Examples 2 in Symmetry about a line 1

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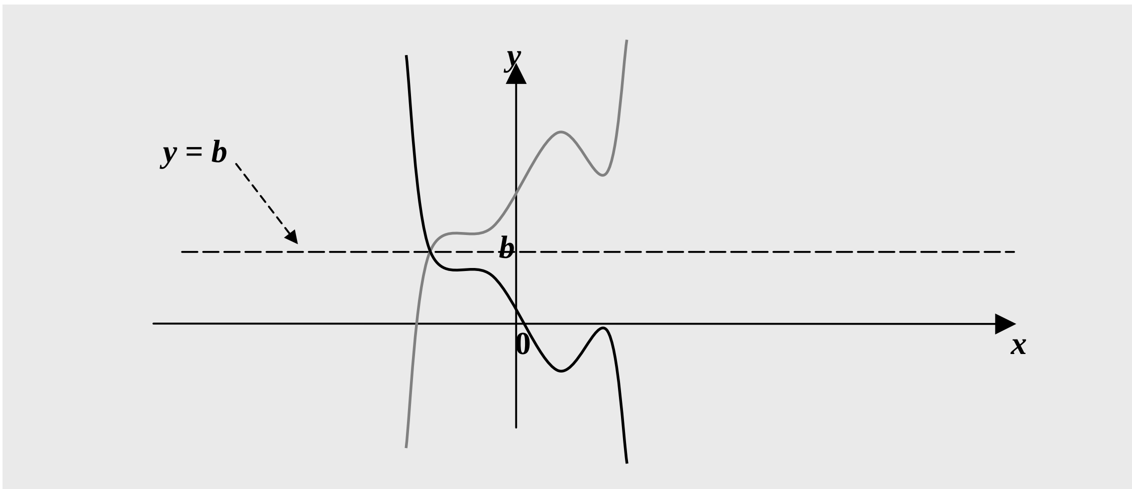


0. Assuming $\boldsymbol{T}$: $(x, y) \longrightarrow (x, 4 - y)$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{T}$.

0.0. $y = f(x) = \sqrt[3]{1 - x} + 2$ for $x \geq 1$.

0.1. $y = f(x) = -\sqrt{x + 1} + 2$ for $-1 \leq x \leq 3$.

0.2. $y = f(x) = (x - 2)^2 + 1$ for $1 \leq x \leq 4$.

0.3. $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$.

1. Assuming $y = f(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq -1$, $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, and $\boldsymbol{T}$: $(x, y) \longrightarrow (x, 6 - y)$, find $g = T \bullet f$, and $h = T \bullet w$.

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4-y)$, and $y = f(x) = \sqrt[3]{1-x}+2$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (x, 4-y)$, we get $s = x \Rightarrow x = s$, and $t = 4-y \Rightarrow y = 4-t$.

So next, we get $y = \sqrt[3]{1-x}+2 \Rightarrow 4-t = \sqrt[3]{1-s}+2 \Rightarrow t = \sqrt[3]{s-1}+2$.

Besides, $x \geq 1 \Rightarrow s \geq 1$, because $x = s$. Thus, we get $T \bullet f = \sqrt[3]{x-1}+2$ for $x \geq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4-y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4-y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4-y_f \Rightarrow y_f = 4-y_g$.

And next, putting $x_f = x_g$ and $y_f = 4-y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4-y_g = f(x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = \sqrt[3]{1-x}+2 \Rightarrow y_f = f(x_f) = \sqrt[3]{1-x_f}+2 \Rightarrow 4-y_g = \sqrt[3]{1-x_g}+2$$

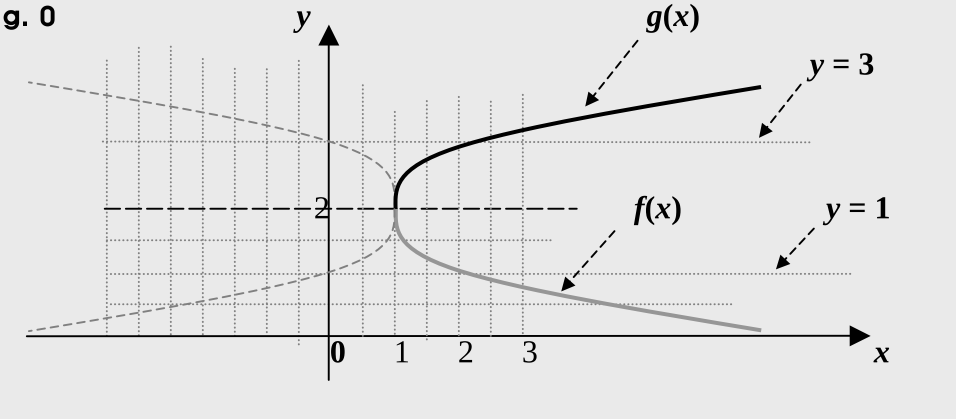
$$\Rightarrow y_g = 2-\sqrt[3]{1-x_g} = \sqrt[3]{x_g-1}+2 \Rightarrow y_g = g(x_g) = \sqrt[3]{x_g-1}+2 \Rightarrow y = g(x) = \sqrt[3]{x-1}+2.$$

Besides, $x_f \geq 1 \Rightarrow x_g \geq 1$, because $x_f = x_g$. And we know $g = T \bullet f$.

So we get $T \bullet f = \sqrt[3]{x-1}+2$ for $x \geq 1$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = -\sqrt{x+1}+2$ for $-1 \le x \le 3$, find $T \bullet f$.

Setting first $(s, t) = (x, 4 - y)$, we get $s = x \Rightarrow x = s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = -\sqrt{x+1}+2 \ \Rightarrow 4 - t = -\sqrt{s+1}+2 \ \Rightarrow t = \sqrt{s+1}+2$

And $-1 \le x \le 3 \Rightarrow -1 \le s \le 3$, because $x = s$. So we get $T \bullet f = \sqrt{x+1}+2$ for $-1 \le x \le 3$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4 - y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = -\sqrt{x+1}+2 \ \Rightarrow y_f = f(x_f) = -\sqrt{x_f+1}+2 \ \Rightarrow 4 - y_g = -\sqrt{x_g+1}+2$$

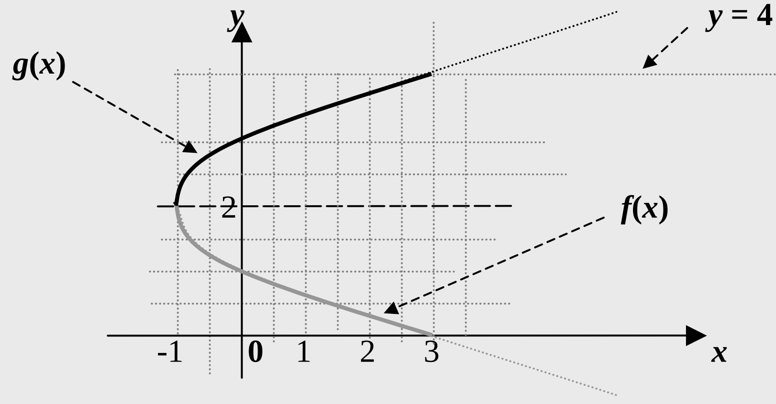
$$\Rightarrow y_g = \sqrt{x_g+1}+2 \ \Rightarrow y_g = g(x_g) = \sqrt{x_g+1}+2 \ \Rightarrow y = g(x) = \sqrt{x+1}+2$$

Besides, $-1 \le x_f \le 3 \Rightarrow -1 \le x_g \le 3$, because $x_f = x_g$. And we know $g = T \bullet f$.

So we get $T \bullet f = \sqrt{x+1}+2$ for $-1 \le x \le 3$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 2 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4 - y)$, and $y = f(x) = (x - 2)^2 + 1$ for $1 \le x \le 4$, find $T \bullet f$.

Setting first $(s, t) = (x, 4 - y)$, we get $s = x \Rightarrow x = s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = (x - 2)^2 + 1 \Rightarrow 4 - t = (s - 2)^2 + 1 \Rightarrow t = 3 - (s - 2)^2$.

And $1 \le x \le 4 \Rightarrow 1 \le s \le 4$, because $x = s$. So we get $T \bullet f = 3 - (x - 2)^2$ for $1 \le x \le 4$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4 - y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$y = f(x) = (x - 2)^2 + 1 \Rightarrow y_f = f(x_f) = (x_f - 2)^2 + 1 \Rightarrow 4 - y_g = (x_g - 2)^2 + 1$

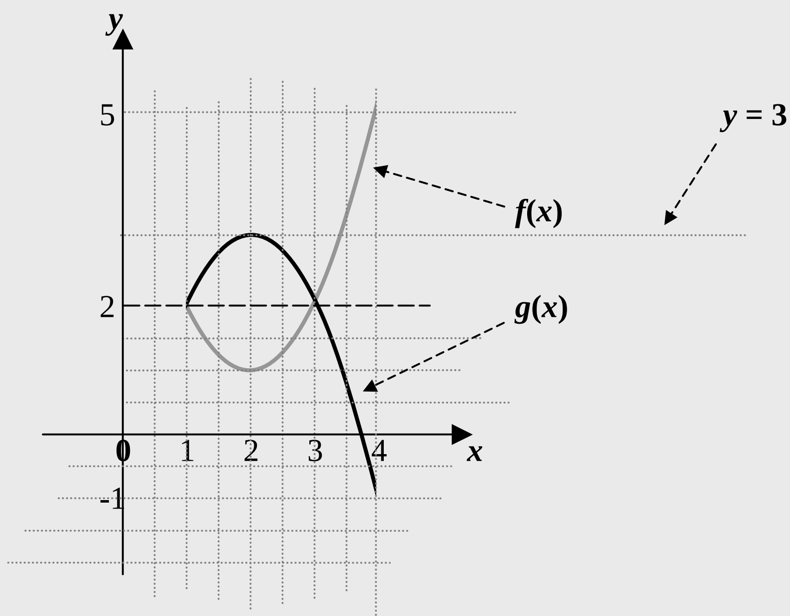
$\Rightarrow y_g = 3 - (x_g - 2)^2 \Rightarrow y_g = g(x_g) = 3 - (x_g - 2)^2 \Rightarrow y = g(x) = 3 - (x - 2)^2$.

Besides, $1 \le x_f \le 4 \Rightarrow 1 \le x_g \le 4$, because $x_f = x_g$. And we know $g = T \bullet f$.

So we get $T \bullet f = 3 - (x - 2)^2$ for $1 \le x \le 4$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 3 in the Example 2

Assuming T: $(x, y) \longrightarrow (x, 4-y)$, and $f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0$, find $T \bullet f$.

Setting first $(s, t) = (x, 4-y)$, we get $s = x \Rightarrow x = s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $(x-2)^2 + (y-1)^2 - 2 = (s-2)^2 + (4-t-1)^2 - 2 = (s-2)^2 + (3-t)^2 - 2$ $= (s-2)^2 + (t-3)^2 - 2 = 0$.

Thus, we get $T \bullet f = (x-2)^2 + (y-3)^2 - 2 = 0$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (x_f, 4-y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (x_f, 4-y_f)$.

So we get $x_g = x_f \Rightarrow x_f = x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) \Rightarrow f(x_g, 4-y_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

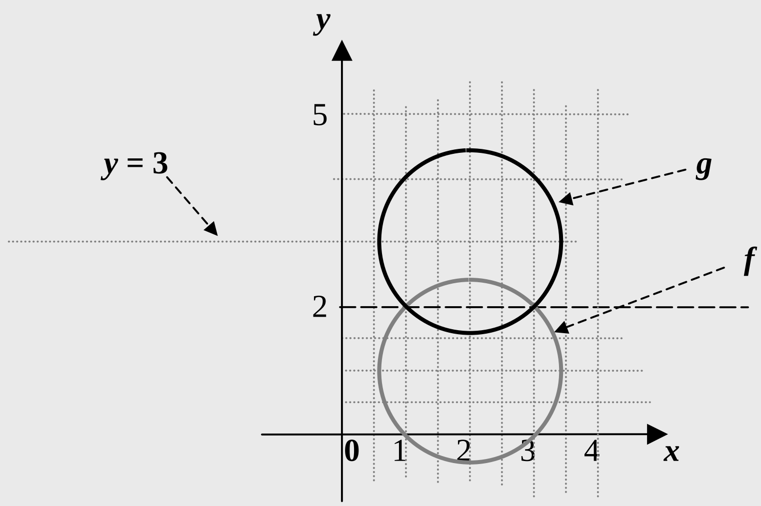
$$f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0 \Rightarrow f(x_f, y_f) = (x_f-2)^2 + (y_f-1)^2 - 2 = 0$$
$$\Rightarrow f(x_g, 4-y_g) = (x_g-2)^2 + (4-y_g-1)^2 - 2 = (x_g-2)^2 + (3-y_g)^2 - 2 = 0$$

$$\Rightarrow g(x_g, y_g) = (x_g-2)^2 + (3-y_g)^2 - 2 = 0 \Rightarrow g(x, y) = (x-2)^2 + (3-y)^2 - 2 = 0.$$

So we get $T \bullet f = (x-2)^2 + (3-y)^2 - 2 = (x-2)^2 + (y-3)^2 - 2 = 0$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Assuming $y = f(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq -1$, $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, and T: $(x, y) \longrightarrow (x, 6 - y)$, find $g = T \bullet f$, and $h = T \bullet w$.

Assuming first B is a transformation symmetric about a line $y = b$, where b is constant, we can put B the way as follows. B: $(x, y) \longrightarrow (x, 2b - y)$. And in this example, b is 3, so the axis of symmetry is a line $y = 3$.

So beginning with g, and assuming (x_g, y_g) is the arbitrary point in the curve of g, and the arbitrary point in the curve of f is (x_f, y_f), we can set $(x_g, y_g) = (x_f, 6 - y_f)$.

Then, we get $x_g = x_f$, and $y_g = 6 - y_f$, and thus, we get $x_f = x_g$, and $y_f = 6 - y_g$.

So next, connecting x_g and y_g, we put the x_f and y_f above into the original equation.
In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 6 - y_g = f(x_g) \Rightarrow y_g = 6 - f(x_g)$.

And we have $y = f(x) = (x + 1)(x - 1)(x - 2)$.

So we get $y_g = 6 - f(x_g) = 6 - (x_g + 1)(x_g - 1)(x_g - 2)$.

Thus, we get $y_g = 6 - (x_g + 1)(x_g - 1)(x_g - 2)$, which is the expression connecting x_g and y_g. And in a transformation symmetric about a line parallel to the x-axis, the new domain is the same as the original domain.

Thus, the new function g is $y = g(x) = 6 - (x + 1)(x - 1)(x - 2)$ for $x \geq -1$.

And next, moving on to $h = T \bullet w$, where $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, we can see that the only difference between the two functions w and f is the domain.

So the new function h is $y = h(x) = 6 - (x + 1)(x - 1)(x - 2)$ for $x \geq 2$.

That's because, in a transformation symmetric about a horizontal line, the domain of the original function remains, and thus, is the same as the domain of the new function.

Fig. 0

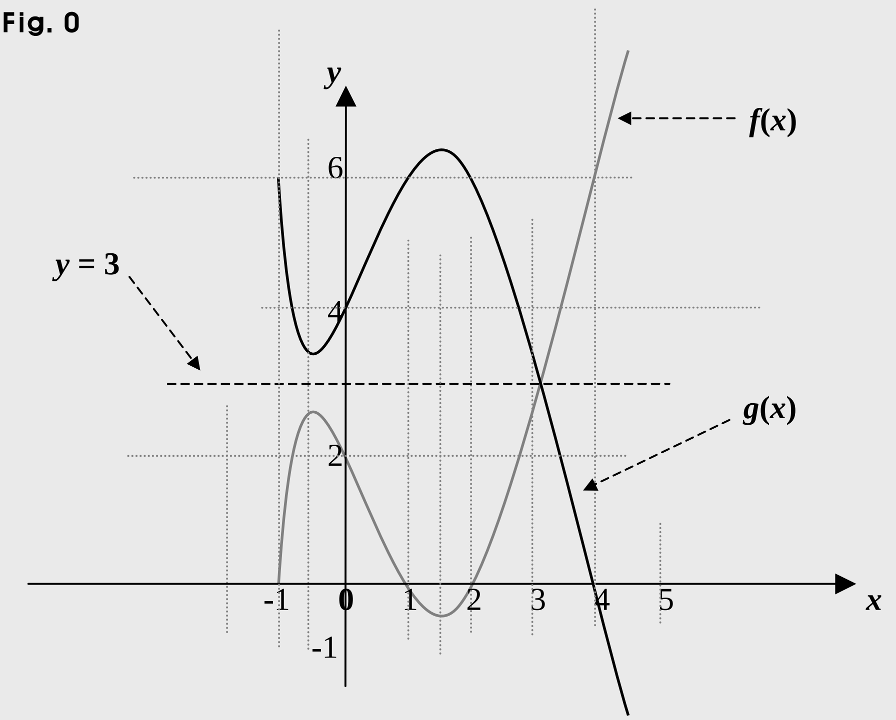


Fig. 1