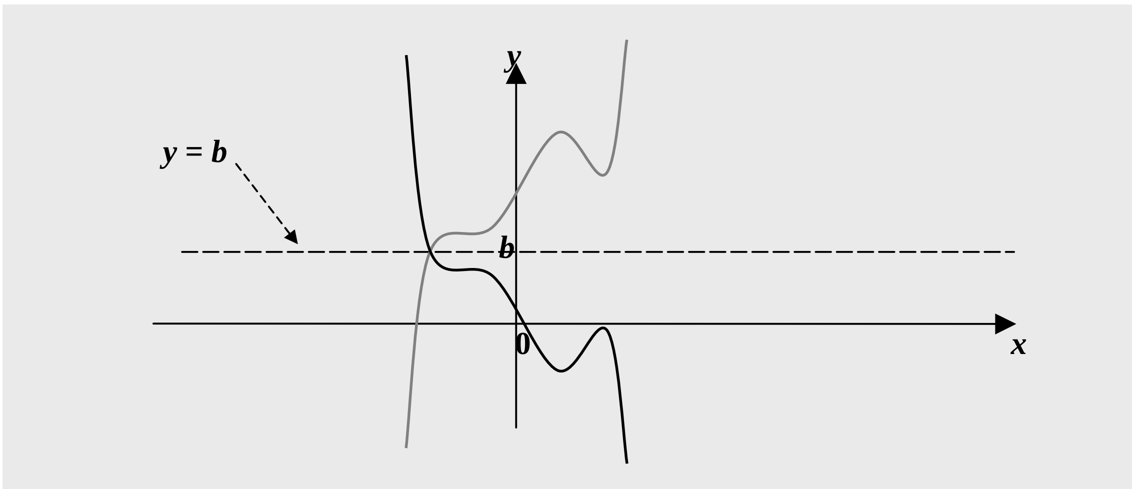
Examples 1 in Symmetry about a line 1

Click this to start.

Examples 1 in Symmetry about a line 1



0. Assuming $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$, transform the function $\boldsymbol{f}$ by a transformation symmetric about a line $\boldsymbol{y = 1}$.

1. Find a function, the curve of which is symmetric about the $\boldsymbol{x}$-axis to the curve of a function $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$.

2. Assuming $\boldsymbol{T: (x, y) \longrightarrow (x, 4 - y)}$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{T}$.

2.0. $\boldsymbol{y = f(x) = x^2 - 2x + 1}$ for $\boldsymbol{x \geq 1}$.

2.1. $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \leq 1}$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming $y = f(x) = (1 - x)^3 + 2$, transform the function f by a transformation symmetric about a line $y = 1$.

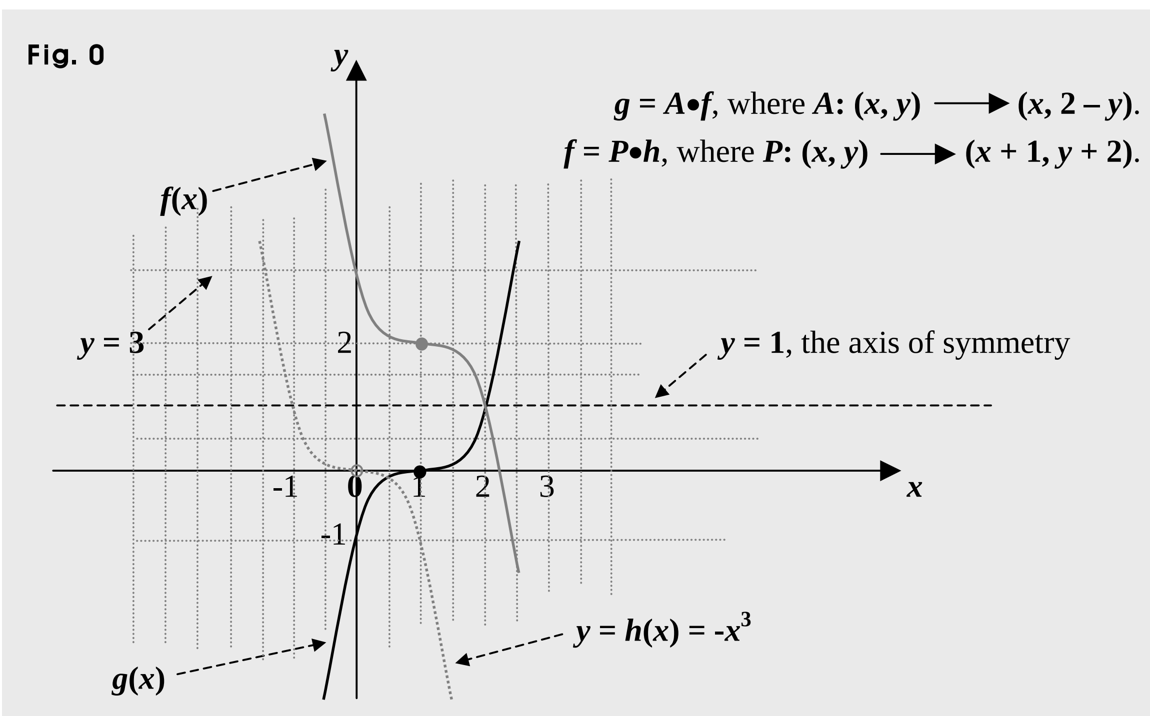
To begin with, assuming $\boldsymbol{B}$ is a transformation symmetric about a line $y = b$, where b is of course, a constant, we can put $\boldsymbol{B}$ the way as follows. $\boldsymbol{B}: (x, y) \longrightarrow (x, 2b - y)$.

So since b is 1 in this example, assuming $\boldsymbol{A}$ is the transformation we want to apply, we can put $\boldsymbol{A}$ the way as follows. $\boldsymbol{A}: (x, y) \longrightarrow (x, 2 - y)$.

Assuming thus first, g is the new function, and (s, t) is the arbitrary point in the curve of g, we can set $(s, t) = (x, 2 - y)$. So we get $s = x$, and $t = 2 - y \Rightarrow y = 2 - t$. Thus, we get $y = (1 - x)^3 + 2 \Rightarrow 2 - t = (1 - s)^3 + 2 \Rightarrow t = -(1 - s)^3 = (s - 1)^3 \Rightarrow t = (s - 1)^3$.

And thus, the new function is $y = g(x) = (x - 1)^3$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Find a function, the curve of which is symmetric about the *x*-axis to the curve of a function $y = f(x) = \sqrt[3]{1-x} + 2$.

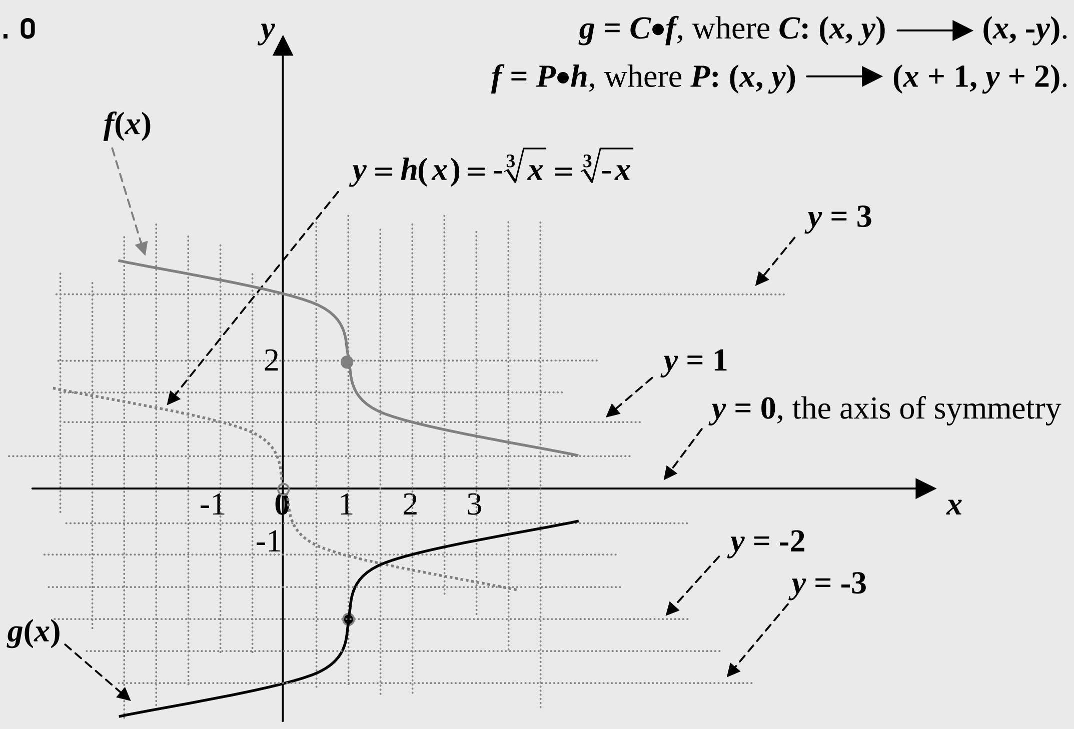
Assuming first ***B*** is a transformation symmetric about a line $y = b$, where b is constant, we can put ***B*** the way as follows. B: $(x, y) \longrightarrow (x, 2b - y)$. And in this example, the axis of symmetry is the ***x***-axis, which is a line $y = 0$, so b is 0 in ***B***, and thus, assuming ***C*** is the transformation we apply, we can put ***C*** the way as follows. C: $(x, y) \longrightarrow (x, -y)$.

So assuming next, g is the new function, and (s, t) is the arbitrary point in the curve of g, we can set $(s, t) = (x, -y)$. So we get $s = x$, and $t = -y \Rightarrow y = -t$.

Thus, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow -t = \sqrt[3]{1-s} + 2 \Rightarrow t = \sqrt[3]{s-1} - 2$.

And thus, the new function is $y = g(x) = \sqrt[3]{x-1} - 2$.

Fig. 0



And we can get the solution the way below, too.

We know the transformation $\boldsymbol{B}$ is as follows. $\boldsymbol{B}$**:** $(x, y) \longrightarrow (x, 2b - y)$, which is the transformation symmetric about a line $y = b$.

So assuming again, $\boldsymbol{g}$ is the new function, we can put $\boldsymbol{B}$ the way below, too.

$\boldsymbol{B}$**:** $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 2b - y_f)$.

And we know $\boldsymbol{C}$ is the transformation symmetric about a line $y = 0$, which is the x-axis.

So we can put $\boldsymbol{C}$ this way, too: $\boldsymbol{C}$**:** $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, -y_f)$.

And briefly, we can just put it this way, too: $\boldsymbol{C}$**:** $(x_f, y_f) \longrightarrow (x_f, -y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = -y_f \Rightarrow y_f = -y_g$. Thus, we can now get the mixture.

That is, putting $x_f = x_g$ and $y_f = -y_g$ into the original equation $y = \sqrt[3]{1-x} + 2$, we get the equation connecting x_g and y_g, that is, the new equation. In other words, we get

$$y = f(x) = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow -y_g = \sqrt[3]{1-x_g} + 2 \Rightarrow y_g = \sqrt[3]{x_g - 1} - 2,$$

which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function $\boldsymbol{g}$. So the new function is $y = g(x) = \sqrt[3]{x-1} - 2$.

In short:

Assuming first, $\boldsymbol{g}$ is the new function, we have $(x_g, y_g) = (x_f, -y_f)$.

So we get $y = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow -y_g = \sqrt[3]{1-x_g} + 2 \Rightarrow y_g = \sqrt[3]{x_g - 1} - 2$

And thus, the new function is $y = g(x) = \sqrt[3]{x-1} - 2$.

And we can get it the way below, too.

Assuming first, $\boldsymbol{g}$ is the new function, and (s, t) is the arbitrary point in the curve of $\boldsymbol{g}$, we can set $(s, t) = (x, -y)$. So we get $s = x$, and $t = -y \Rightarrow y = -t$.

Thus, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow -t = \sqrt[3]{1-s} + 2 \Rightarrow t = \sqrt[3]{s-1} - 2$.

So the new function is $y = g(x) = \sqrt[3]{x-1} - 2$.

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (x, 4 - y)$, we get $s = x \Rightarrow x = s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = x^2 - 2x + 1 \Rightarrow 4 - t = s^2 - 2s + 1 \Rightarrow t = -s^2 + 2s + 3$.

Besides, $x \geq 1 \Rightarrow s \geq 1$, because $x = s$. Thus, we get $T \bullet f = -x^2 + 2x + 3$ for $x \geq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4 - y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = x^2 - 2x + 1 \Rightarrow y_f = f(x_f) = x_f^2 - 2x_f + 1 \Rightarrow 4 - y_g = x_g^2 - 2x_g + 1$$

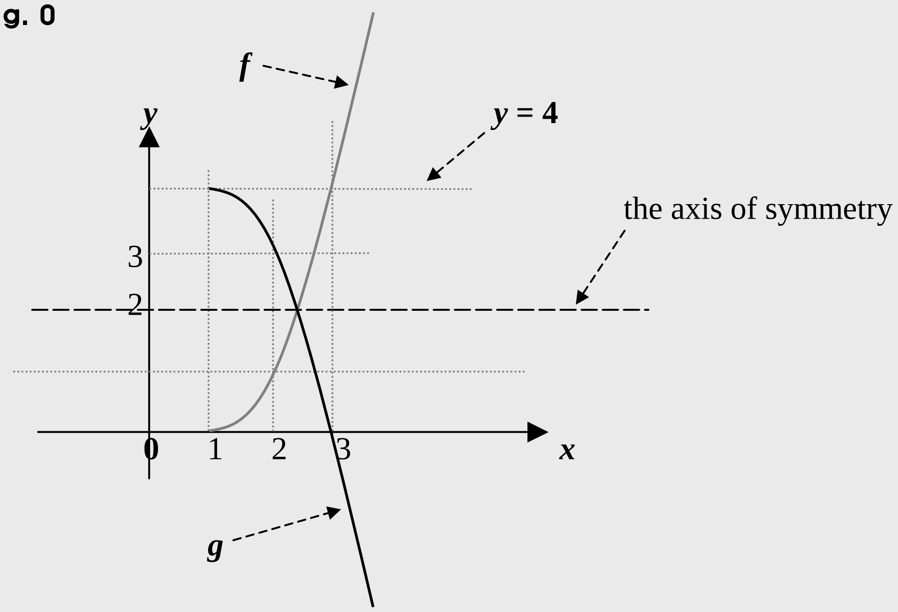
$$\Rightarrow y_g = -x_g^2 + 2x_g + 3 \Rightarrow y_g = g(x_g) = -x_g^2 + 2x_g + 3 \Rightarrow y = g(x) = -x^2 + 2x + 3.$$

Besides, $x_f \geq 1 \Rightarrow x_g \geq 1$, because $x_f = x_g$. And we know $g = T \bullet f$.

So we get $T \bullet f = -x^2 + 2x + 3$ for $x \geq 1$.

And putting the two curves in a graph, we get

Fig. 0



$g = T \bullet f$, where $T: (x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.

Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \le 1$, find $T \bullet f$.

Setting first $(s, t) = (x, 4 - y)$, we get $s = x \Rightarrow x = s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = (1 - x)^3 + 2 \Rightarrow 4 - t = (1 - s)^3 + 2 \Rightarrow t = 2 - (1 - s)^3 = 2 + (s - 1)^3$.

Besides, $x \le 1 \Rightarrow s \le 1$, because $x = s$. Thus, we get $T \bullet f = (x - 1)^3 + 2$ for $x \le 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4 - y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = (1 - x)^3 + 2 \Rightarrow y_f = f(x_f) = (1 - x_f)^3 + 2 \Rightarrow 4 - y_g = (1 - x_g)^3 + 2$$

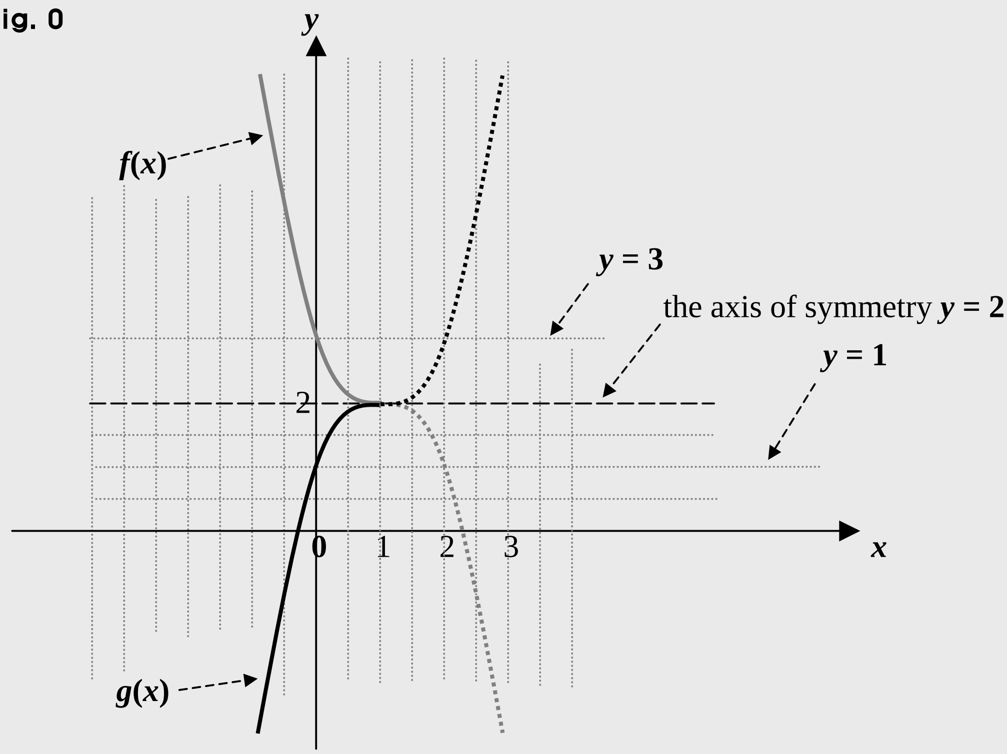
$$\Rightarrow y_g = (x_g - 1)^3 + 2 \Rightarrow y_g = g(x_g) = (x_g - 1)^3 + 2 \Rightarrow y = g(x) = (x - 1)^3 + 2.$$

Besides, $x_f \le 1 \Rightarrow x_g \le 1$, because $x_f = x_g$. And we know $g = T \bullet f$.

So we get $T \bullet f = (x - 1)^3 + 2$ for $x \le 1$.

And putting the two curves in a graph, we get

Fig. 0



$g = T \bullet f$, where T: $(x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$.