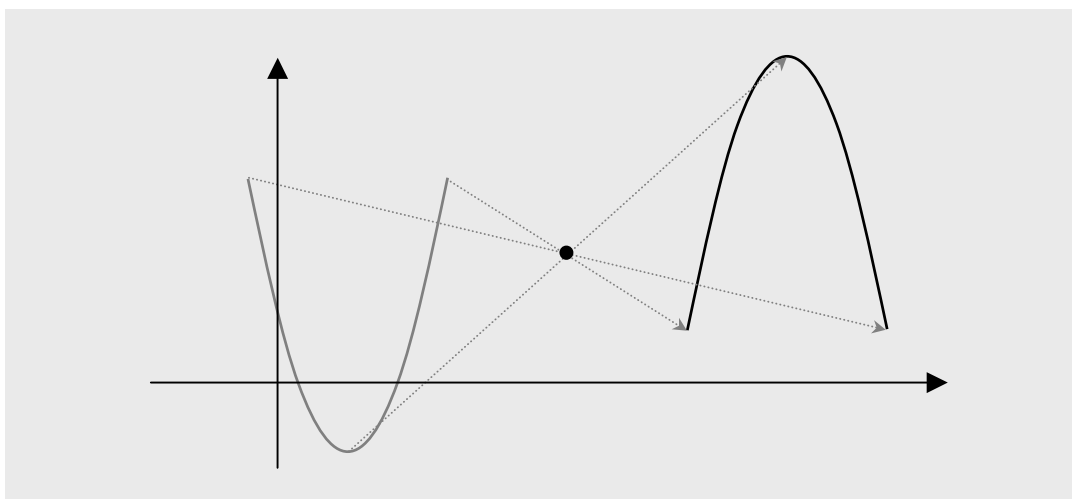


Examples 1 in Symmetry about a point

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Examples 1 in Symmetry about a Point



0. Assuming $y = f(x) = (1 - x)^3 + 2$, transform the function f by a transformation symmetric about a point $(1, 2)$.

1. Find a function, the curve of which is symmetric about the origin to the curve of a function $y = f(x) = \sqrt[3]{1 - x} + 2$.

2. Assuming $T: (x, y) \longrightarrow (2 - x, 4 - y)$, find $T \bullet f$ in each case below.
 - 2.0. $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.
 - 2.1. $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$.

Suggestions or Solutions To the Problem in the Example 0

Assuming $y = f(x) = (1 - x)^3 + 2$, transform the function f by a transformation symmetric about a point $(1, 2)$.

To begin with, assuming C is a transformation symmetric about a point (a, b) , we can put C the way as follows. $C: (x, y) \longrightarrow (2a - x, 2b - y)$.

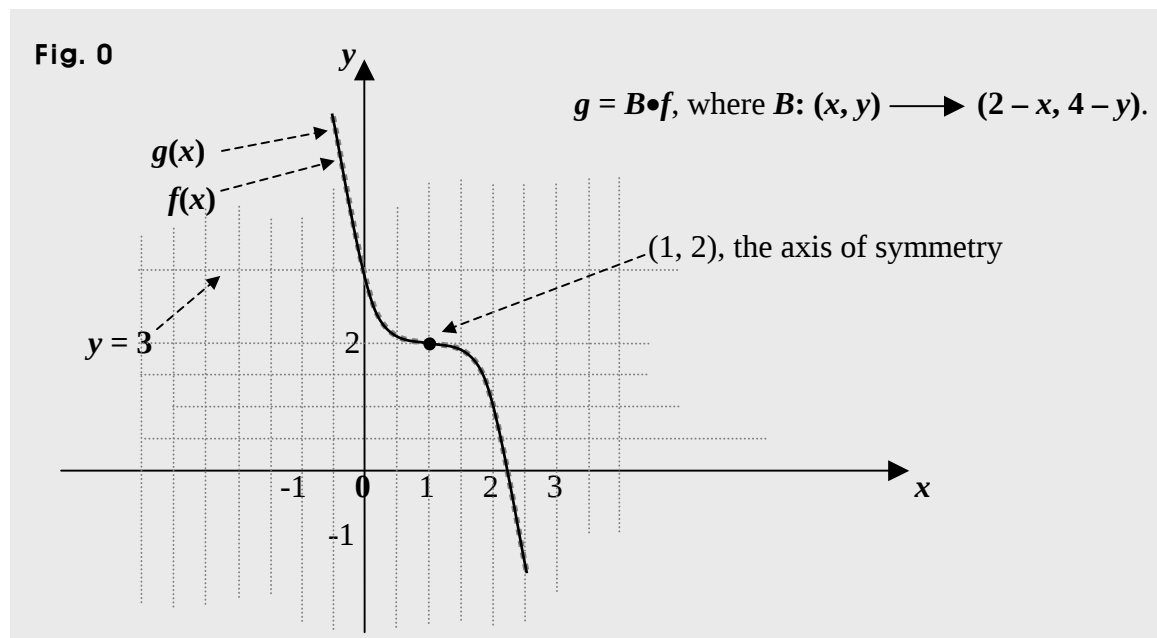
So since $a = 1$, and $b = 2$ in this example, assuming B is the transformation we want to apply, we can put B the way as follows. $B: (x, y) \longrightarrow (2 - x, 4 - y)$.

Assuming thus, first, g is the new function, and (s, t) is the arbitrary point in the curve of g , we can set $(s, t) = (2 - x, 4 - y)$.

So we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$. Thus, we get

$$y = (1 - x)^3 + 2 \Rightarrow 4 - t = \{1 - (2 - s)\}^3 + 2 \Rightarrow t = (1 - s)^3 + 2.$$

Thus, the new function is $y = g(x) = (1 - x)^3 + 2$.



Suggestions or Solutions To the Problem in the Example 1

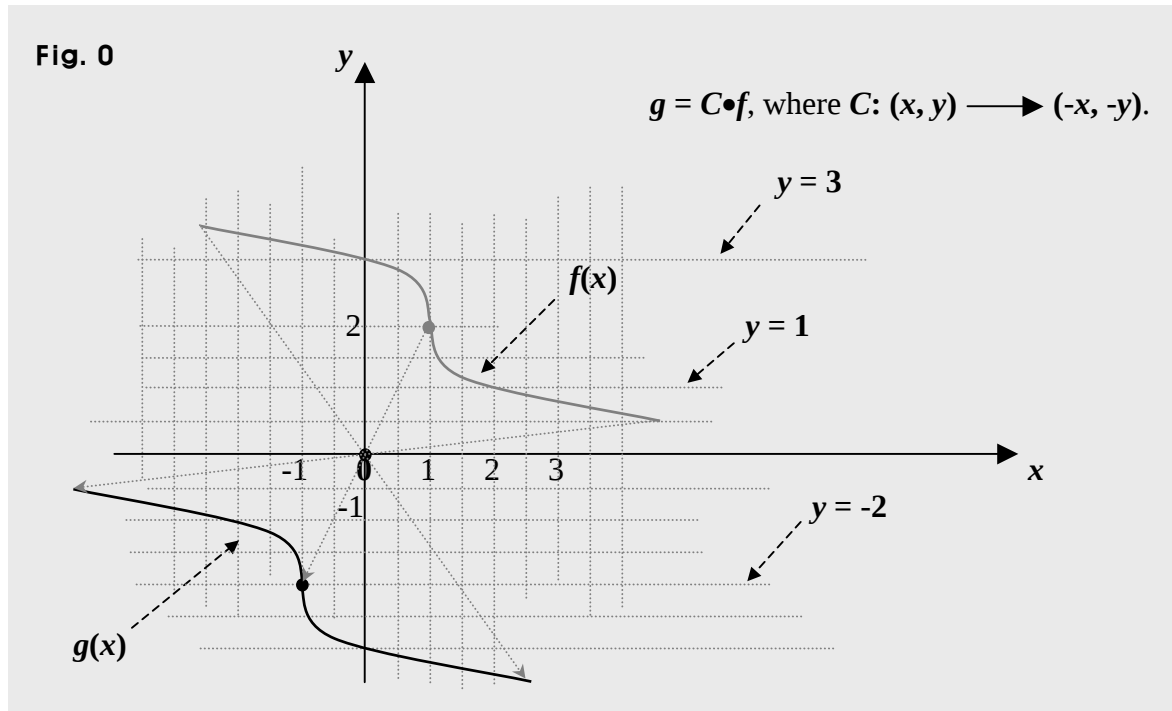
Find a function, the curve of which is symmetric about the origin to the curve of a function $y = f(x) = \sqrt[3]{1-x} + 2$.

Assuming first C is a transformation symmetric about a point (a, b) , we can put A the way as follows. $C: (x, y) \longrightarrow (2a - x, 2b - y)$. And in this example, the axis of symmetry is the origin, which is $(0, 0)$, so assuming B is the transformation we apply, we can put D the way as follows. $C: (x, y) \longrightarrow (-x, -y)$.

So assuming next, g is the new function, and (s, t) is the arbitrary point in the curve of g , we can set $(s, t) = (-x, -y)$. So we get $s = -x \Rightarrow x = -s$, and $t = y \Rightarrow y = -t$.

Thus, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow -t = \sqrt[3]{1+s} + 2 \Rightarrow t = -\sqrt[3]{1+s} - 2$.

So the new function is $y = g(x) = -\sqrt[3]{x+1} - 2$.



And we can get the solution the way below, too.

We know the transformation C is as follows. $C: (x, y) \longrightarrow (2a - x, 2b - y)$, which is the transformation symmetric about a point (a, b) .

So assuming again, g is the new function, we can put C the way below, too.

$$A: (x_f, y_f) \longrightarrow (x_g, y_g) = (2a - x_f, 2b - y_f).$$

And we know B is the transformation symmetric about the origin, which is $(0, 0)$.

$$B: (x_f, y_f) \longrightarrow (x_g, y_g) = (-x_f, -y_f).$$

$$B: (x_f, y_f) \longrightarrow (-x_f, -y_f).$$

So we get $x_g = -x_f \Rightarrow x_f = -x_g$, and $y_g = -y_f \Rightarrow y_f = -y_g$. We can now thus, get the mixture.

That is, putting $x_f = -x_g$ and $y_f = -y_g$ into the original equation $y = \sqrt[3]{1-x} + 2$, we get the equation connecting x_g and y_g , that is, the new equation. In other words, we get

$$y = f(x) = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow -y_g = \sqrt[3]{1+x_g} + 2 \Rightarrow y_g = -\sqrt[3]{x_g+1} - 2,$$

which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g . So the new function is $y = g(x) = -\sqrt[3]{x+1} - 2$.

In short:

Assuming first, g is the new function, we have $(x_g, y_g) = (-x_f, -y_f)$. So we get

$$y = \sqrt[3]{1-x} + 2 \Rightarrow y_f = \sqrt[3]{1-x_f} + 2 \Rightarrow -y_g = \sqrt[3]{1+x_g} + 2 \Rightarrow y_g = -\sqrt[3]{1+x_g} - 2$$

And thus, the new function is $y = g(x) = -\sqrt[3]{x+1} - 2$.

And we can get it the way below, too.

Assuming first, g is the new function, and (s, t) is the arbitrary point in the curve of g , we can set $(s, t) = (-x, -y)$. So we get $s = -x \Rightarrow x = -s$, and $t = y \Rightarrow y = -t$. Thus, we get

$$y = \sqrt[3]{1-x} + 2 \Rightarrow -t = \sqrt[3]{1+s} + 2 \Rightarrow t = -\sqrt[3]{1+s} - 2 \Rightarrow y = g(x) = -\sqrt[3]{x+1} - 2.$$

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming $T: (x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, 4 - y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get

$$y = x^2 - 2x + 1 \Rightarrow 4 - t = (2 - s)^2 - 2(2 - s) + 1 = s^2 - 2s + 1 \Rightarrow t = -s^2 + 2s + 3.$$

Besides, $x \geq 1 \Rightarrow -x \leq -1 \Rightarrow 2 - x \leq 1 \Rightarrow s \leq 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = -x^2 + 2x + 3$ for $x \leq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

$T: (x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, $T: (x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_g = 4 - y_f$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g , that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g .

And putting the idea above more specifically, we get

$$y = f(x) = x^2 - 2x + 1 \Rightarrow y_f = f(x_f) = x_f^2 - 2x_f + 1$$

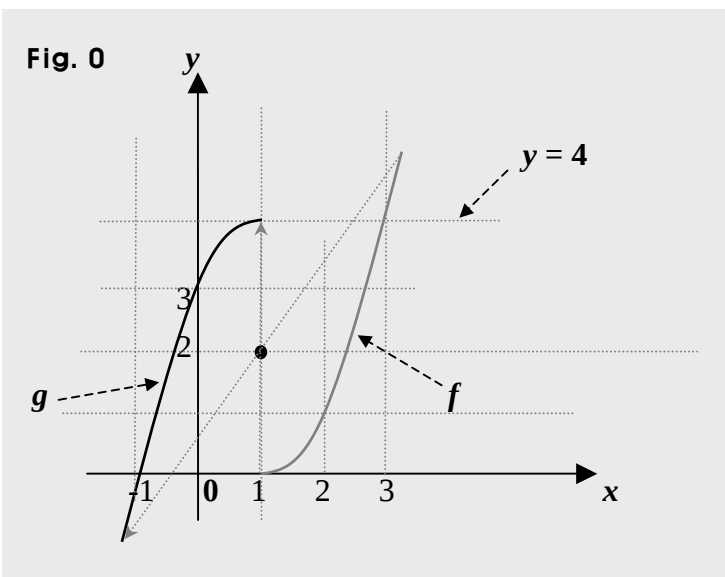
$$\Rightarrow 4 - y_g = f(2 - x_g) = (2 - x_g)^2 - 2(2 - x_g) + 1 \Rightarrow y_g = -(2 - x_g)^2 + 2(2 - x_g) + 3$$

$$\Rightarrow y_g = g(x_g) = -(2 - x_g)^2 + 2(2 - x_g) + 3 = -x_g^2 + 2x_g + 3 \Rightarrow y = g(x) = -x^2 + 2x + 3.$$

Besides, $x_f \geq 1 \Rightarrow -x_f \leq -1 \Rightarrow 2 - x_f \leq 1 \Rightarrow x_g \leq 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = -x^2 + 2x + 3$ for $x \leq 1$.

And putting the two curves in a graph, we get



$g = T \bullet f$, where $T: (x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.

Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming $T: (x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, 4 - y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = (1 - x)^3 + 2 \Rightarrow 4 - t = \{1 - (2 - s)\}^3 + 2 \Rightarrow t = -(s - 1)^3 + 2$.

Besides, $x \leq 1 \Rightarrow -x \geq -1 \Rightarrow 2 - x \geq 1 \Rightarrow s \geq 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = (1 - x)^3 + 2$ for $x \geq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

$T: (x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, $T: (x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g , that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g .

And putting the idea above more specifically, we get

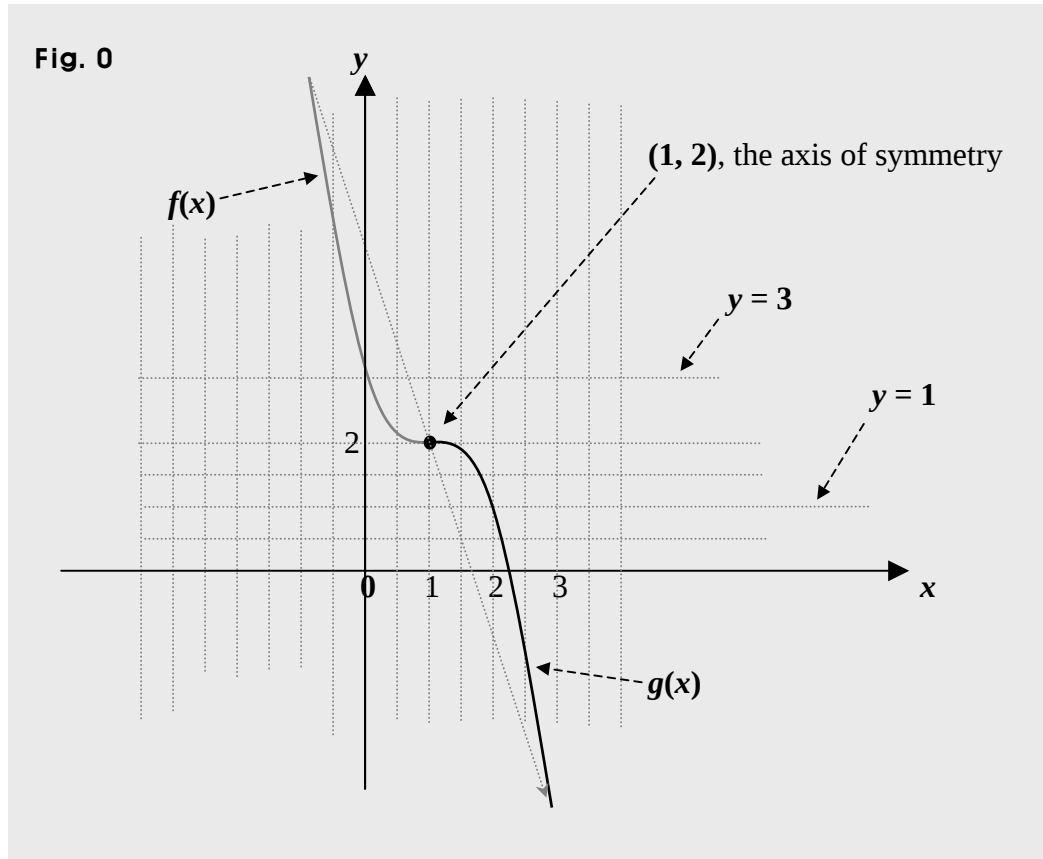
$$y = f(x) = (1 - x)^3 + 2 \Rightarrow y_f = f(x_f) = (1 - x_f)^3 + 2 \Rightarrow 4 - y_g = \{1 - (2 - x_g)\}^3 + 2$$

$$\Rightarrow y_g = -(x_g - 1)^3 + 2 \Rightarrow y_g = g(x_g) = (1 - x_g)^3 + 2 \Rightarrow y = g(x) = (1 - x)^3 + 2.$$

Besides, $x_f \leq 1 \Rightarrow -x_f \geq -1 \Rightarrow 2 - x_f \geq 1 \Rightarrow x_g \geq 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = (1 - x)^3 + 2$ for $x \geq 1$.

And putting the two curves in a graph, we get



$g = T \bullet f$, where $T: (x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$.