Examples in Symmetry about a line 4

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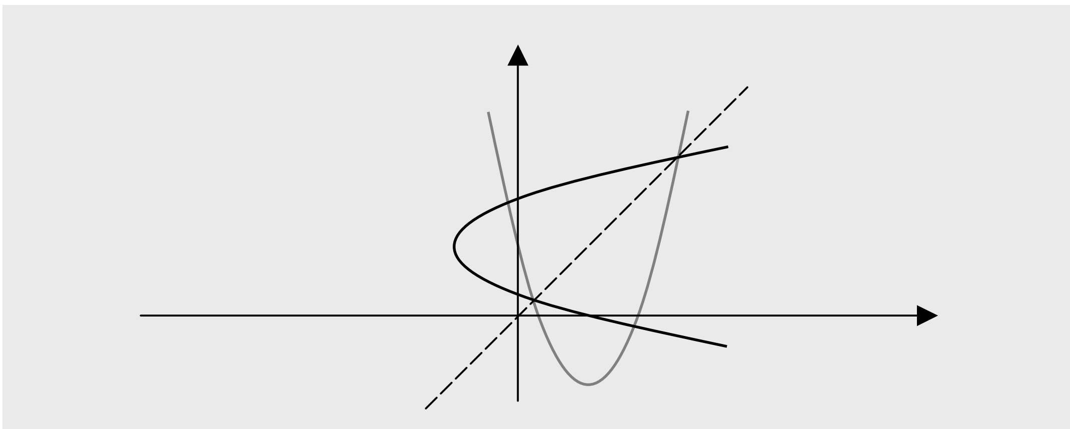


0. Assuming $\boldsymbol{R}$**:** $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (\boldsymbol{y}, \boldsymbol{x})$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{R}$.

0.0. $\boldsymbol{y = f(x) = 2 - \sqrt{x+1}}$ for $\boldsymbol{-1 \le x \le 3}$.

0.1. $\boldsymbol{y = f(x) = (x-2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

0.2. $\boldsymbol{f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0}$.

1. Assuming $\boldsymbol{y = f(x) = (x+1)(x-1)(x-2)}$ for $\boldsymbol{x \ge -1}$, $\boldsymbol{y = w(x) = (x+1)(x-1)(x-2)}$ for $\boldsymbol{x \ge 2}$, and $\boldsymbol{R}$**:** $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (\boldsymbol{y}, \boldsymbol{x})$, find $\boldsymbol{g = R \bullet f}$, and $\boldsymbol{h = R \bullet w}$.

Suggestions or Solutions
To the Problem 0 in the Example 0

Assuming R: $(x, y) \longrightarrow (y, x)$, and $y = f(x) = 2 - \sqrt{x+1}$ for $-1 \le x \le 3$, find $R \bullet f$.

Assuming first, (s, t) is the new arbitrary point, we can set $(s, t) = (y, x)$.
Then, we get $x = t$, and $y = s$.

So next, we get $y = f(x) \Rightarrow s = f(t) = 2 - \sqrt{t+1} \Rightarrow s = 2 - \sqrt{t+1}$
$\Rightarrow \sqrt{t+1} = 2 - s \Rightarrow t + 1 = (2 - s)^2 \Rightarrow t = (s - 2)^2 - 1$.

And we have $y = s$, and $y = 2 - \sqrt{x+1}$ for $-1 \le x \le 3$. So we get

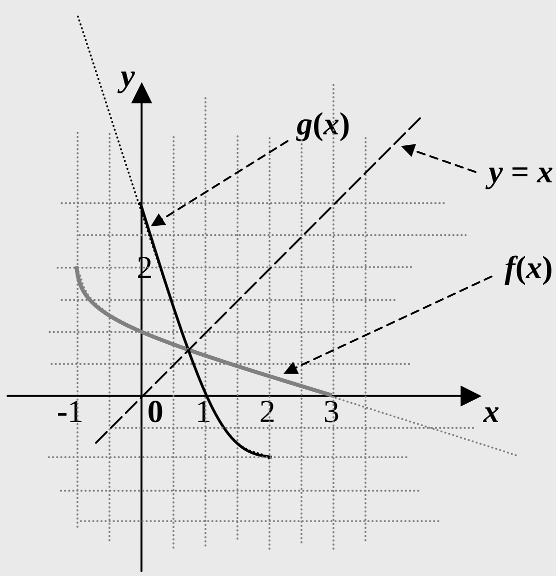
$-1 \le x \le 3 \Rightarrow 0 \le x + 1 \le 4 \Rightarrow 0 \le \sqrt{x+1} \le 2 \Rightarrow -2 \le -\sqrt{x+1} \le 0$
$\Rightarrow 0 \le 2 - \sqrt{x+1} \le 2 \Rightarrow 0 \le y \le 2 \Rightarrow 0 \le s \le 2$.

So assuming g is the new function $R \bullet f$, we get $y = g(x) = (x - 2)^2 - 1$ for $0 \le x \le 2$.

That is, we get $R \bullet f = (x - 2)^2 - 1$ for $0 \le x \le 2$.

And in the original function ***f***, the ***y***-value keeps decreasing as the ***x***-value keeps increasing from -1 to 3, and thus, ***f*** is one-to-one.

Fig. 0



Suggestions or Solutions
To the Problem 1 in the Example 0

Assuming R: $(x, y) \longrightarrow (y, x)$, and $y = f(x) = (x-2)^2 + 1$ for $1 \le x \le 4$, find $R \bullet f$.

Assuming first, g is the new function $R \bullet f$, and (s, t) is the new point, we can set $(s, t) = (y, x)$. Then, we get $y = f(x) \Rightarrow s = f(t) = (t-2)^2 + 1 \Rightarrow s = (t-2)^2 + 1$.

Next, $1 \le x \le 4 \Rightarrow -1 \le x - 2 \le 2 \Rightarrow 0 \le (x-2)^2 \le 4 \Rightarrow 1 \le (x-2)^2 + 1 \le 5 \Rightarrow 1 \le y \le 5$ $\Rightarrow 1 \le s \le 5$. And next, $1 \le x \le 4 \Rightarrow 1 \le t \le 4$.

So the new equation is $x = (y-2)^2 + 1$ for $1 \le x \le 5$, and $1 \le y \le 4$.

And next, getting the new function g, we get first,

$x = (y-2)^2 + 1 \Rightarrow x - 1 = (y-2)^2 \Rightarrow y - 2 = \pm\sqrt{x-1} \Rightarrow y = \pm\sqrt{x-1} + 2$.

Then, first, $1 \le x \le 5 \Rightarrow 0 \le x - 1 \le 4 \Rightarrow 0 \le \sqrt{x-1} \le 2 \Rightarrow 2 \le \sqrt{x-1} + 2 \le 4 \Rightarrow 2 \le y \le 4$.
So we get $y = \sqrt{x-1} + 2$ for $1 \le x \le 5$.

And next, $0 \le \sqrt{x-1} \le 2 \Rightarrow -2 \le -\sqrt{x-1} \le 0 \Rightarrow 0 \le 2 - \sqrt{x-1} \le 2 \Rightarrow 0 \le y \le 2$.
We have this though: $1 \le y \le 4$. So we get $1 \le y \le 2$.

Then first, we get $2 = 2 - \sqrt{x-1} \Rightarrow x = 1$.
And next, we get $1 = 2 - \sqrt{x-1} \Rightarrow \sqrt{x-1} = 1 \Rightarrow x - 1 = 1 \Rightarrow x = 2$.
Thus, we get $y = \sqrt{x-1} + 2$ for $1 \le x \le 2$.

So the new function g is $y = g(x) = \begin{cases} \sqrt{x-1} + 2 \text{ for } 1 \le x \le 5. \\ 2 - \sqrt{x-1} \text{ for } 1 \le x \le 2. \end{cases}$

If not quite sure of the processes above, follow the steps below.

Assuming first, g is the new function $R \bullet f$, and (s, t) is the new point, we can set $(s, t) = (y, x)$. That is, we get $x = t$, and $y = s$.
So next, we get $y = f(x) \Rightarrow s = f(t) = (t-2)^2 + 1 \Rightarrow s = (t-2)^2 + 1$.

And next, we want to get the new domain and range.

To begin with, we have $\boldsymbol{y = s}$, and $\boldsymbol{y = (x - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$. So we get

$\boldsymbol{1 \le x \le 4 \Rightarrow -1 \le x - 2 \le 2 \Rightarrow 0 \le (x - 2)^2 \le 4}$, because the value of $\boldsymbol{(x - 2)^2}$ is minimum when $\boldsymbol{x = 2}$, and the minimum is 0.

So we get $\boldsymbol{1 \le (x - 2)^2 + 1 \le 5 \Rightarrow 1 \le y \le 5 \Rightarrow 1 \le s \le 5}$, since $\boldsymbol{y = s}$.
And next, we have $\boldsymbol{x = t}$, and we know $\boldsymbol{1 \le x \le 4}$. So we get $\boldsymbol{1 \le t \le 4}$.
So the new equation is $\boldsymbol{x = (y - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 5}$, and $\boldsymbol{1 \le y \le 4}$, which indicates the curve of $\boldsymbol{g}$.

And expressing $\boldsymbol{y}$ in terms of $\boldsymbol{x}$, we want to solve for $\boldsymbol{y}$ the new equation above.
Then, we get the new function $\boldsymbol{g}$. So solving it, we get

$\boldsymbol{x = (y - 2)^2 + 1 \Rightarrow x - 1 = (y - 2)^2 \Rightarrow y - 2 = \pm\sqrt{x-1} \Rightarrow y = \pm\sqrt{x-1} + 2}$.

Then, we have two cases. One is $\boldsymbol{y = \sqrt{x-1} + 2}$, and the other is $\boldsymbol{y = 2 - \sqrt{x-1}}$.
So we need to determine the extent of $\boldsymbol{x}$ in each so that each behaves within $\boldsymbol{1 \le x \le 5}$, and $\boldsymbol{1 \le y \le 4}$.

Beginning with $\boldsymbol{y = \sqrt{x-1} + 2}$, we get

$\boldsymbol{1 \le x \le 5 \Rightarrow 0 \le x - 1 \le 4 \Rightarrow 0 \le \sqrt{x-1} \le 2 \Rightarrow 2 \le \sqrt{x-1} + 2 \le 4 \Rightarrow 2 \le y \le 4}$, which is OK, since we have $\boldsymbol{1 \le y \le 4}$. Thus, we get $\boldsymbol{y = \sqrt{x-1} + 2}$ for $\boldsymbol{1 \le x \le 5}$.

And next, moving on to $\boldsymbol{y = 2 - \sqrt{x-1}}$, since we already have $\boldsymbol{0 \le \sqrt{x-1} \le 2}$, we get

$\boldsymbol{0 \le \sqrt{x-1} \le 2 \Rightarrow -2 \le -\sqrt{x-1} \le 0 \Rightarrow 0 \le 2 - \sqrt{x-1} \le 2 \Rightarrow 0 \le y \le 2}$, which is not OK, since we have $\boldsymbol{1 \le y \le 4}$, which is the new range. The $\boldsymbol{y}$-value has to be in the range.

So it has to be the case where $\boldsymbol{1 \le y \le 2}$, because it satisfies both $\boldsymbol{0 \le y \le 2}$ and $\boldsymbol{1 \le y \le 4}$.
What then, is the extent of $\boldsymbol{x}$, that is, the domain of $\boldsymbol{y = 2 - \sqrt{x-1}}$ for $\boldsymbol{1 \le y \le 2}$?

We know that the value of ($\boldsymbol{2 - \sqrt{x-1}}$), that is, $\boldsymbol{y}$ keeps decreasing as $\boldsymbol{x}$ keeps increasing.
And we know $\boldsymbol{1 \le y \le 2}$. So we can say that $\boldsymbol{y}$ keeps decreasing from 2 to 1.

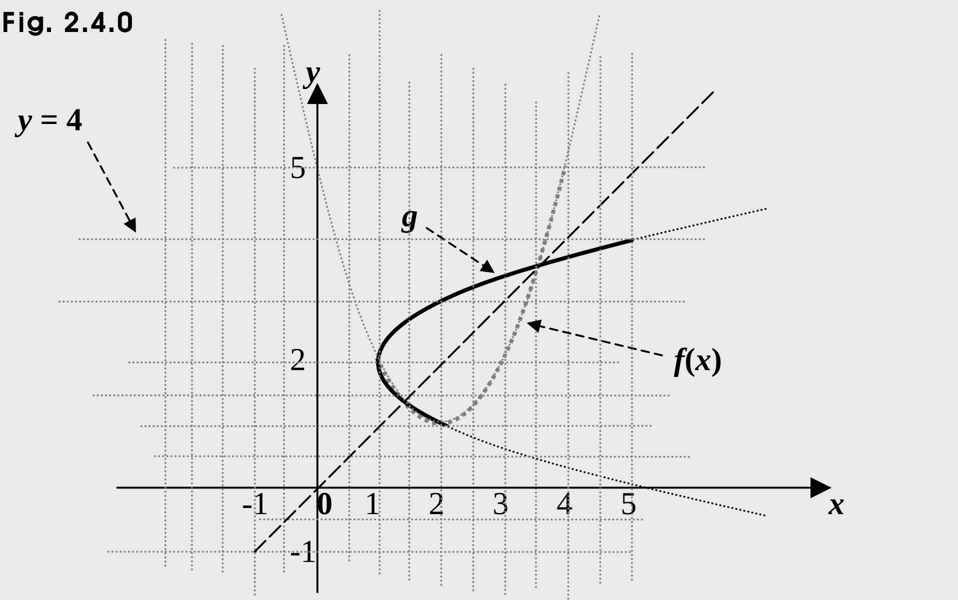
And we know since we have $\boldsymbol{1 \le x \le 5}$, $\boldsymbol{x}$ begins with 1, and then, increases up to 5.
And we have $\boldsymbol{y = 2 - \sqrt{x-1}}$, so when $\boldsymbol{x = 1}$, $\boldsymbol{y = 2}$. So we now want to find $\boldsymbol{x}$ for which we get $\boldsymbol{y = 1}$. So finding it, we get $\boldsymbol{1 = 2 - \sqrt{x-1} \Rightarrow \sqrt{x-1} = 1 \Rightarrow x - 1 = 1 \Rightarrow x = 2}$.

Thus, we get $\boldsymbol{y = \sqrt{x-1} + 2}$ for $\mathbf{1 \le x \le 2}$.

So the new function $\boldsymbol{g}$ is $\boldsymbol{y = g(x)} = \begin{cases} \sqrt{x-1} + 2 \text{ for } 1 \le x \le 5. \\ 2 - \sqrt{x-1} \text{ for } 1 \le x \le 2. \end{cases}$

And of course, we can put it the way below, too.
$\boldsymbol{y = g(x) = \sqrt{x-1} + 2}$ for $\mathbf{1 \le x \le 5}$, and $\boldsymbol{y = g(x) = 2 - \sqrt{x-1}}$ for $\mathbf{1 \le x \le 2}$.

Fig. 2.4.0



How do we know though, algebracally if $\boldsymbol{f}$ is not one-to-one?

If $\boldsymbol{a \ne b}$, and $\boldsymbol{f(a) \ne f(b)}$, $\boldsymbol{f}$ is one-to-one.
So to begin with, we have $\boldsymbol{f(a) = (a-2)^2 + 1}$, and $\boldsymbol{f(b) = (b-2)^2 + 1}$.

Next, taking the difference, we get
$\boldsymbol{f(a) - f(b) = (a-2)^2 + 1 - \{(b-2)^2 + 1\} = (a-2)^2 - (b-2)^2 = (a+b-4)(a-b)}$.

Then first, we have $\boldsymbol{a - b \ne 0}$ since $\boldsymbol{a \ne b}$.
And we have $\mathbf{1 \le \boldsymbol{a} \le 4}$, and $\mathbf{1 \le \boldsymbol{b} \le 4}$, because $\mathbf{1 \le \boldsymbol{x} \le 4}$.
So we get $\mathbf{2 \le \boldsymbol{a + b} \le 8}$, and thus, we get $\mathbf{-2 \le \boldsymbol{a + b} - 4 \le 4}$.
So we can get $\boldsymbol{a + b - 4 = 0}$ for some $\boldsymbol{a}$ and $\boldsymbol{b}$ even if $\boldsymbol{a \ne b}$.
And thus, we can get $\boldsymbol{(s-t)(s^2 + st + t^2) = 0 \Rightarrow f(a) - f(b) = 0 \Rightarrow f(a) = f(b)}$. So $\boldsymbol{f}$ is not one-to-one.

Suggestions or Solutions
To the Problem 2 in the Example 0

Assuming R: $(x, y) \longrightarrow (y, x)$, and $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$, find $R \bullet f$.

Assuming first, g is the new equation, and (x_g, y_g) is the new point, and (x_f, y_f) is the original point, we can set $(x_g, y_g) = (y_f, x_f)$. Then, we get $x_f = y_g$, and $y_f = x_g$.

So next, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) = 0 \Rightarrow f(y_g, x_g) = (y_g - 2)^2 + (x_g - 1)^2 - 2 = 0$.
And thus, the new equation g is $g(x, y) = (x - 1)^2 + (y - 2)^2 - 2 = 0$.

The transformation R we apply here is the transformation symmetric about the line $y = x$. So the new curve we get is the curve symmetric to the original curve about the line $y = x$. In this example the curve we apply the symmetric transformation R to is a circle, where the center is $(2, 1)$, and the radius is $\sqrt{2}$.

Thus, f is the equation of the circle above, and assuming g is the new equation, we can say that the two equations f and g are symmetric about the line $y = x$.

And since R is a symmetric transformation, the curve itself of g, that is, the new circle itself is the same as the original circle itself. The circle g can however, have a center different from the center of the circle f.

If a circle gets transformed symmetrically, its center gets transformed the same manner, too. So the center of g is symmetric to the center of f about the line $y = x$.

And knowing the center and the radius of a circle, we can readily get the circle. That is, using the standard form for the equation for circles, we can easily get the equation of the circle. The equation of the circle f, that is, the original equation is given in fact, in the standard form. So let's first, get the new circle finding the center of the new circle.

We can get the new center applying the transformation R to the original center, that is, the center of the circle f.   So let's now apply R to the original center $(2, 1)$.

To begin with, assuming (x, y) is the original point, (s, t) is the new, we get $(s, t) = (y, x)$.

So since the original center is **(2, 1)**, the new center is **(1, 2)**.

And thus, the new circle is simply $(x - 1)^2 + (y - 2)^2 - 2 = 0$.

So the new equation g can be put this way: $g(x, y) = (x - 1)^2 + (y - 2)^2 - 2 = 0$.

And let's next, get the new circle using the method we used for the previous examples.

Assuming again, (s, t) is the new point, and (x, y) is the original point, we can set $(s, t) = (y, x)$. Thus, we get $x = t$, and $y = s$.

So next, connecting s and t, we put $x = t$ and $y = s$ into the original equation.

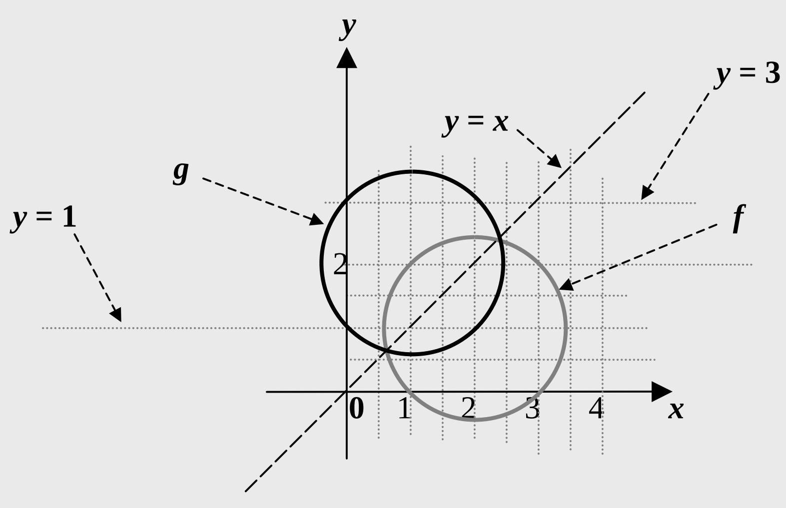
In other words, we get $f(x, y) = 0 \Rightarrow f(t, s) = 0$, which is the new equation g.
That is, we get $g(x, y) = f(y, x) = 0$.

And we have $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$.

So we get $g(x, y) = f(y, x) = (y - 2)^2 + (x - 1)^2 - 2 = 0$.

And thus, the new equation g is $g(x, y) = (x - 1)^2 + (y - 2)^2 - 2 = 0$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Assuming $y = f(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq -1$, $y = w(x) = (x + 1)(x - 1)(x - 2)$ for $x \geq 2$, and R: $(x, y) \longrightarrow (y, x)$, find $g = R \bullet f$, and $h = R \bullet w$.

To begin with, ***R*** is the transformation symmetric about the line $y = x$, which is called the axis of symmetry in the transformation ***R***.

So if the original function ***f*** is one-to-one, applying ***R*** to ***f***, we get a new function, which is therefore, symmetric to ***f*** about the line $y = x$.

However, in the original function ***f***, the ***y***-value does not keep either decreasing or increasing as the ***x***-value keeps increasing from -1, and thus, ***f*** is not one-to-one.

We can show ***f*** is not one-to-one the way below, too.

Putting 1 into ***x***, we get 0. That is, we get $y = f(1) = (1 + 1)(1 - 1)(1 - 2) = 0$.
And also, putting 2 into ***x***, we get 0, too. So for two different ***x***-values, the same ***y***-value gets produced. And thus, in the function ***f***, there are many cases where for two different ***x***-values, the same ***y***-value gets produced.   Why?

Suppose the ***y***-value increases as the ***x***-value increases beginning with 1.
Then, the ***y***-value cannot keep increasing as the ***x***-value changes from 1 to 2, because we have $f(1) = f(2)$. And thus, for some ***x***-value between 1 and 2, the ***y***-value has to start decreasing. And the same is true, too, for the case where the ***y***-value decreases as the ***x***-value increases beginning with 1. That is, the ***y***-value cannot keep decreasing as the ***x***-value changes from 1 to 2, because we have $f(1) = f(2)$. And thus, for some ***x***-value between 1 and 2, the ***y***-value has to start increasing. So ***f*** is not one-to-one.

And producing such a proof as above, we say we produce a counter example.
So we can disprove a statement producing a counter example.
In this case, $f(1) = f(2)$ is a counter example against the statement that ***f*** is one-to-one.

And thus, ***g***, that is, the transformation of ***f*** by ***R*** is not a function but an equation.
And the curve of the function ***f*** and the curve of the equation ***g*** are symmetric about the line $y = x$.

In short, the original curve $\boldsymbol{f}$ and the new curve $\boldsymbol{g}$ are symmetric about the line $\boldsymbol{y = x}$. What then about $\boldsymbol{h}$?

We have $\boldsymbol{h = R \bullet w}$, where $\boldsymbol{y = w(x) = (x + 1)(x - 1)(x - 2)}$ for $\boldsymbol{x \geq 2}$.
So in the original function $\boldsymbol{w}$, the $\boldsymbol{y}$-value keeps increasing as the $\boldsymbol{x}$-value keeps increasing from 2, and thus, $\boldsymbol{w}$ is one-to-one. And we can check to see if $\boldsymbol{f}$ is one-to-one the way below, too.

If $\boldsymbol{f(a) \neq f(b)}$ where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constant, and $\boldsymbol{a \neq b}$, $\boldsymbol{f}$ is one-to-one. Otherwise, $\boldsymbol{f}$ is not, and thus, is many-to-one.

To begin with, simplifying $\boldsymbol{w}$, we get
$\boldsymbol{y = w(x) = (x + 1)(x - 1)(x - 2) = (x^2 - 1)(x - 2) = x^3 - 2x^2 - x + 2}$.

Next, taking the difference, we get

$$\boldsymbol{f(a) - f(b) = a^3 - 2a^2 - a + 2 - (b^3 - 2b^2 - b + 2) = a^3 - b^3 - 2(a^2 - b^2) - (a - b)}$$

$$\boldsymbol{= (a - b)(a^2 + ab + b^2) - 2(a + b)(a - b) - (a - b) = (a - b)\{a^2 + ab + b^2 - 2(a + b) - 1\}}$$

Then first, we know $\boldsymbol{a - b \neq 0}$ since $\boldsymbol{a \neq b}$.
And next, we have $\boldsymbol{a^2 + ab + b^2 - 2(a + b) - 1 = a^2 + (a + b)b - 2(a + b) - 1}$
$\boldsymbol{= a^2 + (a + b)(b - 2) - 1}$.

Now, we have $\boldsymbol{x \geq 2}$. So we get $\boldsymbol{a \geq 2}$, and then, $\boldsymbol{b > 2}$, since $\boldsymbol{a \neq b}$.

Thus, we get $\boldsymbol{a^2 \geq 4}$, $\boldsymbol{a + b > 4}$, and $\boldsymbol{b - 2 > 0}$.

So we get $\boldsymbol{a^2 + (a + b)(b - 2) > 4}$, and thus, we get $\boldsymbol{a^2 + (a + b)(b - 2) - 1 > 3}$.

So we get $\boldsymbol{a^2 + (a + b)(b - 2) - 1 \neq 0}$ anyway.

And therefore, we get $\boldsymbol{f(a) - f(b) \neq 0}$, that is, $\boldsymbol{f(a) \neq f(b)}$. So $\boldsymbol{f}$ is one-to-one.

And thus, $\boldsymbol{h}$, that is, the transformation of $\boldsymbol{w}$ by $\boldsymbol{R}$ is a function, the inverse of $\boldsymbol{w}$.

Let's now get $\boldsymbol{g}$ and $\boldsymbol{h}$.
Beginning with $\boldsymbol{g}$, and assuming $\boldsymbol{(s, t)}$ is the new point, we can set $\boldsymbol{(s, t) = (y, x)}$.
Then, we get $\boldsymbol{x = t}$, and $\boldsymbol{y = s}$.

And a transformation, the original domain can change, too. So we want to find the new domain, that is, the extent of $\boldsymbol{s}$, which is the variable $\boldsymbol{x}$ in the new equation.

First, we have $\boldsymbol{y = s}$. So the extent of $\boldsymbol{s}$ is the same as the extent of $\boldsymbol{y}$ in the original equation. So we want to find the extent of $\boldsymbol{y}$, that is, the original range. Finding it, we want to do differential calculus. So doing it, we can get $\boldsymbol{y \geq f(c)}$ where $\boldsymbol{c} = \frac{2+\sqrt{7}}{3} \approx 1.55$.
So setting $\boldsymbol{k = f(c)}$ where $\boldsymbol{c} = \frac{2+\sqrt{7}}{3}$, we get $\boldsymbol{s \geq k}$, which is the new domain.

And if we want to find the new range, that is, the extent of $\boldsymbol{t}$, we can use $\boldsymbol{x = t}$.
We know $\boldsymbol{x \geq -1}$. So we get $\boldsymbol{t \geq -1}$, which is the new range. What then, is the next?

We want to get the equation connecting $\boldsymbol{s}$ and $\boldsymbol{t}$, that is, the new equation.
Connecting $\boldsymbol{s}$ and $\boldsymbol{t}$, we put $\boldsymbol{x = t}$ and $\boldsymbol{y = s}$ into the original equation.
So doing the replacements, we get $\boldsymbol{y = f(x) \Rightarrow s = f(t) = (t + 1)(t - 1)(t - 2)}$.
And we know $\boldsymbol{s}$ is $\boldsymbol{x}$ in the new equation, and $\boldsymbol{t}$ is $\boldsymbol{y}$ in the new.
So the new equation is $\boldsymbol{x = (y + 1)(y - 1)(y - 2)}$ for $\boldsymbol{x \geq k}$ where $\boldsymbol{k = f(c)}$ where $\boldsymbol{c} = \frac{2+\sqrt{7}}{3}$.
And since $\boldsymbol{g}$ is the new, we can put it this way: $\boldsymbol{g(x, y) = (y + 1)(y - 1)(y - 2) - x = 0}$ for $\boldsymbol{x \geq k}$ where $\boldsymbol{k = f(c)}$ where $\boldsymbol{c} = \frac{2+\sqrt{7}}{3}$.

And next, moving on to $\boldsymbol{h = R \bullet w}$, where $\boldsymbol{y = w(x) = (x + 1)(x - 1)(x - 2)}$ for $\boldsymbol{x \geq 2}$.
Then, the expression of the function $\boldsymbol{w}$ is the same as that of the function $\boldsymbol{f}$.

What makes then, $\boldsymbol{w}$ different from $\boldsymbol{f}$?

The difference is in their domains, and thus, the ranges are different, too.

Applying $\boldsymbol{R}$ to $\boldsymbol{w}$, we get a new function. And the new function is the function $\boldsymbol{h}$.
And the only difference between the two functions $\boldsymbol{f}$ and $\boldsymbol{w}$ is in their domains and ranges, so the expression of $\boldsymbol{h}$ will be $\boldsymbol{x = (y + 1)(y - 1)(y - 2)}$, too.

What then, are the extents of $\boldsymbol{x}$ and $\boldsymbol{y}$ used in $\boldsymbol{h}$?

We can begin with this: $\boldsymbol{(s, t) = (y, x)}$.
Assuming this time, for practice purposes, $\boldsymbol{(x_h, y_h)}$ is the new point, and the original point is $\boldsymbol{(x_w, y_w)}$, we can set $\boldsymbol{(x_h, y_h) = (y_w, x_w)}$. Then, we get $\boldsymbol{x_w = y_h}$, and $\boldsymbol{y_w = x_h}$.

And beginning with the new domain, we can use $y_w = x_h$, where y_w is y in w. And thus, the original range, that is, the range of w is the new domain, that is, the extent of x in h.

So we want to get the original range, that is, the range of w first.
Getting the range of w, we can use $y = w(x) = (x + 1)(x – 1)(x – 2)$ for $x \geq 2$. How?

We can get it keeping track of the variables. We can see that the y-value just keeps increasing as the x-value increases from 2.

And we know $y = w(2) = 0$. So we get $y = (x + 1)(x – 1)(x – 2) \geq 0$ for $x \geq 2$.

And thus, the range of w is $y \geq 0$, that is, $y_w \geq 0$. And we have $y_w = x_h$, so we get $x_h \geq 0$, which is the new domain.

And if we want to find the new range, too, we can use $x_w = y_h$, and $x_w \geq 2$.
Then, we get $y_h \geq 2$, which is the new range.

And thus, the new equation is $x = (y + 1)(y – 1)(y – 2)$ for $x \geq 0$, and $y \geq 2$.

What then, about the new function h, that is, how do we get the expression of h?

Finding the expression of h, we want to solve for y the new equation. We cannot however, solve it for y, at least for now. We may be able to solve it though, using a method often called Cardano's formula, which is beyond high school math, though.

Though not analytic, that is, not algebraic, a tool called a numerical method can be used. And using such a method, we normally use a computer or calculator.

And though we cannot put the function h in an expression, we can put it in a graph.

Using the fact that the curve of the new function h is symmetric about the line $y = x$ to the curve of the original function w, we can put the curve of h in a graph.

And then, we can get a good approximation of the y-value for each x-value in the function h.

So graphing matters. And putting the curves in graphs, we can put them the way below.

Fig. 0

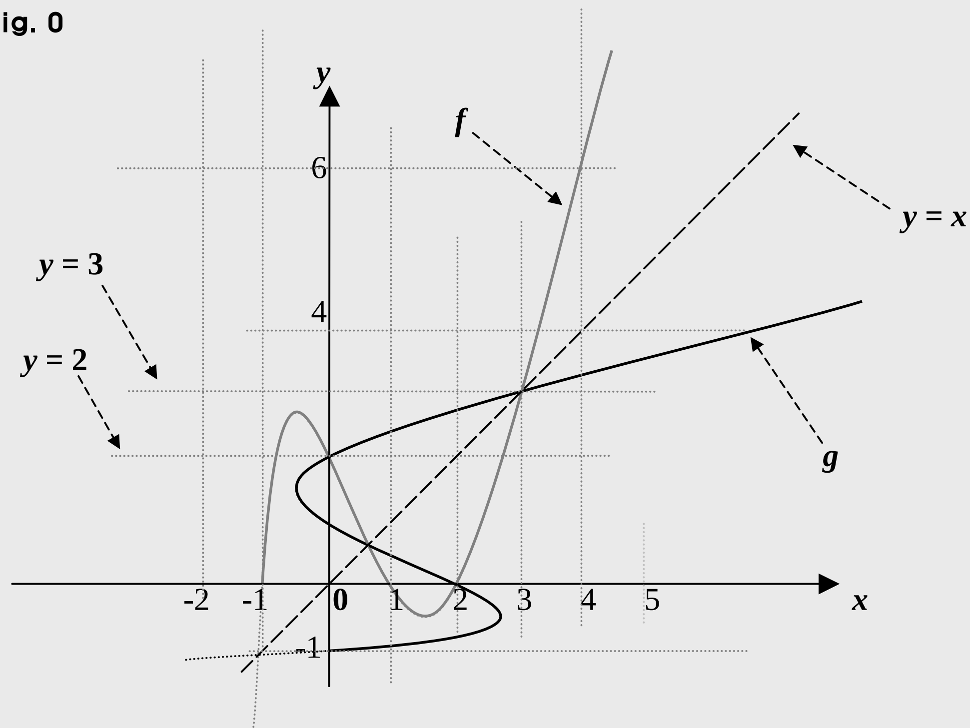


Fig. 1