Examples in Symmetry about a line 5

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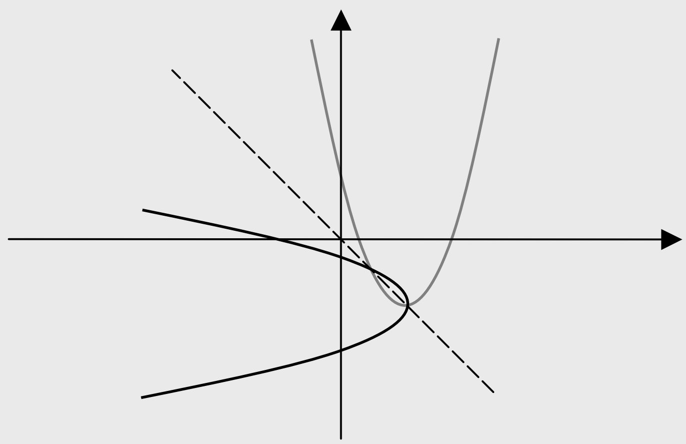
Examples in Symmetry about a line 5

Fig. 0



0. Find $\boldsymbol{g = N \bullet f}$, where $\boldsymbol{N: (x, y) \longrightarrow (-y, -x)}$, and $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$.

1. Find a function, the curve of which is symmetric about the line $\boldsymbol{y = -x}$ to the curve of a function $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x}$ real.

2. Assuming $\boldsymbol{N: (x, y) \longrightarrow (-y, -x)}$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{N}$.

2.0. $\boldsymbol{y = f(x) = x^2 - 2x + 1}$ for $\boldsymbol{x \geq 1}$.

2.1. $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \leq 1}$.

2.2. $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x \geq 1}$.

Suggestions or Solutions
To the Problem in the Example 0

Find $g = N \bullet f$, where N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = (1 - x)^3 + 2$.

Assuming first, g is the new function $N \bullet f$, we can set $(x_g, y_g) = (-y_f, -x_f)$.

So we get $x_f = -y_g$, and $y_f = -x_g$. So next, we get

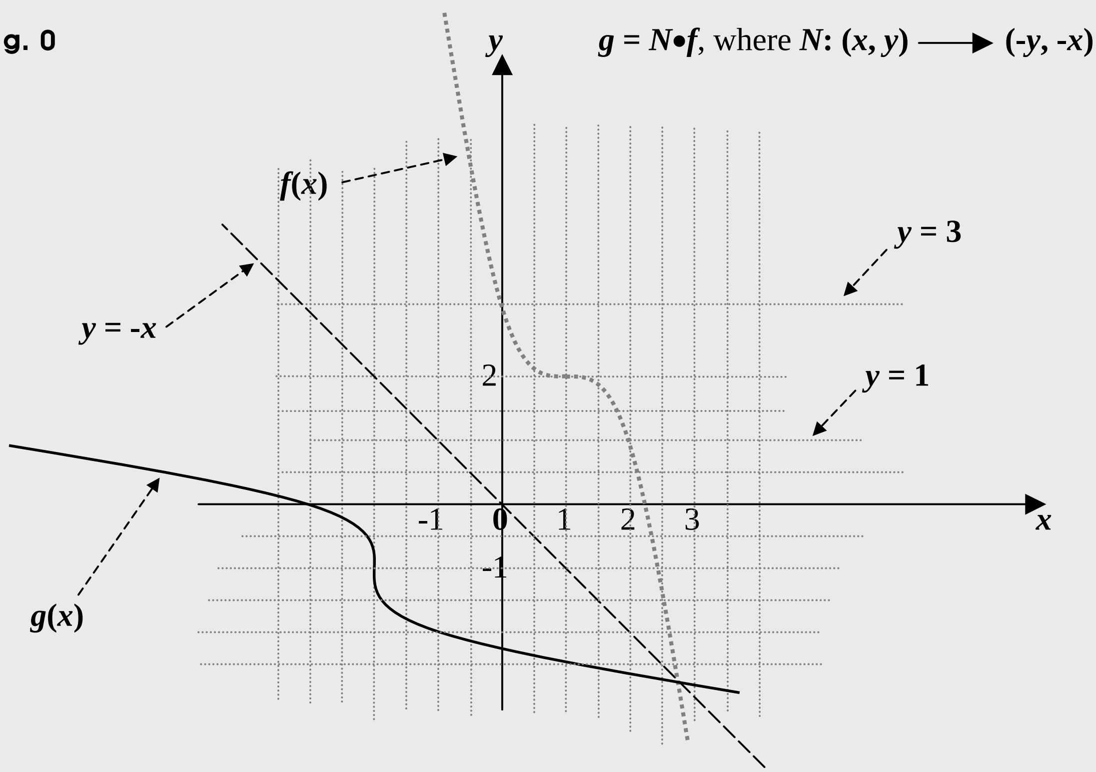
$$y = f(x) = (1 - x)^3 + 2 \Rightarrow y_f = f(x_f) = (1 - x_f)^3 + 2 \Rightarrow -x_g = (1 + y_g)^3 + 2$$

$$\Rightarrow -(2 + x_g) = (1 + y_g)^3 \Rightarrow 1 + y_g = -\sqrt[3]{x_g + 2} \Rightarrow y_g = -1 - \sqrt[3]{x_g + 2}$$

$$\Rightarrow y_g = g(x_g) = -1 - \sqrt{x_g + 2} \Rightarrow y = g(x) = -1 - \sqrt{x + 2}.$$

So we get $g = R \bullet f = -1 - \sqrt[3]{x + 2}$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Find a function, the curve of which is symmetric about the line $y = x$ to the curve of a function $y = f(x) = \sqrt[3]{1-x} + 2$.

Assuming first, g is the new function, we can set $(x_g, y_g) = (-y_f, -x_f)$.

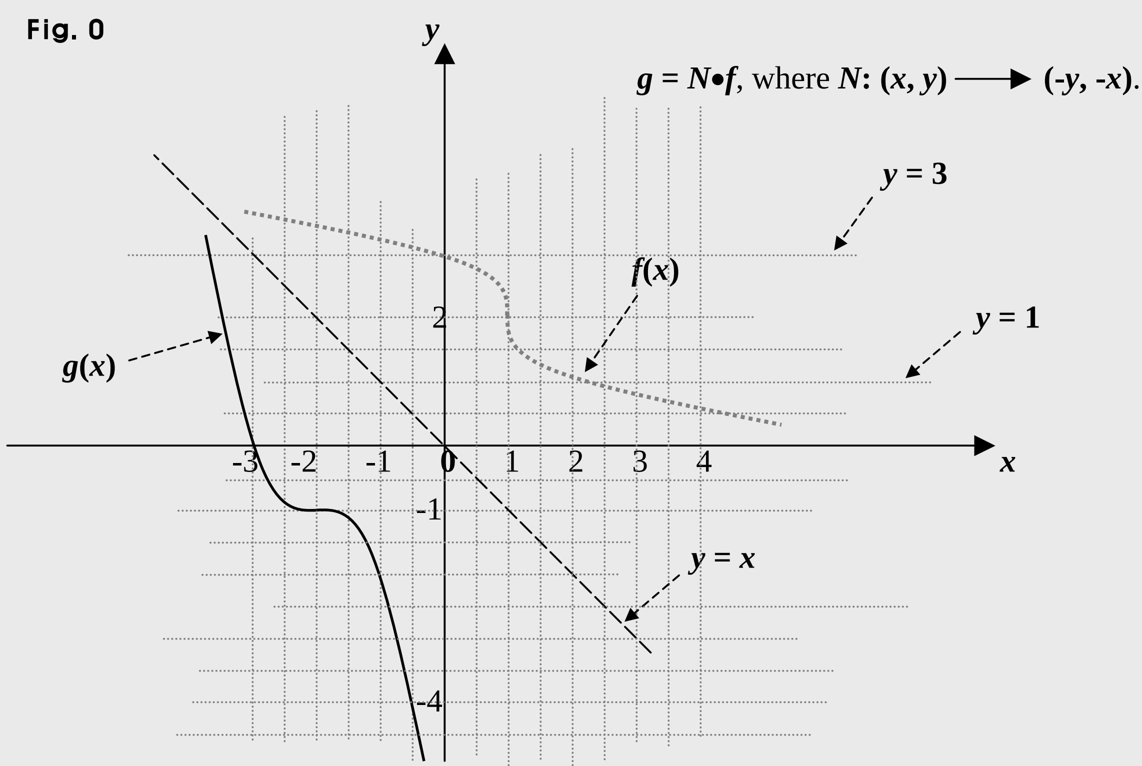
So we get $x_f = -y_g$, and $y_f = -x_g$. So next, we get

$$y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow -x_g = f(-y_g) = \sqrt[3]{1+y_g} + 2 \Rightarrow -x_g = \sqrt[3]{1+y_g} + 2$$

$$\Rightarrow -x_g - 2 = \sqrt[3]{1+y_g} \Rightarrow -(x_g + 2)^3 = 1 + y_g \Rightarrow y_g = -1 - (x_g + 2)^3$$

$$\Rightarrow y_g = g(x_g) = -1 - (x_g + 2)^3 \Rightarrow y = g(x) = -1 - (x + 2)^3.$$

Fig. 0



Suggestions or Solutions
To the Problem 0 in the Example 2

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$, find $N \bullet f$.

Assuming first, g is the new function $R \bullet f$, we can set $(x_g, y_g) = (-y_f, -x_f)$.

So we get $x_f = -y_g$, and $y_f = -x_g$. So next, we get

$$y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow -x_g = f(-y_g) = (y_g^2 + 2y_g + 1) = (y_g + 1)^2 \Rightarrow x_g = -(y_g + 1)^2$$

$$\Rightarrow y_g + 1 = \pm\sqrt{-x_g} \Rightarrow y_g = -1 \pm \sqrt{-x_g}.$$

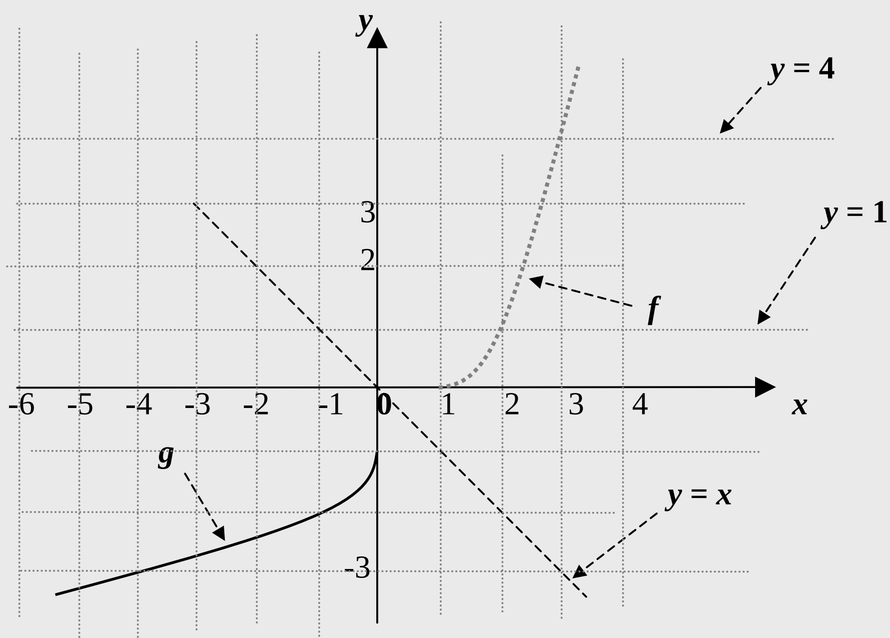
And we have $y_g = -x_f$. And we know $x_f \geq 1 \Rightarrow -x_f \leq -1$. So we get $y_g \leq -1$.

Thus, we get $y_g = -1 - \sqrt{-x_g}$.

And we have $x_g = -y_f$, and we get $y_f = f(x_f) = x_f^2 - 2x_f + 1 = (x_f - 1)^2 \geq 0$, since $x_f \geq 1$.
So we get $y_f \geq 0 \Rightarrow -y_f \leq 0$, and thus, $x_g \leq 0$, since $x_g = -y_f$.

And therefore, the new function g is $y = g(x) = -1 - \sqrt{-x}$ for $x \leq 0$.

Fig. 0



Suggestions or Solutions
To the Problem 1 in the Example 2

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \le 1$, find $N \bullet f$.

Assuming first, g is the new function $N \bullet f$, we can set $(x_g, y_g) = (-y_f, -x_f)$.

So next, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow -x_g = f(-y_g) = (1 + y_g)^3 + 2 \Rightarrow -x_g - 2 = (1 + y_g)^3$

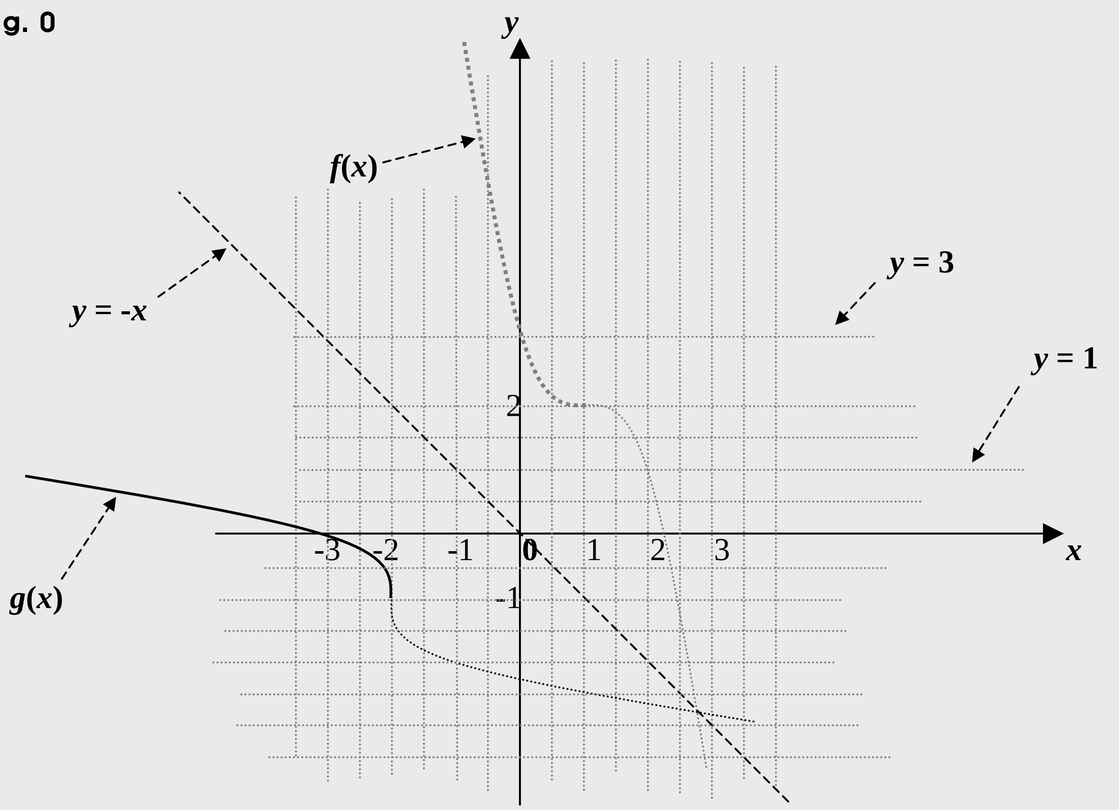
$\Rightarrow y_g + 1 = -\sqrt[3]{2 + x_g} \Rightarrow y_g = -1 - \sqrt[3]{x_g + 2}$. And we have $x_g = -y_f$. Also, we get

$y_f = (1 - x_f)^3 + 2 \ge 2$, since $x_f \le 1$. So we get $-y_f \le -2 \Rightarrow x_g \le -2$, since $x_g = -y_f$.

Thus, we get $y_g = -1 - \sqrt[3]{x_g + 2}$ for $x_g \le -2$.

So the new function g is $y = g(x) = -1 - \sqrt[3]{x + 2}$ for $x \le -2$.

Fig. 0



Suggestions or Solutions
To the Problem 2 in the Example 2

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = \sqrt[3]{1-x} + 2$ for $x \geq 1$, find $N \bullet f$.

Assuming first, g is the new function $N \bullet f$, we can set $(x_g, y_g) = (-y_f, -x_f)$.

So next, we get $y = f(x) \Rightarrow y_f = f(x_f) \Rightarrow -x_g = f(-y_g) = \sqrt[3]{1+y_g} + 2 \Rightarrow -x_g - 2 = \sqrt[3]{1+y_g}$ $\Rightarrow -(x_g + 2)^3 = 1 + y_g \Rightarrow y_g = -1 - (x_g + 2)^3$.

And we have $x_g = -y_f$, and we get $y_f = f(x_f) = \sqrt[3]{1-x_f} + 2 \leq 2$, since $x_f \geq 1$.

So we get $y_f \leq 2 \Rightarrow -y_f \geq -2 \Rightarrow x_g \geq -2$, since we have $x_g = -y_f$.

And therefore, the new function g is $y = g(x) = -1 - (x + 2)^3$ for $x \geq -2$.

Fig. 0