Examples 5 in Standard Forms

Copyright © 2018 by Seong Ryeol Kim. All rights reserved

Click this to start.

Examples 5 in Standard Forms

Find the center and the two main axes of each hyperbola below.

0. $\mathbf{x^2 + 2x - 2y^2 + 2y - 1 = 0}$

1. $\mathbf{36y^2 + 36y - 100x^2 - 300x + 9 = 0}$

2. $\mathbf{x^2 + 12x - 9y^2 - 90y + 252 = 0}$

3. $\mathbf{2x - 2x^2 + 3y + y^2 + 1 = 0}$

4. $\mathbf{25x^2 + 250x - 16y^2 + 16y + 521 = 0}$

5. $\mathbf{9y^2 - 4x^2 + 24x = 0}$

Suggestions or Solutions
To the Problems in the Examples

0. $x^2 + 2x - 2y^2 + 2y - 1 = 0 \Rightarrow x^2 + 2x + 1 - 2(y^2 - y) - 1 = 0$

$$\Rightarrow (x+1)^2 - 2\{y^2 - y + (\tfrac{1}{2})^2 - (\tfrac{1}{2})^2\} - 1 = 0 \Rightarrow (x+1)^2 - 2\{y^2 - y + (\tfrac{1}{2})^2\} - \tfrac{1}{2} - 1 = 0$$

$$\Rightarrow (x+1)^2 - 2(y-\tfrac{1}{2})^2 - \tfrac{3}{2} = 0 \Rightarrow (x+1)^2 - 2(y-\tfrac{1}{2})^2 = \tfrac{3}{2} \Rightarrow \frac{(x+1)^2}{\frac{3}{2}} - \frac{2(y-\frac{1}{2})^2}{\frac{3}{2}} = 1$$

$$\Rightarrow \frac{(x+1)^2}{(\sqrt{\frac{3}{2}})^2} - \frac{(y-\frac{1}{2})^2}{\frac{3}{4}} = 1 \Rightarrow \frac{(x+1)^2}{(\sqrt{\frac{3}{2}})^2} - \frac{(y-\frac{1}{2})^2}{(\frac{\sqrt{3}}{2})^2} = 1.$$

So the hyperbola is horizontal, the center is $(-1, \tfrac{1}{2})$, the transverse axis is $2\sqrt{\tfrac{3}{2}}$, which is $\sqrt{6}$, and the conjugate axis is $\sqrt{3}$.

1. $36y^2 + 36y - 100x^2 - 300x + 9 = 0 \Rightarrow 36(y^2 + y) - 100(x^2 + 3x) + 9 = 0$

$$\Rightarrow 36(y^2 + y + \tfrac{1}{4} - \tfrac{1}{4}) - 100(x^2 + 3y + \tfrac{9}{4} - \tfrac{9}{4}) + 9 = 0$$

$$\Rightarrow 36(y^2 + y + \tfrac{1}{4}) - 9 - 100(x^2 + 3x + \tfrac{9}{4}) - 225 + 9 = 0$$

$$\Rightarrow 36(y+\tfrac{1}{2})^2 - 100(x+\tfrac{3}{2})^2 - 225 = 0 \Rightarrow 36(y+\tfrac{1}{2})^2 - 100(x+\tfrac{3}{2})^2 = 225$$

$$\Rightarrow \frac{36(y+\frac{1}{2})^2}{225} - \frac{100(x+\frac{3}{2})^2}{225} = 1 \Rightarrow \frac{4(y+\frac{1}{2})^2}{25} - \frac{4(x+\frac{3}{2})^2}{9} = 1 \Rightarrow \frac{(y+\frac{1}{2})^2}{\frac{25}{4}} - \frac{(x+\frac{3}{2})^2}{\frac{9}{4}} = 1$$

$$\Rightarrow \frac{(y+\frac{1}{2})^2}{(\frac{5}{2})^2} - \frac{(x+\frac{3}{2})^2}{(\frac{3}{2})^2} = 1.$$

So the hyperbola is vertical, the center is $(-\tfrac{3}{2}, -\tfrac{1}{2})$, the transverse axis is 5, and the conjugate axis is 3.

2. $x^2 + 12x - 9y^2 - 90y + 252 = 0$

$\Rightarrow x^2 + 12x - 9y^2 - 90y + 252 = x^2 + 12x - 9(y^2 + 10y) + 252$

$= x^2 + 12x + 36 - 36 - 9(y^2 + 10y + 25 - 25) + 252$

$= (x + 6)^2 - 36 - 9(y^2 + 10y + 25) - 225 + 252$

$= (x + 6)^2 - 9(y + 5)^2 - 9 = 0$

$$\Rightarrow \frac{(x+6)^2}{9} - (y+5)^2 = 1 \Rightarrow \frac{\{x-(-6)\}^2}{3^2} - \frac{\{y-(-5)\}^2}{1^2} = 1.$$

So the hyperbola is horizontal, the center is (-6, -5), the transverse axis is 6, and the conjugate axis is 2.

3. $2x - 2x^2 + 3y + y^2 + 1 = 0 \Rightarrow -2x^2 + 2x + y^2 + 3y + 1 = 0$

$$\Rightarrow -2x^2 + 2x + y^2 + 3y + 1 = -2(x^2 - x + \tfrac{1}{4} - \tfrac{1}{4}) + (y^2 + 3y + \tfrac{9}{4} - \tfrac{9}{4}) + 1$$

$$= -2(x - \tfrac{1}{2})^2 + \tfrac{1}{2} + (y + \tfrac{3}{2})^2 - \tfrac{9}{4} + 1 = -2(x - \tfrac{1}{2})^2 + (y + \tfrac{3}{2})^2 - \tfrac{3}{4} = 0$$

$$\Rightarrow -2(x - \tfrac{1}{2})^2 + (y + \tfrac{3}{2})^2 = \tfrac{3}{4} \Rightarrow \frac{(y + \frac{3}{2})^2}{\frac{3}{4}} - \frac{2(x - \frac{1}{2})^2}{\frac{3}{4}} = 1 \Rightarrow \frac{(y + \frac{3}{2})^2}{\frac{3}{4}} - \frac{(x - \frac{1}{2})^2}{\frac{3}{8}} = 1$$

$$\Rightarrow \frac{(y + \frac{3}{2})^2}{(\frac{\sqrt{3}}{2})^2} - \frac{(x - \frac{1}{2})^2}{(\frac{\sqrt{3}}{2\sqrt{2}})^2} = 1 \Rightarrow \frac{(y + \frac{3}{2})^2}{(\frac{\sqrt{3}}{2})^2} - \frac{(x - \frac{1}{2})^2}{(\frac{\sqrt{6}}{4})^2} = 1.$$

So the hyperbola is vertical, the center is $(\frac{1}{2}, -\frac{3}{2})$, the transverse axis is $\sqrt{3}$, and the conjugate axis is $\frac{\sqrt{6}}{2}$.

4. $25x^2 + 250x - 16y^2 + 16y + 521 = 0$

$\Rightarrow 25x^2 + 250x - 16y^2 + 16y + 521 = 25(x^2 + 10x) - 16(y^2 - y) + 521$

$$= 25(x^2 + 10x + 25 - 25) - 16(y^2 - y + \tfrac{1}{4} - \tfrac{1}{4}) + 521$$

$$= 25(x^2 + 10x + 25) - 625 - 16(y^2 - y + \tfrac{1}{4}) + 4 + 521$$

$$= 25(x+5)^2 - 16(y - \tfrac{1}{2})^2 - 100 = 0 \Rightarrow 25(x+5)^2 - 16(y - \tfrac{1}{2})^2 = 100$$

$$\Rightarrow \frac{25(x+5)^2}{100} - \frac{16(y-\frac{1}{2})^2}{100} = 1 \Rightarrow \frac{(x+5)^2}{4} - \frac{4(y-\frac{1}{2})^2}{25} = 1 \Rightarrow \frac{(x+5)^2}{4} - \frac{(y-\frac{1}{2})^2}{\frac{25}{4}} = 1$$

$$\Rightarrow \frac{\{x-(-5)\}^2}{2^2} - \frac{(y-\frac{1}{2})^2}{(\frac{5}{2})^2} = 1.$$

So the hyperbola is horizontal, the center is $(-5, \tfrac{1}{2})$, the transverse axis is 4, and the conjugate axis is 5.

5. $9y^2 - 4x^2 + 24x - 72 = 0$

$\Rightarrow 9y^2 - 4x^2 + 24x - 72 = 9y^2 - 4(x^2 - 6x) - 72 = 9y^2 - 4(x^2 - 6x + 9 - 9) - 72$

$= 9y^2 - 4(x^2 - 6y + 9) + 36 - 72 = 9y^2 - 4(x - 3)^2 - 36 = 0 \Rightarrow 9y^2 - 4(x - 3)^2 = 36$

$$\Rightarrow \frac{9y^2}{36} - \frac{4(x-3)^2}{36} = 1 \Rightarrow \frac{y^2}{4} - \frac{(x-3)^2}{9} = 1 \Rightarrow \frac{y^2}{2^2} - \frac{(x-3)^2}{3^2} = 1.$$

So the hyperbola is vertical, the center is (3, 0), the transverse axis is 4, and the conjugate axis is 6.