Examples 2 in Symmetry about a line 2

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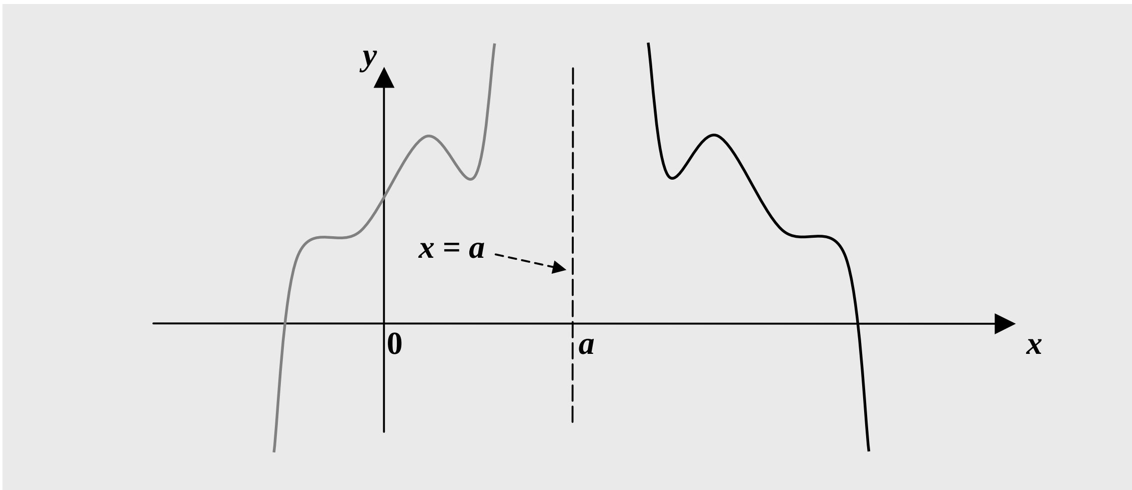


0. Assuming $\boldsymbol{T}$**:** $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (\mathbf{2} - \boldsymbol{x}, \boldsymbol{y})$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{T}$.

0.0. $y = f(x) = \sqrt[3]{1-x} + 2$ for $x \geq 1$.

0.1. $y = f(x) = -\sqrt{x+1} + 2$ for $-1 \leq x \leq 3$.

0.2. $y = f(x) = (x-2)^2 + 1$ for $1 \leq x \leq 4$.

0.3. $f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0$.

1. Assuming $y = f(x) = (x+1)(x-1)(x-2)$ for $x \geq -1$, $y = w(x) = (x+1)(x-1)(x-2)$ for $x \geq 2$, and T: $(x, y) \longrightarrow (2 - x, y)$, find $g = T \bullet f$, and $h = T \bullet w$.

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = \sqrt[3]{1-x} + 2$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow t = \sqrt[3]{1-(2-s)} + 2 \Rightarrow t = \sqrt[3]{s-1} + 2$.

Besides, $x \geq 1 \Rightarrow -x \leq -1 \Rightarrow 2 - x \leq 1 \Rightarrow s \leq 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = \sqrt[3]{x-1} + 2$ for $x \leq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_f = y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$y = f(x) = \sqrt[3]{1-x} + 2 \Rightarrow y_f = f(x_f) = \sqrt[3]{1-x_f} + 2 \Rightarrow y_g = \sqrt[3]{1-(2-x_g)} + 2$

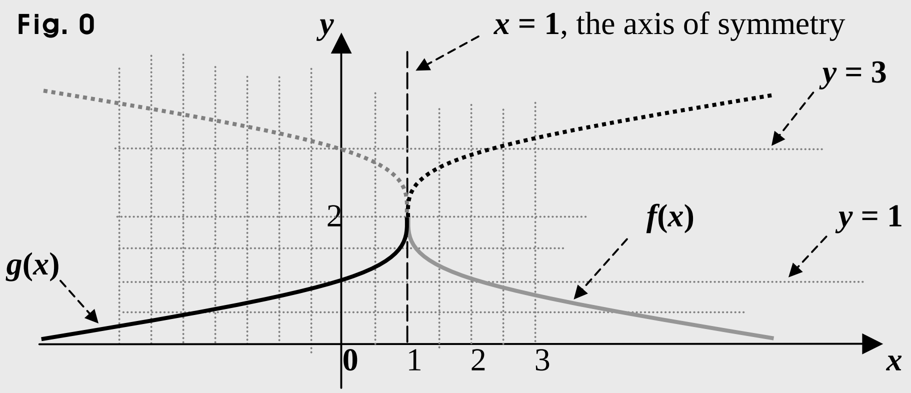
$\Rightarrow y_g = \sqrt[3]{x_g - 1} + 2 \Rightarrow y_g = g(x_g) = \sqrt[3]{x_g - 1} + 2 \Rightarrow y = g(x) = \sqrt[3]{x-1} + 2$.

Besides, $x_f \geq 1 \Rightarrow -x_f \leq -1 \Rightarrow 2 - x_f \leq 1 \Rightarrow x_g \leq 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = \sqrt[3]{x-1} + 2$ for $x \leq 1$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = -\sqrt{x+1}+2$ for $-1 \le x \le 3$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get $y = -\sqrt{x+1}+2 \;\Rightarrow t = -\sqrt{2-s+1}+2 \;\Rightarrow t = 2-\sqrt{3-s}$

And $-1 \le x \le 3 \Rightarrow 1 \ge -x \ge -3 \Rightarrow 3 \ge 2 - x \ge -1 \Rightarrow -1 \le s \le 3$, because $s = 2 - x$.

So we get $T \bullet f = 2-\sqrt{3-x}$ for $-1 \le x \le 3$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_f = y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = -\sqrt{x+1}+2 \;\Rightarrow y_f = f(x_f) = -\sqrt{x_f+1}+2 \;\Rightarrow y_g = -\sqrt{2-x_g+1}+2$$

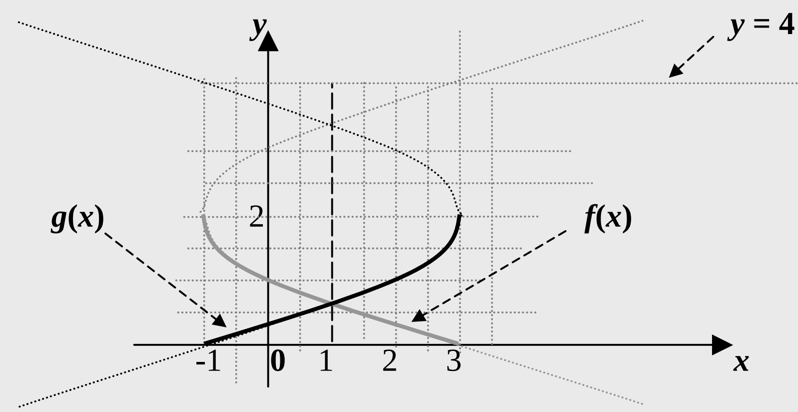
$$\Rightarrow y_g = 2-\sqrt{3-x_g} \;\Rightarrow y_g = g(x_g) = 2-\sqrt{3-x_g} \;\Rightarrow y = g(x) = 2-\sqrt{3-x}$$

Besides, $-1 \le x_f \le 3 \Rightarrow 1 \ge -x_f \ge -3 \Rightarrow 3 \ge 2 - x_f \ge -1 \Rightarrow -1 \le x_g \le 3$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = 2-\sqrt{3-x}$ for $-1 \le x \le 3$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 2 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $y = f(x) = (x - 2)^2 + 1$ for $1 \le x \le 4$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get $y = (x - 2)^2 + 1 \Rightarrow t = (2 - s - 2)^2 + 1 \Rightarrow t = s^2 + 1$.

And $1 \le x \le 4 \Rightarrow -1 \ge -x \ge -4 \Rightarrow 1 \ge 2 - x \ge -2 \Rightarrow -2 \le s \le 1$, because $s = 2 - x$.

So we get $T \bullet f = x^2 + 1$ for $-2 \le x \le 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_f = y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$y = f(x) = (x - 2)^2 + 1 \Rightarrow y_f = f(x_f) = (x_f - 2)^2 + 1 \Rightarrow y_g = (2 - x_g - 2)^2 + 1$

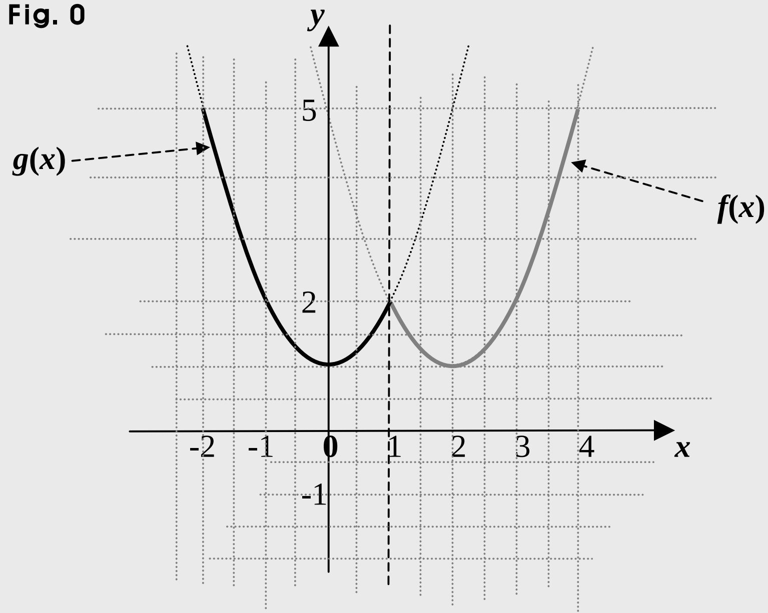
$\Rightarrow y_g = x_g^2 + 1 \Rightarrow y_g = g(x_g) = x_g^2 + 1 \Rightarrow y = g(x) = x^2 + 1$.

Besides, $1 \le x_f \le 4 \Rightarrow -1 \ge -x_f \ge -4 \Rightarrow 1 \ge 2 - x_f \ge -2 \Rightarrow -2 \le x_g \le 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = x^2 + 1$ for $-2 \le x \le 1$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem 3 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, y)$, and $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = y$.

So next, we get
$(x - 2)^2 + (y - 1)^2 - 2 = (2 - s - 2)^2 + (t - 1)^2 - 2 = s^2 + (t - 1)^2 - 2 = s^2 + (t - 1)^2 - 2 = 0$.

Thus, we get $T \bullet f = x^2 + (y - 1)^2 - 2 = 0$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = y_f$.

And next, putting $x_f = 2 - x_g$ and $y_f = y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) \Rightarrow f(2 - x_g, y_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

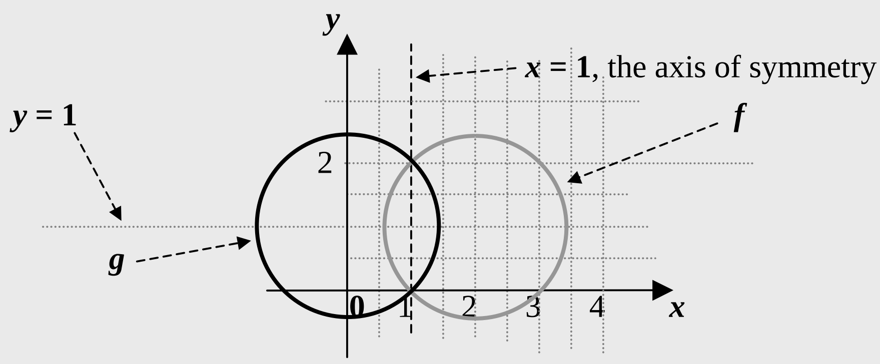
$$f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0 \Rightarrow f(x_f, y_f) = (x_f - 2)^2 + (y_f - 1)^2 - 2 = 0$$
$$\Rightarrow f(2 - x_g, y_g) = (2 - x_g - 2)^2 + (y_g - 1)^2 - 2 = x_g^2 + (y_g - 1)^2 - 2 = 0$$

$$\Rightarrow g(x_g, y_g) = x_g^2 + (y_g - 1)^2 - 2 = 0 \Rightarrow g(x, y) = x^2 + (y - 1)^2 - 2 = 0.$$

So we get $T \bullet f = x^2 + (y - 1)^2 - 2 = 0$.

And putting the two curves in a graph, we get

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Assuming $y = f(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq -1$, $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, and T: $(x, y) \longrightarrow (2 - x, y)$, find $g = T \bullet f$, and $h = T \bullet w$.

Assuming first A is a transformation symmetric about a line $x = a$, where a is constant, we can put A the way as follows. A: $(x, y) \longrightarrow (2a - x, y)$. And in this example, a is 1, so the axis of symmetry is a line $x = 1$.

So beginning with g, and assuming (x_g, y_g) is the arbitrary point in the curve of g, and the arbitrary point in the curve of f is (x_f, y_f), we can set $(x_g, y_g) = (2 - x_f, y_f)$.

Then, we get $x_g = 2 - x_f$, and $y_g = y_f$, and thus, we get $x_f = 2 - x_g$, and $y_f = y_g$.

So next, connecting x_g and y_g, we put the x_f and y_f above into the original equation.
In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow y_g = f(2 - x_g)$.

And we have $y = f(x) = (x + 1)(x - 1)(x - 2)$.

So we get $y_g = f(2 - x_g) = (2 - x_g + 1)(2 - x_g - 1)(2 - x_g - 2)$.

Thus, we get $y_g = (3 - x_g)(1 - x_g)x_g$, which is the expression connecting x_g and y_g. And in a transformation symmetric about a line parallel to the y-axis, the new domain can be different from the original domain.

So finding the new domain, we get $x_f \geq -1 \Rightarrow -x_f \leq 1 \Rightarrow 2 - x_f \leq 3 \Rightarrow x_g \leq 3$, because we have $x_g = 2 - x_f$. And thus, the new function g is $y = g(x) = x(x - 3)(x - 1)$ for $x \leq 3$.

And next, moving on to $h = T \bullet w$, where $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, we can see that the only difference between the two functions w and f is the domain.

So finding the domain in h, we get $x_f \geq 2 \Rightarrow -x_f \leq -2 \Rightarrow 2 - x_f \leq 0 \Rightarrow x_h \leq 0$.

So the new function h is $y = h(x) = x(x - 3)(x - 1)$ for $x \leq 0$.

Fig. 0

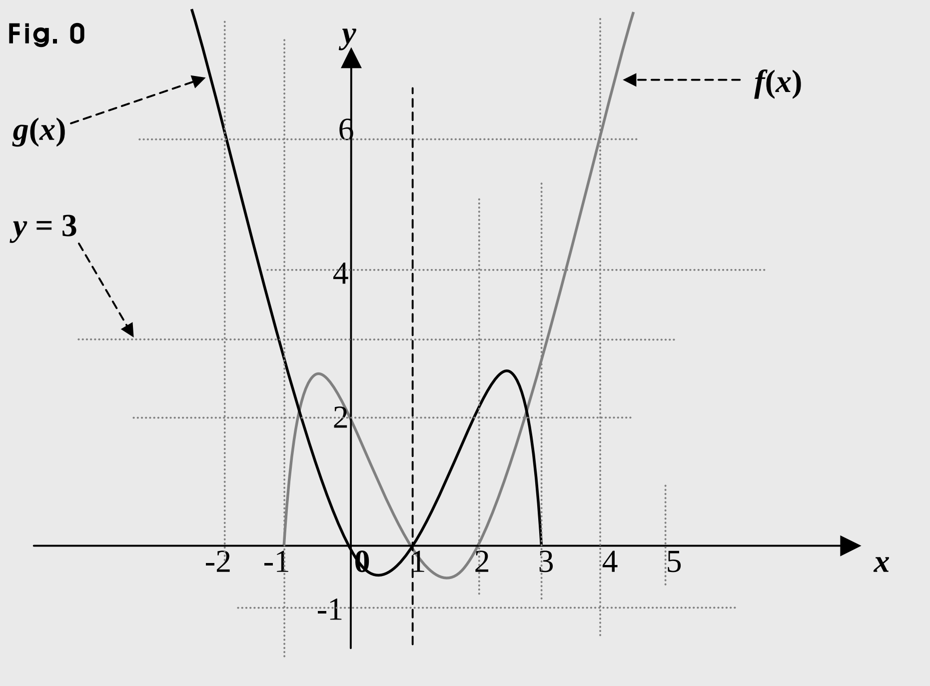


Fig. 1