Examples 1 in Parallel Transformations

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0. Assuming in the ***x-y*** plane, a point **(3, 8)** gets moved to a point **(8, -3)** by a parallel transformation ***T***, specify the transformation ***T***, and then, find the point that can be transformed to **(3, 8)** by the transformation ***T***.

1. Assuming that $\boldsymbol{T}$**:** $\mathbf{(x, y) \longrightarrow (x - 2, y + 1)}$, and that the equation of the curve of a function $\boldsymbol{y = f(x)}$ is $\mathbf{12x^2 - 3y - 12x - 1 = 0}$, find ***T•f***.

2. Find ***T•f***, assuming $\boldsymbol{T}$**:** $\mathbf{(x, y) \longrightarrow (x + 6, y - 2)}$, and the function ***f*** is as follows.
$\boldsymbol{y = f(x)} = \mathbf{0.2x^5 - 2.25x^4 + 9x^3 - 15.5x^2 + 12x + 2}$, find ***T•f***.

3. Suppose that ***g*** is ***T•f*** found in the example 1 above, and that if another transformation ***Q*** is applied to ***g***, the origin in the graph of ***g*** gets moved to (2, -3). Then, assuming ***h* = *Q•g***, put in one graph all the three curves of ***f***, ***g***, and ***h***.

4. Assuming $\boldsymbol{T}$**:** $\mathbf{(x, y) \longrightarrow (x + 2, y - 1)}$, find ***T•f*** where $\boldsymbol{y = f(x)} = \mathbf{\sqrt{x - 1} + 2}$ for $\mathbf{x \geq 1}$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming in the *x*-*y* plane, a point (3, 8) gets moved to a point (8, -3) by a parallel transformation *T*, specify the transformation *T*, and then, find the point that can be transformed to (3, 8) by the transformation *T*.

$\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (x + 5, y - 11)}$. And the point that can be transformed is **(-2, 19)**.

If not sure of the idea behind the solution above, follow the steps below.

First, since ***T*** is a parallel transformation, we can put it the way below.

$\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (x + a, y + b)}$, which can be the definition for parallel transformations.

And we can put the definition this way, too: $\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (x + \Delta x, y + \Delta y)}$.

If a point moves, its coordinates get changed.
So in the definition above, we can say that $\boldsymbol{\Delta x}$ is the amount of change in the ***x***-coordinate, and $\boldsymbol{\Delta y}$ is the amount of change in the ***y***-coordinate.

And we know that the point at **(3, 8)** moves to **(8, -3)**.

So we get $\boldsymbol{\Delta x = 8 - 3}$, which is 5, and $\boldsymbol{\Delta y = -3 - 8 = -11}$.

And thus, we get $\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (x + 5, y - 11)}$.

Suppose now that the point that can be transformed is $\boldsymbol{(p, q)}$.

Then, applying ***T*** to $\boldsymbol{(p, q)}$, we get $\boldsymbol{(p, q) \longrightarrow (p + 5, q - 11)}$.

And we know the fact that applying ***T*** to $\boldsymbol{(p, q)}$, we get **(3, 8)**, which is therefore, the transformation of $\boldsymbol{(p, q)}$ by ***T***.

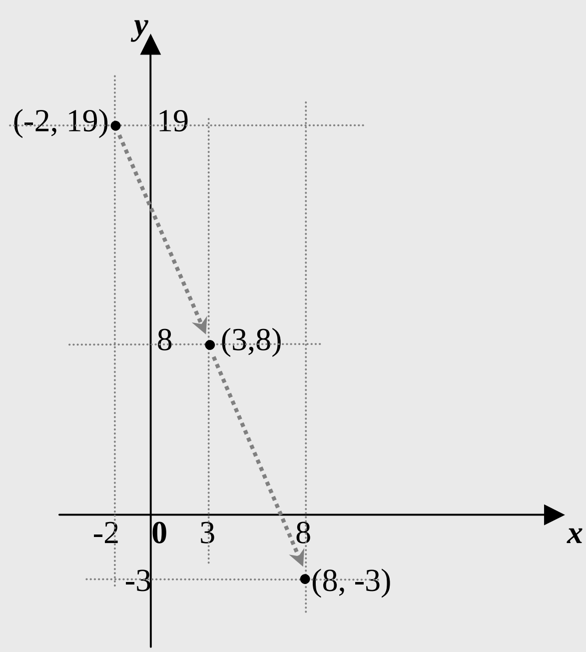
That is to say that $\boldsymbol{(p, q)}$ is the original, and **(3, 8)** is the new.

So we get **(3, 8) = (*p* + 5, *q* – 11)**.

Thus, we get **3 = *p* + 5 ⇒ *p* = -2**, and **8 = *q* – 11 ⇒ *q* = 19**.

Therefore, by the transformation ***T***, the point that gets moved to **(3, 8)** is **(-2, 19)**.

Fig. 0



The three points above are in the same line, where the slope is $\frac{\Delta y}{\Delta x}$, which is $-\frac{11}{5}$.

So we can notice that applying a parallel transformation ***P*: (*x*, *y*)** ⟶ **(*x* + *a*, *y* + *b*)**, every point in the original curve will move along a line $y = \frac{a}{b}x + c$, where ***c*** is constant.

And note that **(3, 8)** is the midpoint between the other two (-2, 19) and (8, -3).

So we can notice that if **(*m*, *n*)** is the new point we get applying the transformation ***T*** to the point (8, -3), the point (8, -3) will be the midpoint between (3, 8) and **(*m*, *n*)**.

Suggestions or Solutions
To the Problem in the Example 1

Assuming that T: $(x, y) \longrightarrow (x - 2, y + 1)$, and that the equation of the curve of a function $y = f(x)$ is $12x^2 - 3y - 12x - 1 = 0$, find $T \bullet f$.

Setting first $(s, t) = (x - 2, y + 1)$, we get $s = x - 2 \Rightarrow x = s + 2$, and $t = y + 1 \Rightarrow y = t - 1$.

So next, we get $12x^2 - 3y - 12x - 1 = 12(s + 2)^2 - 3(t - 1) - 12(s + 2) - 1 = 0$.

And simplifying the equation above, we get

$$12(s + 2)^2 - 3(t - 1) - 12(s + 2) - 1 = 0 \Rightarrow 4(s + 2)^2 - (t - 1) - 4(s + 2) - \tfrac{1}{3} = 0$$
$$\Rightarrow t - 1 = 4(s + 2)^2 - 4(s + 2) - \tfrac{1}{3} = 4s^2 + 16s + 16 - 4s - 8 - \tfrac{1}{3} = 4s^2 + 12s + 8 - \tfrac{1}{3}$$
$$\Rightarrow t = 4s^2 + 12s + \tfrac{26}{3}.$$

And thus, we get $T \bullet f = 4x^2 + 12x + \frac{26}{3}$.

If not sure of the idea behind the processes above, follow the steps below.

To begin with, the definition for parallel transformations is as follows.
P: $(x, y) \longrightarrow (x + a, y + b)$, where a and b are constant.

So T is a parallel transformation, since setting $a = -2$, and $b = 1$, we get T.

Next, applying T to f, we get $T \bullet f$. So it is the transformation of the function f by T. And $f(x)$ is the original function, and $T \bullet f(x)$, that is, $T \bullet f$ is the new function.

Next, assuming g is the new function, and applying T to f, we get first, the equation of the curve of g, that is, the new equation. And then, we get the function g. So let's now get the new equation.

In the original equation however, the variable y is not on the other side of the equation. So do we need to move it to the other side?

Not necessarily
What we need is the equation connecting the variables in the new arbitrary point.

It's because the equation is the new equation indicating the new curve.

In other words, using $(s, t) = (x - 2, y + 1)$, along with $12x^2 - 3y - 12x - 1 = 0$, we can get the equation, which is the new equation, indicating the curve of the new function g.

So let's connect now, the two coordinates s and t.

Then first, we can set $s = x - 2$, and $t = y + 1$. So we get $x = s + 2$, and $y = t - 1$.

And thus, we can now connect s and t putting $x = s + 2$, and $y = t - 1$ into the original equation $12x^2 - 3y - 12x - 1 = 0$.

So putting them into the original equation, we get

$12x^2 - 3y - 12x - 1 = 12(s + 2)^2 - 3(t - 1) - 12(s + 2) - 1 = 0$, which is the connective equation between s and t. And simplifying the equation above, we get

$12(s + 2)^2 - 3(t - 1) - 12(s + 2) - 1 = 0 \Rightarrow 4(s + 2)^2 - (t - 1) - 4(s + 2) - \frac{1}{3} = 0$
$\Rightarrow t - 1 = 4(s + 2)^2 - 4(s + 2) - \frac{1}{3} = 4s^2 + 16s + 16 - 4s - 8 - \frac{1}{3} = 4s^2 + 12s + 8 - \frac{1}{3}$
$\Rightarrow t = 4s^2 + 12s + \frac{26}{3}$, which is the equation connecting the coordinates of (s, t), which is the arbitrary point in the curve of g, which is in the x-y system.

And we know s is the x-coordinate, and t is the y-coordinate.
So in the equation where $t = 4s^2 + 12s + \frac{26}{3}$, just replacing s with x, and t with y, we get the new equation, that is, the equation of the curve of g.

Then, we get $y = 4x^2 + 12x + \frac{26}{3}$, which is the curve of the new function $y = g(x)$, which is $T \bullet f$. And thus, we get $T \bullet f = 4x^2 + 12x + \frac{26}{3}$. And putting it more specifically, we can put it this way, too: $y = T \bullet f(x) = 4x^2 + 12x + \frac{26}{3}$. If the domain is not specified, it is assumed to be a set of all real numbers.

Let's now put both functions in the same graph.

Then, finding the vertex of the curve of $\boldsymbol{f}$ first, we get

$$\mathbf{12}x^2 - 3y - 12x - 1 = 0 \Rightarrow 3y = 12x^2 - 12x - 1 \Rightarrow y = 4x^2 - 4x - \tfrac{1}{3}$$
$$= 4(x^2 - x + \tfrac{1}{4} - \tfrac{1}{4}) - \tfrac{1}{3} = 4(x - \tfrac{1}{2})^2 - 1 - \tfrac{1}{3} = 4(x - \tfrac{1}{2})^2 - \tfrac{4}{3} \Rightarrow y = 4(x - \tfrac{1}{2})^2 - \tfrac{4}{3}.$$

So the vertex is at $(\frac{1}{2}, -\frac{4}{3})$, and the axis of symmetry is $x = \frac{1}{2}$.

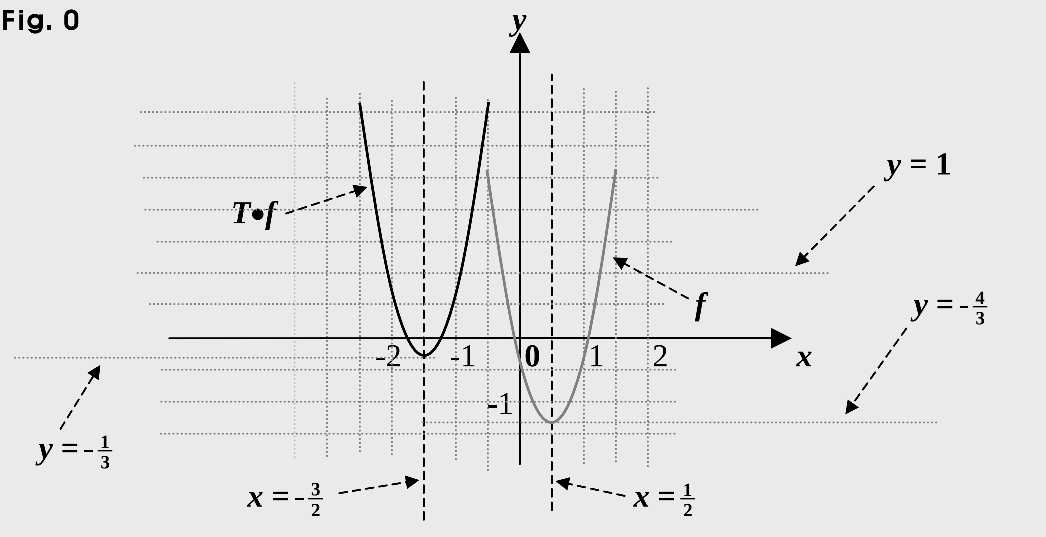
And next, moving on to the curve of $\boldsymbol{T \bullet f}$, we get

$$y = 4x^2 + 12x + \tfrac{26}{3} = 4(x^2 + 3x + \tfrac{9}{4} - \tfrac{9}{4}) + \tfrac{26}{3} = 4(x + \tfrac{3}{2})^2 - \tfrac{4}{3} \Rightarrow y = 4(x + \tfrac{3}{2})^2 - \tfrac{1}{3}.$$

So the vertex is at $(-\frac{3}{2}, -\frac{1}{3})$, and the axis of symmetry is $x = -\frac{3}{2}$.

And we can put both functions in a graph the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 2

Find $T\bullet f$, assuming T: $(x, y) \longrightarrow (x + 6, y - 2)$, and a function is as follows.
$y = f(x) = 0.2x^5 - 2.25x^4 + 9x^3 - 15.5x^2 + 12x + 2$.

Setting first $(s, t) = (x + 6, y - 2)$, we get $s = x + 6 \Rightarrow x = s - 6$, and $t = y - 2 \Rightarrow y = t + 2$.

So next, we get $y = 0.2x^5 - 2.25x^4 + 9x^3 - 15.5x^2 + 12x + 2 \Rightarrow$

$t + 2 = 0.2(s - 6)^5 - 2.25(s - 6)^4 + 9(s - 6)^3 - 15.5(s - 6)^2 + 12(s - 6) + 2 \Rightarrow$

$t = 0.2(s - 6)^5 - 2.25(s - 6)^4 + 9(s - 6)^3 - 15.5(s - 6)^2 + 12(s - 6)$.

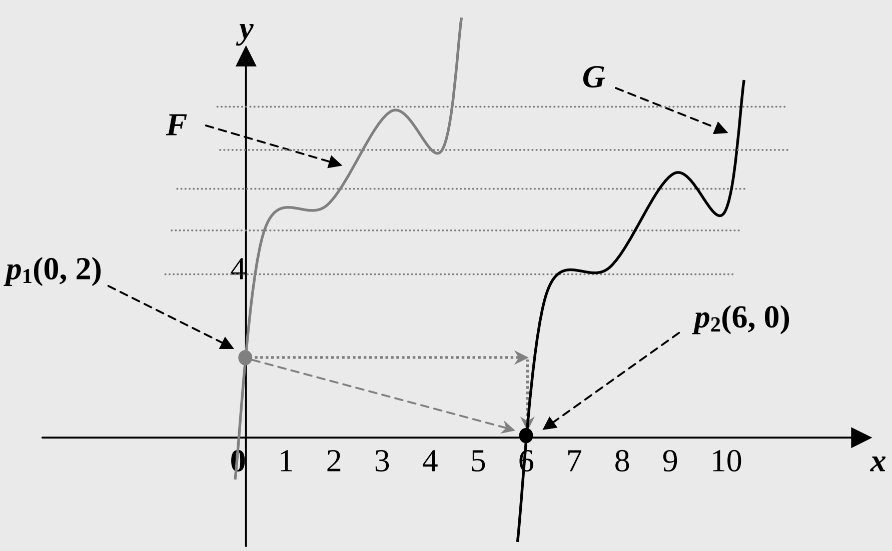
So we get $T\bullet f = 0.2(x - 6)^5 - 2.25(x - 6)^4 + 9(x - 6)^3 - 15.5(x - 6)^2 + 12(x - 6)$.

And assuming $g = T\bullet f$, we can put it the way below, too.

$y = g(x) = 0.2(x - 6)^5 - 2.25(x - 6)^4 + 9(x - 6)^3 - 15.5(x - 6)^2 + 12(x - 6)$.

And assuming F is the original curve, G is the new curve, we can put them in a graph the way below.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 3

Suppose that *g* is *T•f* found in the example 1 above, and that if another transformation *Q* is applied to *g*, the origin in the graph of *g* gets moved to (2, -3). Then, assuming *h* = *Q•g*, put in one graph all the three curves of *f*, *g*, and *h*.

Just moving the origin in a graph, we apply a parallel transformation to the curve in the graph. And since the origin moves to **(2, -3)**, we can put the transformation ***Q*** this way:

$\boldsymbol{Q: (x, y) \longrightarrow (x - 2, y + 3)}$.

So what do we get applying the transformation ***Q*** to ***g***?

We get the function ***h***. And in ***Q*** therefore, ***g*** is the original function, and the new function is ***h***.   So let's now get the function ***h***.

Assuming first, the new arbitrary point is $\boldsymbol{(s, t)}$, we can set $\boldsymbol{(s, t) = (x - 2, y + 3)}$.

Then, we get $\boldsymbol{s = x - 2 \Rightarrow x = s + 2}$, and $\boldsymbol{t = y + 3 \Rightarrow y = t - 3}$.

So next, putting into the original equation, $\boldsymbol{x = s + 2}$ and $\boldsymbol{y = t - 3}$, we can get the equation connecting ***s*** and ***t***.   And the original equation is $\boldsymbol{y = 4x^2 + 12x + \frac{26}{3}}$.

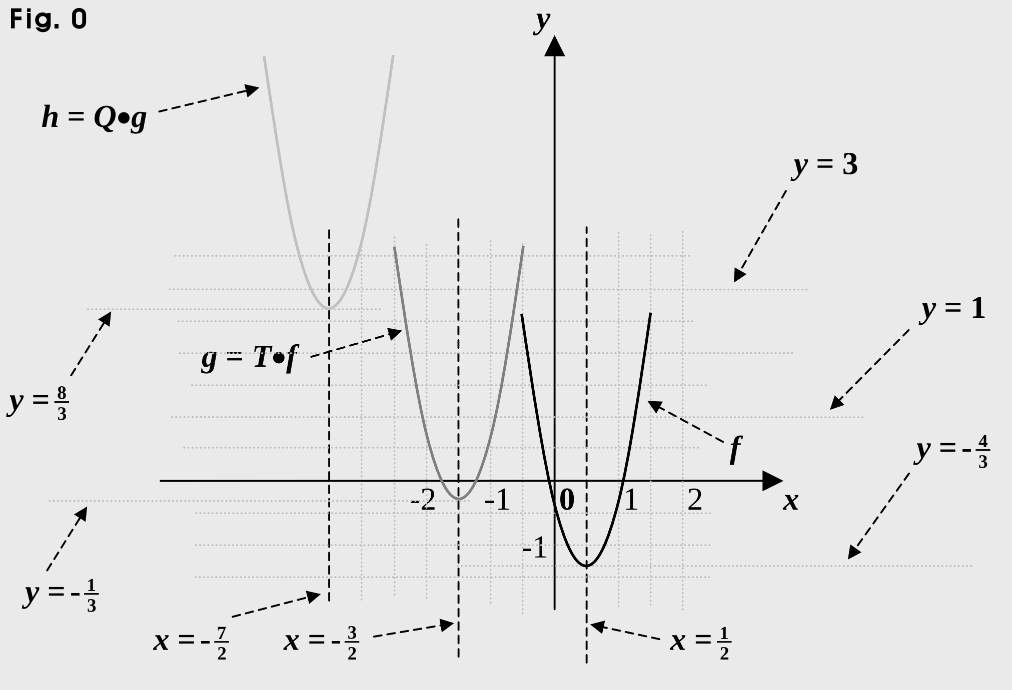
So we get $\boldsymbol{t - 3 = 4(s + 2)^2 + 12(s + 2) + \frac{26}{3} = 4s^2 + 16s + 16 + 12s + 24 + \frac{26}{3}}$
$\boldsymbol{= 4s^2 + 28s + 40 + \frac{26}{3} \Rightarrow t = s^2 + 28s + 43 + \frac{26}{3} = s^2 + 28s + \frac{155}{3} \Rightarrow t = s^2 + 28s + \frac{155}{3}}$, which is the new equation, indicating the curve of ***h***, which is to be in the ***x-y*** plane.

So replacing ***s*** with ***x***, and ***t*** with ***y***, we get $\boldsymbol{y = x^2 + 28x + \frac{155}{3}}$.

Thus, we get $\boldsymbol{y = h(x) = x^2 + 28x + \frac{155}{3}}$.

And thus, putting in one graph the three functions, ***f***, ***g***, and ***h***, we get

Fig. 0



And if the original curve is already in a graph, we don’t need to find the equation of the new curve if we have only to put the new curve in the graph.

That’s because a parallel transformation does not change the curve itself.
And if the curve is a parabola, we can readily move it moving its vertex.

Suggestions or Solutions
To the Problem in the Example 4

Assuming T: $(x, y) \longrightarrow (x + 2, y - 1)$, find $T \bullet f$ where $y = f(x) = \sqrt{x-1} + 2$ for $x \geq 2$.

Assuming first, (s, t) is the new arbitrary point, we can set $(s, t) = (x + 2, y - 1)$.

Then next, we can set $s = x + 2$, and $t = y - 1$. So we get $x = s - 2$, and $y = t + 1$.

Thus, we get $y = \sqrt{x-1} + 2 \Rightarrow t + 1 = \sqrt{(s-2)-1} + 2 \Rightarrow t = \sqrt{s-3} + 1$, which is the connective equation between the coordinates of (s, t), which is the arbitrary point in the curve of g, which is in the x-y system.

And we know that the domain in f is $x \geq 2$, and s is the x-coordinate in the new arbitrary point. So getting the new domain, we get $x \geq 2 \Rightarrow x + 2 \geq 4 \Rightarrow s \geq 4$.

And thus, the domain in the new function g is $x \geq 4$. So the curve of the new function g is $y = \sqrt{x-3} + 1$ for $x \geq 4$, and thus, the new function g is $y = g(x) = \sqrt{x-3} + 1$ for $x \geq 4$.

So we get $T \bullet f = \sqrt{x-3} + 1$ for $x \geq 4$.

Fig. 0