Examples 2 in Symmetry about a point

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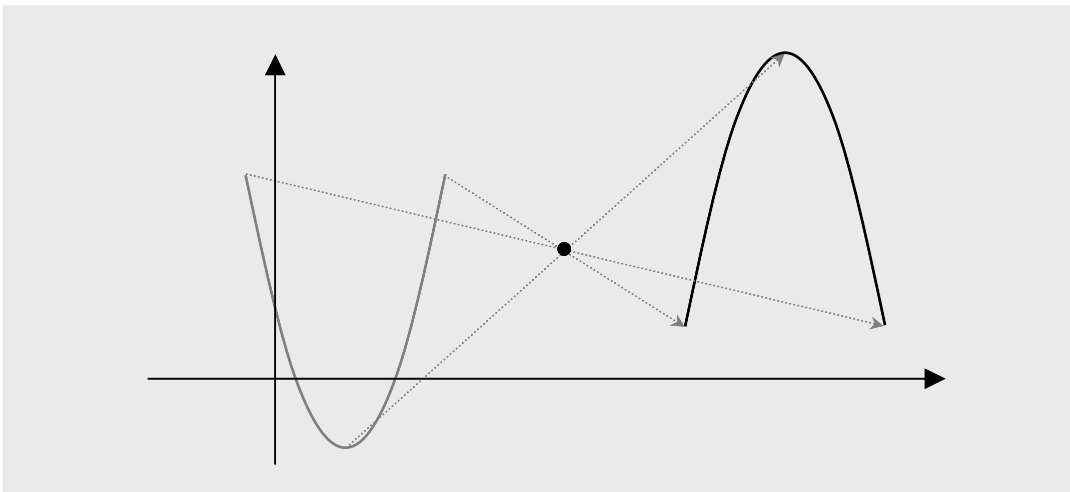


0. Assuming $\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (2 - x, 4 - y)}$, find $\boldsymbol{T \bullet f}$ in each case below.

0.0. $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x \geq 1}$.

0.1. $\boldsymbol{y = f(x) = -\sqrt{x + 1} + 2}$ for $\boldsymbol{-1 \leq x \leq 3}$.

0.2. $\boldsymbol{y = f(x) = (x - 2)^2 + 1}$ for $\boldsymbol{1 \leq x \leq 4}$.

0.3. $\boldsymbol{f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0}$.

1. Find an equation, the curve of which is symmetric about a point **(1, 2)** to the curve of an equation $\boldsymbol{f(x, y) = \frac{x^2y^2}{xy + \frac{3}{2}} - 2 = 0}$.

Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = \sqrt[3]{1 - x} + 2$ for $x \geq 1$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, 4 - y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = \sqrt[3]{1-x} + 2 \Rightarrow 4 - t = \sqrt[3]{1-(2-s)} + 2 \Rightarrow t = -\sqrt[3]{s-1} + 2 = \sqrt[3]{1-s} + 2$.

Besides, $x \geq 1 \Rightarrow -x \leq -1 \Rightarrow 2 - x \leq 1 \Rightarrow s \leq 1$, because $s = 2 - x$.

Thus, we get $T \bullet f = \sqrt[3]{1-x} + 2$ for $x \leq 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = \sqrt[3]{1-x} + 2 \;\Rightarrow y_f = f(x_f) = \sqrt[3]{1-x_f} + 2 \;\Rightarrow 4 - y_g = \sqrt[3]{1-(2-x_g)} + 2$$

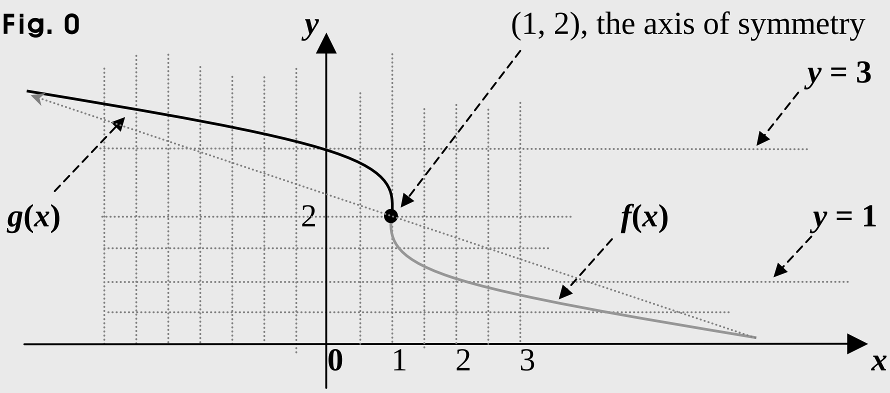
$$\Rightarrow y_g = -\sqrt[3]{x_g - 1} + 2 \;\Rightarrow y_g = g(x_g) = \sqrt[3]{1-x_g} + 2 \;\Rightarrow y = g(x) = \sqrt[3]{1-x} + 2.$$

Besides, $x_f \geq 1 \Rightarrow -x_f \leq -1 \Rightarrow 2 - x_f \leq 1 \Rightarrow x_g \leq 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = \sqrt[3]{1-x} + 2$ for $x \leq 1$.

And putting the two curves in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (2 - x, 4 - y)}$, and $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x \geq 1}$.

Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = -\sqrt{x+1}+2$ for $-1 \le x \le 3$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, 4 - y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = -\sqrt{x+1}+2 \Rightarrow 4 - t = -\sqrt{2-s+1}+2 \Rightarrow t = 2+\sqrt{3-s}$

And $-1 \le x \le 3 \Rightarrow 1 \ge -x \ge -3 \Rightarrow 3 \ge 2 - x \ge -1 \Rightarrow -1 \le s \le 3$, because $s = 2 - x$.

So we get $T \bullet f = 2+\sqrt{3-x}$ for $-1 \le x \le 3$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = -\sqrt{x+1}+2 \Rightarrow y_f = f(x_f) = -\sqrt{x_f+1}+2 \Rightarrow 4 - y_g = -\sqrt{2-x_g+1}+2$$

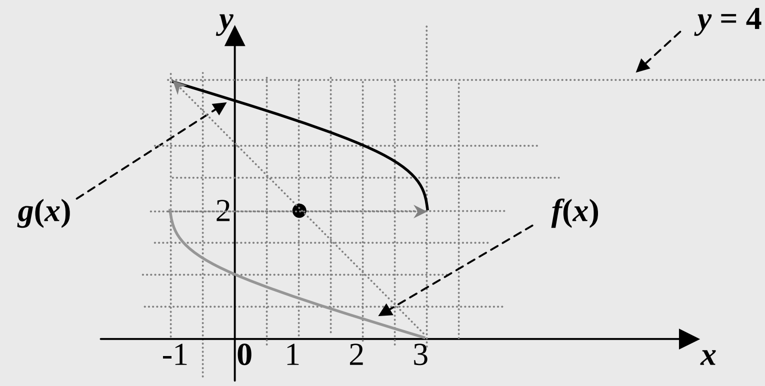
$$\Rightarrow y_g = 2+\sqrt{3-x_g} \Rightarrow y_g = g(x_g) = 2+\sqrt{3-x_g} \Rightarrow y = g(x) = 2+\sqrt{3-x}$$

Besides, $-1 \le x_f \le 3 \Rightarrow 1 \ge -x_f \ge -3 \Rightarrow 3 \ge 2 - x_f \ge -1 \Rightarrow -1 \le x_g \le 3$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$. So we get $T \bullet f = 2+\sqrt{3-x}$ for $-1 \le x \le 3$.

And putting the two curves in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (2 - x, 4 - y)}$, and $\boldsymbol{y = f(x) = -\sqrt{x+1} + 2}$ for $\boldsymbol{-1 \leq x \leq 3}$.

Suggestions or Solutions
To the Problem 2 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, 4 - y)$, and $y = f(x) = (x - 2)^2 + 1$ for $1 \le x \le 4$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get $y = (x - 2)^2 + 1 \Rightarrow 4 - t = (2 - s - 2)^2 + 1 \Rightarrow t = -s^2 + 3$.

And $1 \le x \le 4 \Rightarrow -1 \ge -x \ge -4 \Rightarrow 1 \ge 2 - x \ge -2 \Rightarrow -2 \le s \le 1$, because $s = 2 - x$.

So we get $T \bullet f = -x^2 + 3$ for $-2 \le x \le 1$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow 4 - y_g = f(2 - x_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$y = f(x) = (x - 2)^2 + 1 \Rightarrow y_f = f(x_f) = (x_f - 2)^2 + 1 \Rightarrow 4 - y_g = (2 - x_g - 2)^2 + 1$$

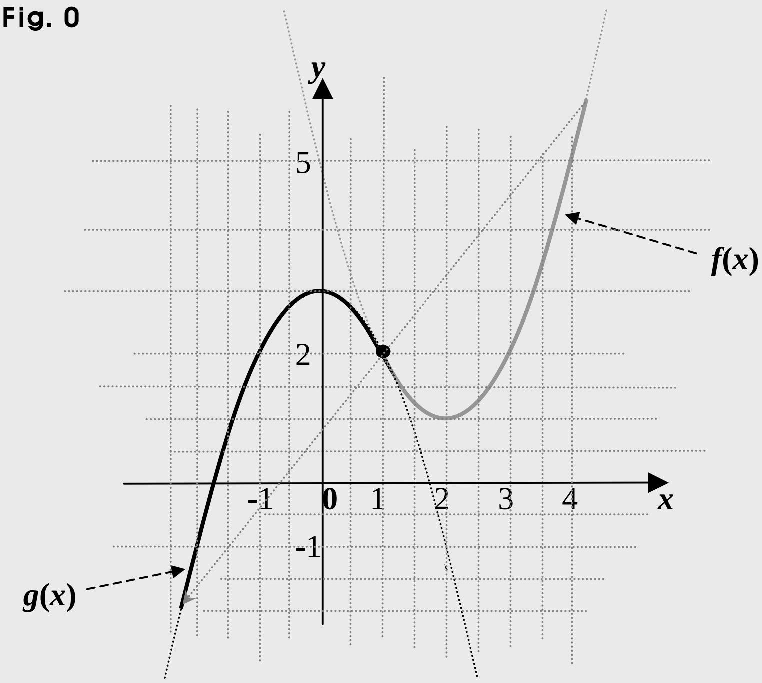
$$\Rightarrow y_g = -x_g^2 + 3 \Rightarrow y_g = g(x_g) = -x_g^2 + 3 \Rightarrow y = g(x) = -x^2 + 3.$$

Besides, $1 \le x_f \le 4 \Rightarrow -1 \ge -x_f \ge -4 \Rightarrow 1 \ge 2 - x_f \ge -2 \Rightarrow -2 \le x_g \le 1$, because $x_g = 2 - x_f$.

And we know $g = T \bullet f$.   So we get $T \bullet f = -x^2 + 3$ for $-2 \le x \le 1$.

And putting the two curves in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (2 - x, 4 - y)}$, and $\boldsymbol{y = f(x) = (x - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

Suggestions or Solutions
To the Problem 3 in the Example 2

Assuming T: $(x, y) \longrightarrow (2 - x, 4 - y)$, and $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$, find $T \bullet f$.

Setting first $(s, t) = (2 - x, 4 - y)$, we get $s = 2 - x \Rightarrow x = 2 - s$, and $t = 4 - y \Rightarrow y = 4 - t$.

So next, we get
$(x - 2)^2 + (y - 1)^2 - 2 = (2 - s - 2)^2 + (4 - t - 1)^2 - 2 = s^2 + (t - 3)^2 - 2 = 0$.

Thus, we get $T \bullet f = x^2 + (y - 3)^2 - 2 = 0$.

And we can get the solution the way below, too.

Assuming first, g is the new function $T \bullet f$, we can put T the way below, too.

T: $(x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, T: $(x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_f = 4 - y_g$ into the original equation $y = f(x)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) \Rightarrow f(2 - x_g, 4 - y_g)$, which is the connective equation, that is, the new equation, which indicates thus, the curve of the new function g.

And putting the idea above more specifically, we get

$$f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0 \Rightarrow f(x_f, y_f) = (x_f - 2)^2 + (y_f - 1)^2 - 2 = 0$$

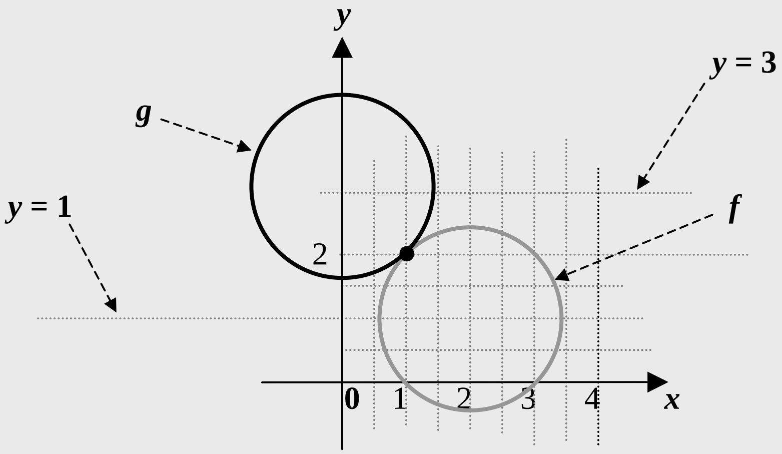
$$\Rightarrow f(2 - x_g, 4 - y_g) = (2 - x_g - 2)^2 + (4 - y_g - 1)^2 - 2 = x_g^2 + (3 - y_g)^2 - 2 = 0$$

$$\Rightarrow g(x_g, y_g) = x_g^2 + (3 - y_g)^2 - 2 = 0 \Rightarrow g(x, y) = x^2 + (3 - y)^2 - 2 = 0.$$

So we get $T \bullet f = x^2 + (3 - y)^2 - 2 = x^2 + (y - 3)^2 - 2 = 0$.

And putting the two curves in a graph, we get

Fig. 0



The axis of symmetry is a point **(1, 2)**.

$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (2 - x, 4 - y)}$, and $\boldsymbol{f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0}$.

Suggestions or Solutions
To the Problem in the Example 1

Find an equation, the curve of which is symmetric about a point (1, 2) to the curve of an equation $f(x, y) = \frac{x^2y^2}{xy+\frac{3}{2}} - 2 = 0$.

Assuming first C is a transformation symmetric about a point (a, b), we can put C the way as follows. $C: (x, y) \longrightarrow (2a - x, 2b - y)$.

And since $a = 1$, and $b = 2$ in this example, assuming B is the transformation we want to apply, we can put B the way as follows. $B: (x, y) \longrightarrow (2 - x, 4 - y)$.

So assuming next, g is the equation we want, that is the new equation, we can put B the way below, too.

$B: (x_f, y_f) \longrightarrow (x_g, y_g) = (2 - x_f, 4 - y_f)$. And briefly, $B: (x_f, y_f) \longrightarrow (2 - x_f, 4 - y_f)$.

So we get $x_g = 2 - x_f \Rightarrow x_f = 2 - x_g$, and $y_g = 4 - y_f \Rightarrow y_f = 4 - y_g$.

And next, putting $x_f = 2 - x_g$ and $y_g = 4 - y_f$ into the original equation $y = f(x, y)$, we get the equation connecting x_g and y_g, that is, the new equation.

In other words, we get $y = f(x, y) \Rightarrow y_f = f(x_f, y_f) \Rightarrow 4 - y_g = f(2 - x_g, 4 - y_g)$, which is the connective equation, that is, the new equation g.

And putting the idea above more specifically, we get

$$f(x, y) = \frac{x^2y^2}{xy+\frac{3}{2}} - 2 = 0 \Rightarrow f(x_f, y_f) = \frac{{x_f}^2{y_f}^2}{x_f y_f+\frac{3}{2}} - 2 = 0 \Rightarrow f(2 - x_g, 4 - y_g) = \frac{(2-x_g)^2(4-y_g)^2}{(2-x_g)(4-y_g)+\frac{3}{2}} - 2 = 0$$

$$\Rightarrow g(x_g, y_g) = \frac{(2-x_g)^2(4-y_g)^2}{(2-x_g)(4-y_g)+\frac{3}{2}} - 2 = 0 \Rightarrow g(x, y) = \frac{(2-x)^2(4-y)^2}{(2-x)(4-y)+\frac{3}{2}} - 2 = 0.$$

And setting $g = T \bullet f$, we get $g = T \bullet f = \frac{(2-x)^2(4-y)^2}{(2-x)(4-y)+\frac{3}{2}} - 2 = 0$.

And let's now take a look at the two curves, the curves of f and g.

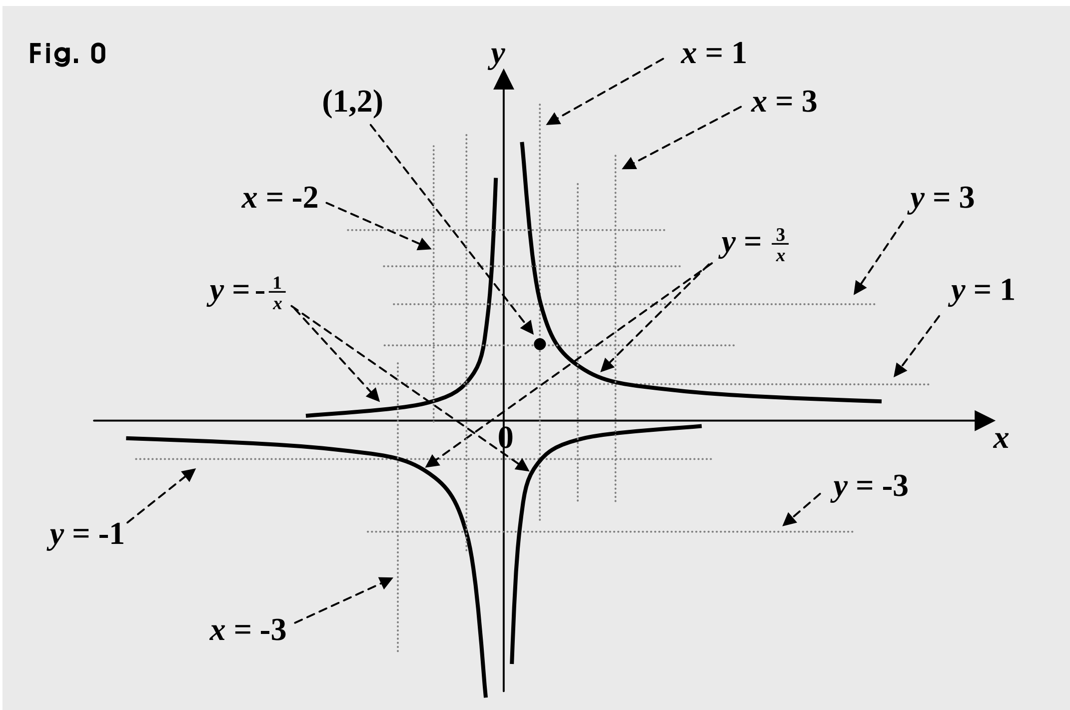
Beginning with the curve of ***f***, we can put its equation the way below.

We have $\boldsymbol{f(x, y)} = \frac{x^2y^2}{xy+\frac{3}{2}} - 2 = 0$. So we get

$$\frac{x^2y^2}{xy+\frac{3}{2}} - 2 = 0 \Rightarrow \frac{x^2y^2}{xy+\frac{3}{2}} = 2 \Rightarrow x^2y^2 = 2(xy+\tfrac{3}{2}) = 2xy+3 \Rightarrow x^2y^2 - 2xy - 3 = 0$$

$\Rightarrow (xy + 1)(xy - 3) = 0$. So we get $xy = -1$ or $xy = 3$. Thus, we get $y = -\frac{1}{x}$ or $y = \frac{3}{x}$.

So putting the curve of ***f*** in a graph, we can get



And next, moving on to the curve of **g**, we can put its equation the way below.

We have $\boldsymbol{g(x, y)} = \frac{(2-x)^2(4-y)^2}{(2-x)(4-y)+\frac{3}{2}} - 2 = 0$. So we get

$$\frac{(2-x)^2(4-y)^2}{(2-x)(4-y)+\frac{3}{2}} - 2 = 0 \Rightarrow \frac{(2-x)^2(4-y)^2}{(2-x)(4-y)+\frac{3}{2}} = 2 \Rightarrow (2-x)^2(4-y)^2 = 2(2-x)(4-y)+3$$

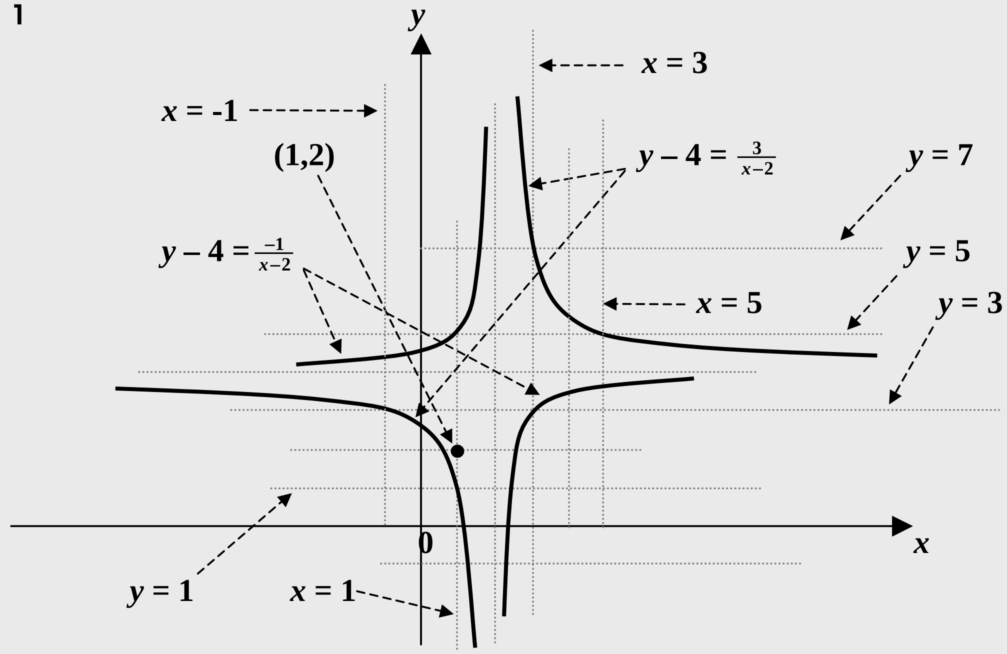
$\Rightarrow (2 - x)^2(4 - y)^2 - 2(2 - x)(4 - y) - 3 = 0$. And setting $u = (2 - x)(4 - y)$, we get

$u^2 - 2u - 3 = 0$. Factorizing it, we get $(u + 1)(u - 3) = 0$. So we get $u = -1$ or 3.

And thus, we get $(2 - x)(4 - y) = -1$ or 3. In other words, we get $(x - 2)(y - 4) = -1$ or 3

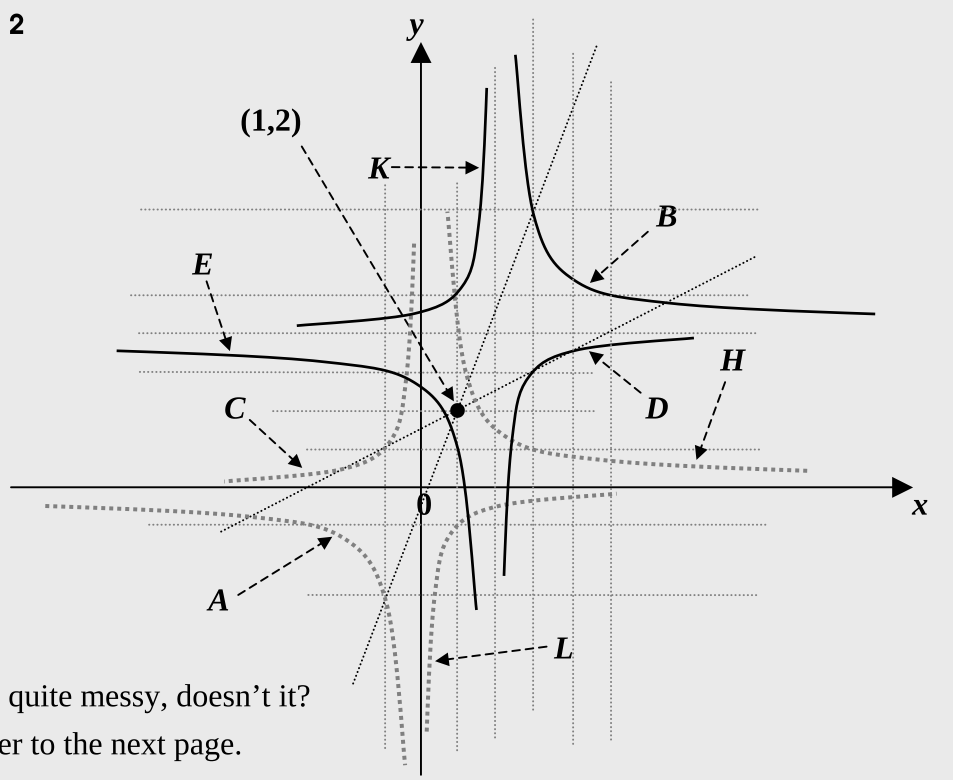
So we get $y - 4 = \frac{-1}{x-2}$ or $y - 4 = \frac{3}{x-2}$. Putting thus, the curve of ***g*** in a graph, we can get

Fig. 1



Now, let's put in one graph the two curves of ***f*** and ***g***.

Fig. 2



Looks quite messy, doesn't it?

So refer to the next page.

Each curve has four parts called branches.

The four branches ***A***, ***L***, ***C***, and ***H*** belong to the curve of ***f***, and the curve ***g*** is made of the four branches ***E***, ***K***, ***B***, and ***D***.

And the branch ***A*** corresponds to the branch ***B***, and thus, is transformed to the branch ***B***.

The branch ***C*** corresponds to the branch ***D***.

The branch ***L*** corresponds to the branch ***K***.

And the branch ***H*** corresponds to the branch ***E***.

And let's now put the equations together the way below.

For ***f***, we have $y = -\frac{1}{x}$ or $y = \frac{3}{x}$.
For ***g***, we have $y - 4 = \frac{-1}{x-2}$ or $y - 4 = \frac{3}{x-2}$.

Then, comparing the two sets of equations above, we can relate the two curves the way below, too.

Translating the curve of ***f*** in the amount of 2 in the direction of the ***x***-axis, and in the amount of 4 in the direction of the ***y***-axis, we get the curve of ***g***.