Examples 1 in Symmetry about a line 2

Click this to start.

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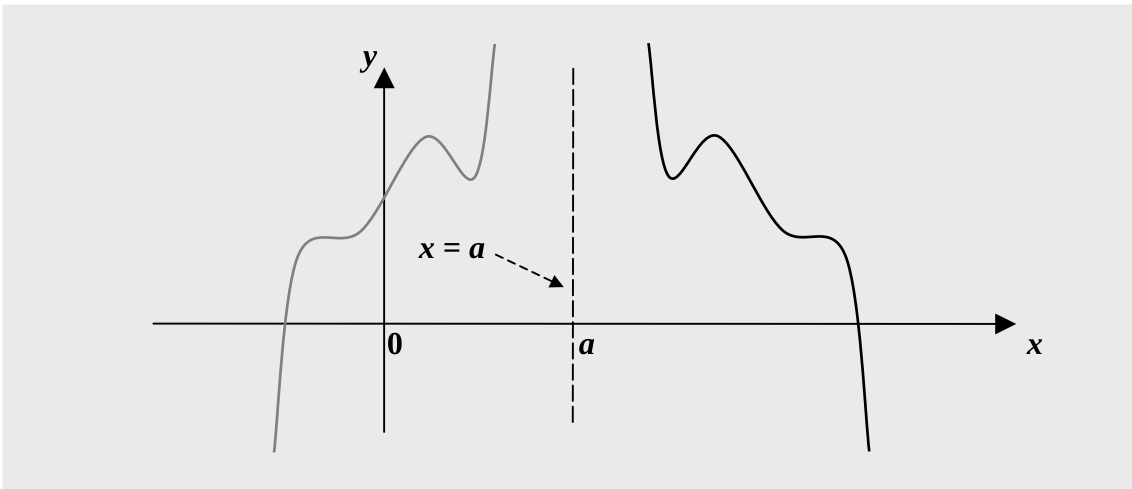


0. Assuming $y = f(x) = (1 - x)^3 + 2$, transform the function f by a transformation symmetric about a line $x = 2$.

1. Find a function, the curve of which is symmetric about the y-axis to the curve of a function $y = f(x) = \sqrt[3]{1 - x} + 2$.

2. Assuming $T: (x, y) \longrightarrow (2 - x, y)$, find the transformation of each f below by T.

2.0. $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.

2.1. $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming $y = f(x) = (1 - x)^3 + 2$, transform the function f by a transformation symmetric about a line $x = 2$.

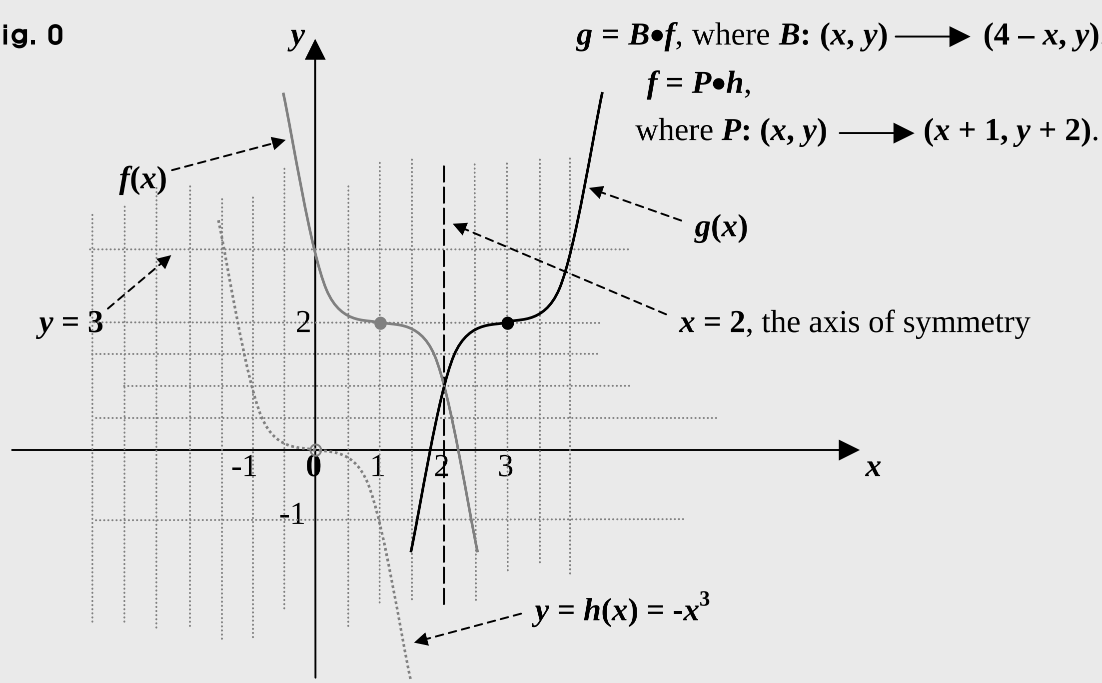
To begin with, assuming $\boldsymbol{A}$ is a transformation symmetric about a line $\boldsymbol{x = a}$, where $\boldsymbol{a}$ is constant, we can put $\boldsymbol{A}$ the way as follows. $\boldsymbol{A}$**:** $\boldsymbol{(x, y) \longrightarrow (2a - x, y)}$.

So since $\boldsymbol{a}$ is 2 in this example, assuming $\boldsymbol{B}$ is the transformation we want to apply, we can put $\boldsymbol{B}$ the way as follows. $\boldsymbol{B}$**:** $\boldsymbol{(x, y) \longrightarrow (4 - x, y)}$.

Assuming thus first, $\boldsymbol{g}$ is the new function, and $\boldsymbol{(s, t)}$ is the arbitrary point in the curve of $\boldsymbol{g}$, we can set $\boldsymbol{(s, t) = (4 - x, y)}$. So we get $\boldsymbol{s = 4 - x \Rightarrow x = 4 - s}$, and $\boldsymbol{t = y}$. Thus, we get $\boldsymbol{y = (1 - x)^3 + 2 \Rightarrow t = \{1 - (4 - s)\}^3 + 2 \Rightarrow t = (s - 3)^3 + 2}$.

Thus, the new function is $\boldsymbol{y = g(x) = (x - 3)^3 + 2}$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Find a function, the curve of which is symmetric about the *y*-axis to the curve of a function $y = f(x) = \sqrt[3]{1-x} + 2$.

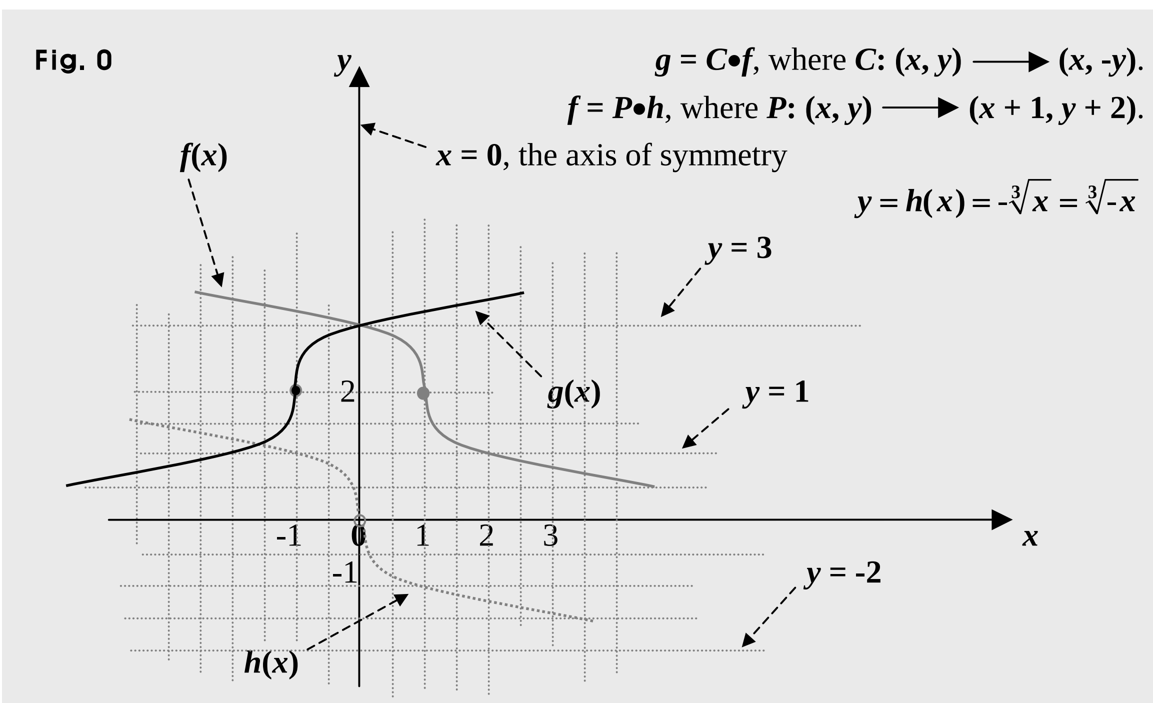
Assuming first ***A*** is a transformation symmetric about a line $\boldsymbol{x = a}$, where $\boldsymbol{a}$ is constant, we can put ***A*** the way as follows. $\boldsymbol{A: (x, y) \longrightarrow (2a - x, y)}$. And in this example, the axis of symmetry is the ***y***-axis, which is a line $\boldsymbol{x = 0}$, so $\boldsymbol{a}$ is 0 in ***A***, and thus, assuming ***C*** is the transformation we apply, we can put ***C*** the way as follows. $\boldsymbol{C: (x, y) \longrightarrow (-x, y)}$.

So assuming next, ***g*** is the new function, and $\boldsymbol{(s, t)}$ is the arbitrary point in the curve of ***g***, we can set $\boldsymbol{(s, t) = (-x, y)}$. So we get $\boldsymbol{s = -x \Rightarrow x = -s}$, and $\boldsymbol{t = y}$.

Thus, we get $\boldsymbol{y = \sqrt[3]{1-x} + 2 \Rightarrow t = \sqrt[3]{1+s} + 2}$.

Thus, the new function is $\boldsymbol{y = g(x) = \sqrt[3]{x+1} + 2}$.

Fig. 0



And we can get the solution the way below, too.

We know the transformation $\boldsymbol{A}$ is as follows. $\boldsymbol{A}$**:** $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (2\boldsymbol{a} - \boldsymbol{x}, \boldsymbol{y})$, which is the transformation symmetric about a line $\boldsymbol{x} = \boldsymbol{a}$.

So assuming again, $\boldsymbol{g}$ is the new function, we can put $\boldsymbol{A}$ the way below, too.

$\boldsymbol{A}$**:** $(x_f, y_f) \longrightarrow (x_g, y_g) = (2a - x_f, y_f)$.

And we know $\boldsymbol{C}$ is the transformation symmetric about a line $\boldsymbol{x} = \mathbf{0}$, which is the $\boldsymbol{y}$-axis.

So we can put $\boldsymbol{C}$ this way, too. $\boldsymbol{C}$**:** $(x_f, y_f) \longrightarrow (x_g, y_g) = (-x_f, y_f)$.

And briefly, we can just put it this way, too. $\boldsymbol{C}$**:** $(x_f, y_f) \longrightarrow (-x_f, y_f)$.

So we get $x_g = -x_f \Rightarrow x_f = -x_g$, and $y_g = y_f$. Thus, we can now get the mixture.

That is, putting $x_f = -x_g$ and $y_f = y_g$ into the original equation $y = \sqrt[3]{1-x} + 2$, we get the equation connecting x_g and y_g, that is, the new equation. In other words, we get

$y = f(x) = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow y_g = \sqrt[3]{1+x_g} + 2$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function $\boldsymbol{g}$. So the new function is $y = g(x) = \sqrt[3]{x+1} + 2$.

In short:

Assuming first, $\boldsymbol{g}$ is the new function, we have $(x_g, y_g) = (-x_f, y_f)$.

So we get $y = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow y_g = \sqrt[3]{1+x_g} + 2$

Thus, the new function is $y = g(x) = \sqrt[3]{x+1} + 2$.

And we can get it the way below, too.

Assuming first, $\boldsymbol{g}$ is the new function, and $(\boldsymbol{s}, \boldsymbol{t})$ is the arbitrary point in the curve of $\boldsymbol{g}$, we can set $(s, t) = (-x, y)$. So we get $s = -x \Rightarrow x = -s$, and $t = y$.

Thus, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow t = \sqrt[3]{1+s} + 2 \Rightarrow y = g(x) = \sqrt[3]{x+1} + 2$.

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get $y = x^2 - 2x + 1 \Rightarrow t = (2 - s)^2 - 2(2 - s) + 1 = s^2 - 2s + 1$.

Besides, $x \geq 1 \Rightarrow -x \leq -1 \Rightarrow 2 - x \leq 1 \Rightarrow s \leq 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = x^2 - 2x + 1$ for $x \leq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_g = y_f$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$y = f(x) = x^2 - 2x + 1 \Rightarrow y_f = f(x_f) = x_f^2 - 2x_f + 1$

$\Rightarrow y_g = f(2 - x_g) = (2 - x_g)^2 - 2(2 - x_g) + 1 \Rightarrow y_g = (2 - x_g)^2 - 2(2 - x_g) + 1$

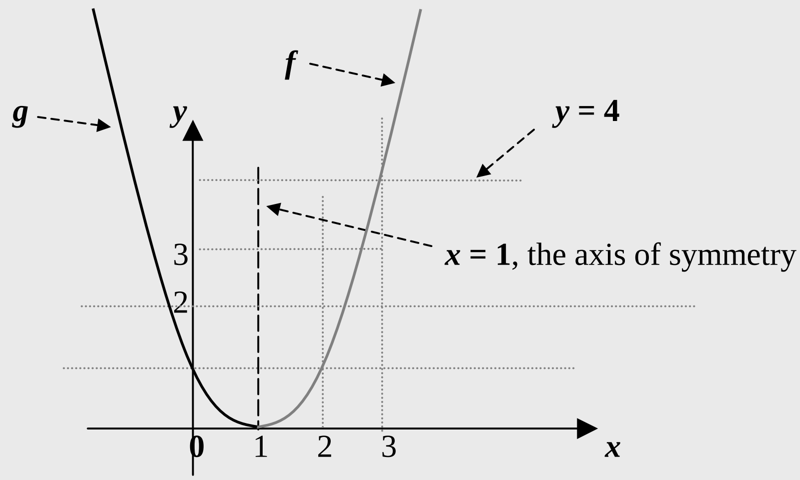
$\Rightarrow y_g = g(x_g) = (2 - x_g)^2 - 2(2 - x_g) + 1 = x_g^2 - 2x_g + 1 \Rightarrow y = g(x) = x^2 - 2x + 1$.

Besides, $x_f \geq 1 \Rightarrow -x_f \leq -1 \Rightarrow 2 - x_f \leq 1 \Rightarrow x_g \leq 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = x^2 - 2x + 1$ for $x \leq 1$.

And putting the two curves in a graph, we get

Fig. 0



$g = T \bullet f$, where T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.

Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \le 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get $y = (1 - x)^3 + 2 \Rightarrow t = \{1 - (2 - s)\}^3 + 2 \Rightarrow t = (s - 1)^3 + 2$.

Besides, $x \le 1 \Rightarrow -x \ge -1 \Rightarrow 2 - x \ge 1 \Rightarrow s \ge 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = (x - 1)^3 + 2$ for $x \ge 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_g = y_f$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$y = f(x) = (1 - x)^3 + 2 \Rightarrow y_f = f(x_f) = (1 - x_f)^3 + 2 \Rightarrow y_g = \{1 - (2 - x_g)\}^3 + 2$

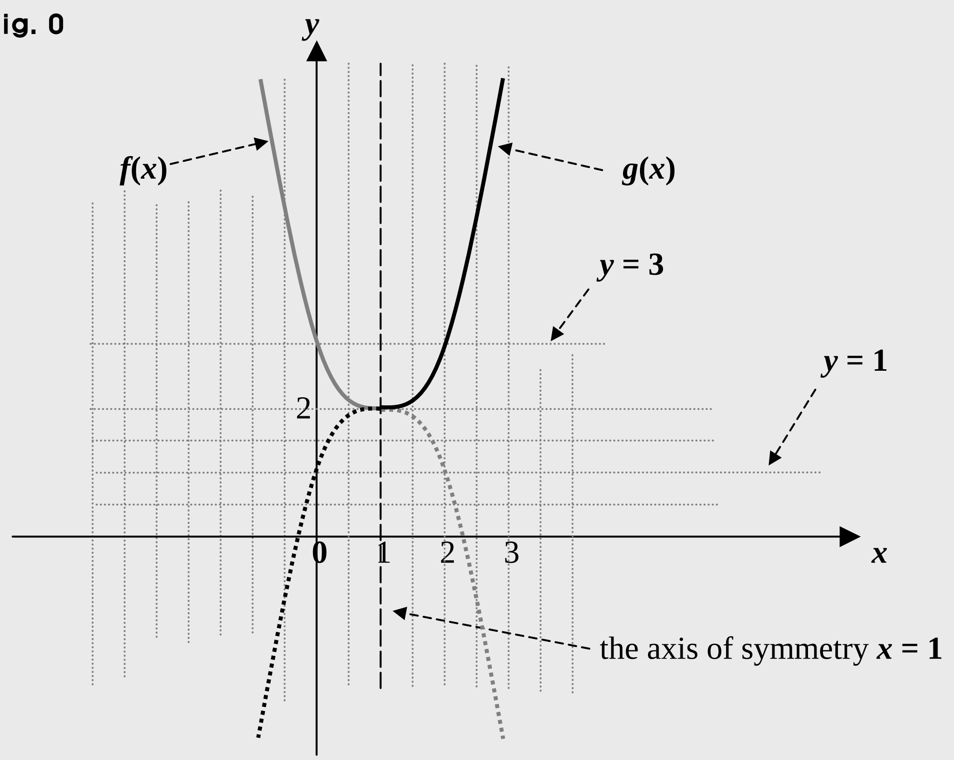
$\Rightarrow y_g = (x_g - 1)^3 + 2 \Rightarrow y_g = g(x_g) = (x_g - 1)^3 + 2 \Rightarrow y = g(x) = (x - 1)^3 + 2$.

Besides, $x_f \le 1 \Rightarrow -x_f \ge -1 \Rightarrow 2 - x_f \ge 1 \Rightarrow x_g \ge 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = (x - 1)^3 + 2$ for $x \ge 1$.

And putting the two curves in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (2 - x, y)}$, and $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \leq 1}$.