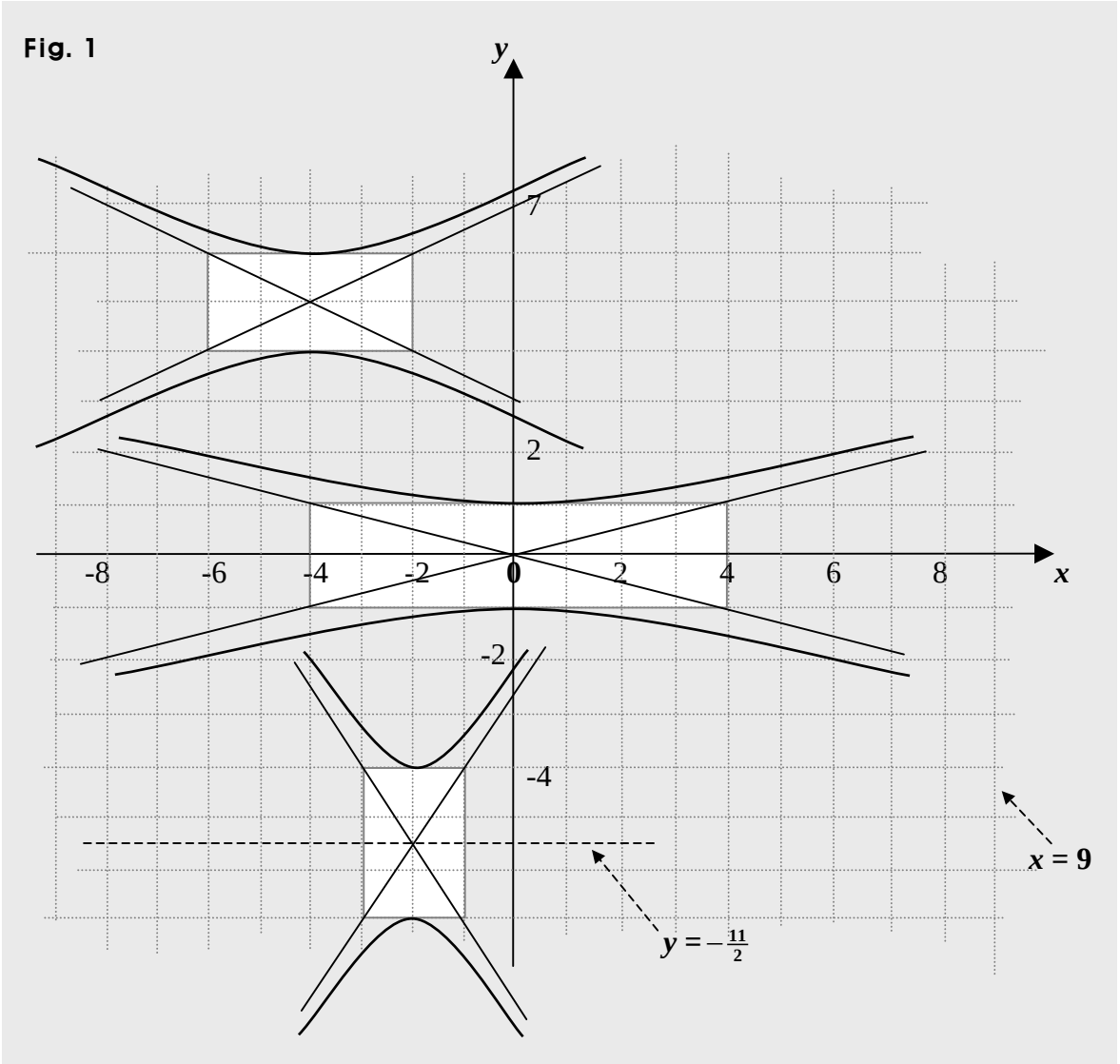
7. $144(y-\frac{5}{2})^2-100(x-\frac{9}{2})^2=225 \Rightarrow \frac{16(y-\frac{5}{2})^2}{25}-\frac{4(x-\frac{9}{2})^2}{9}=1$

$\Rightarrow \frac{16(y-\frac{5}{2})^2}{25}-\frac{4(x-\frac{9}{2})^2}{9}=\frac{4(y-\frac{5}{2})^2}{\frac{25}{4}}-\frac{(x-\frac{9}{2})^2}{\frac{9}{4}}=\frac{4(y-\frac{5}{2})^2}{(\frac{5}{2})^2}-\frac{(x-\frac{9}{2})^2}{(\frac{3}{2})^2}=1$

So the hyperbola is vertical, the center is $(\frac{9}{2}, \frac{5}{2})$, the transverse axis is 5, and the conjugate axis is 3. And the rectangle is 3 by 5, that is, the width is 3, and the height is 5.

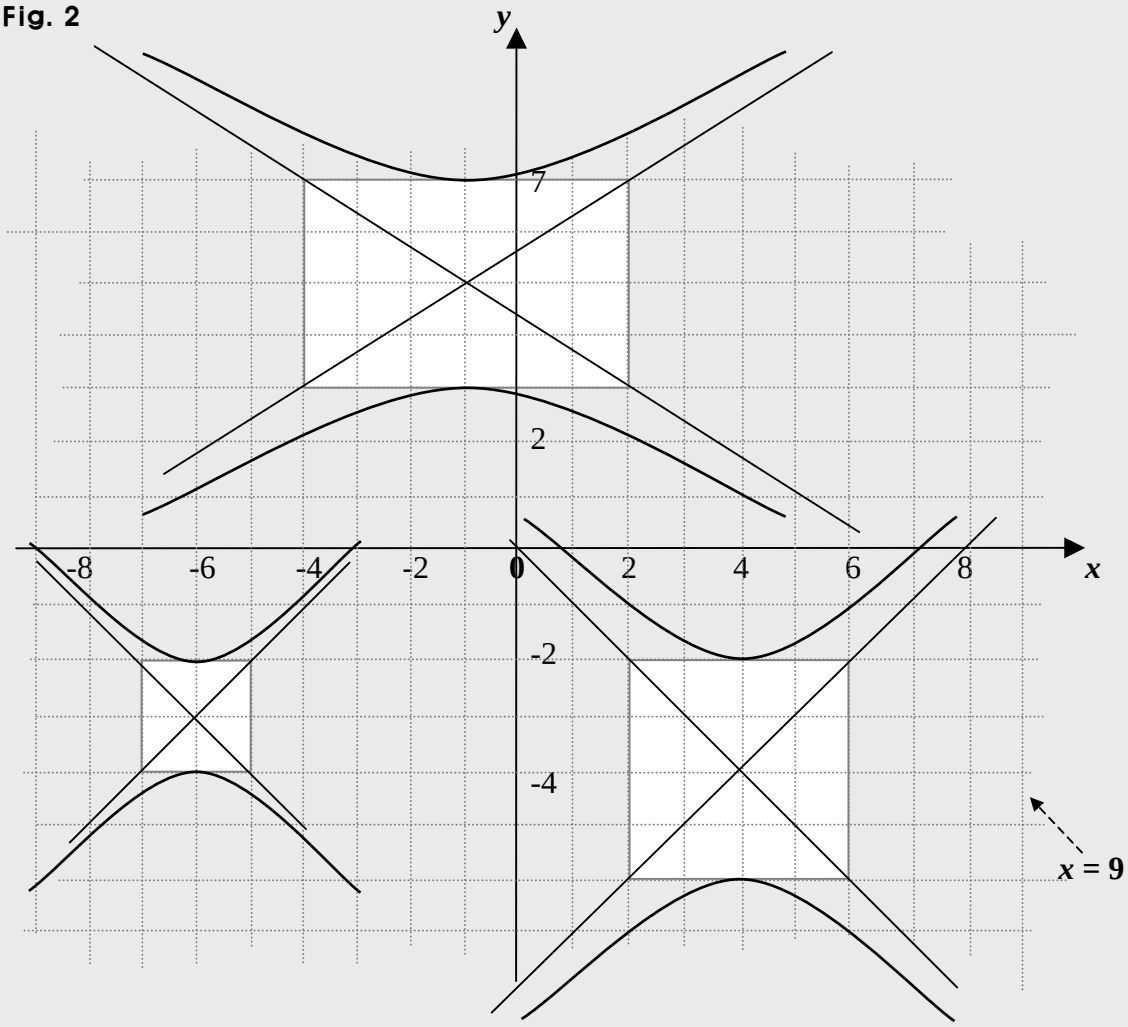
And graphing the equations, we get

Fig. 1



There are more hyperbolas in the next page.

Fig. 2



And the next page shows the other hyperbolas.

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