Examples in Symmetry about a line 6

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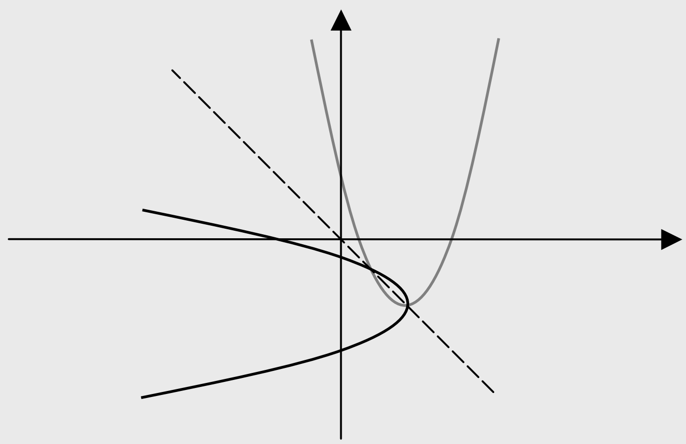
Fig. 0



Assuming $\boldsymbol{N}$**:** $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (\boldsymbol{-y}, \boldsymbol{-x})$, find the transformation of each $\boldsymbol{f}$ below by $\boldsymbol{N}$.

0. $\boldsymbol{y = f(x) = 2 - \sqrt{x+1}}$ for $\boldsymbol{-1 \le x \le 3}$.

1. $\boldsymbol{y = f(x) = (x-2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

2. $\boldsymbol{f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0}$.

3. $\boldsymbol{f(x, y) = \frac{y^2}{1-x} + 2 = 0}$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = 2 - \sqrt{x+1}$ for $-1 \le x \le 3$, find $N \bullet f$.

Assuming first, (s, t) is the new arbitrary point, we can set $(s, t) = (-y, -x)$.
Then, we get $x = -t$, and $y = -s$.

So next, we get $y = f(x) \Rightarrow -s = f(-t) = 2 - \sqrt{-t+1} \Rightarrow \sqrt{1-t} = s + 2 \Rightarrow 1 - t = (s + 2)^2$
$\Rightarrow t = 1 - (s + 2)^2$.

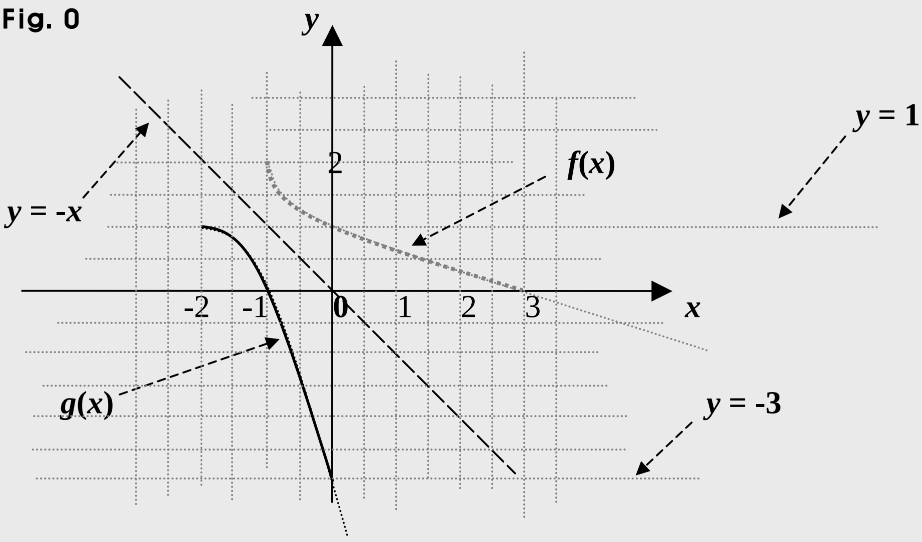
And we have $s = -y$, and $y = 2 - \sqrt{x+1}$ for $-1 \le x \le 3$. So we get

$-1 \le x \le 3 \Rightarrow 0 \le x + 1 \le 4 \Rightarrow 0 \le \sqrt{x+1} \le 2 \Rightarrow -2 \le -\sqrt{x+1} \le 0$
$\Rightarrow 0 \le 2 - \sqrt{x+1} \le 2 \Rightarrow 0 \le y \le 2 \Rightarrow -2 \le -y \le 0 \Rightarrow -2 \le s \le 0$.

So assuming g is the new function $N \bullet f$, we get $y = g(x) = 1 - (x + 2)^2$ for $-2 \le x \le 0$.

That is, we get $N \bullet f = 1 - (x + 2)^2$ for $-2 \le x \le 0$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $y = f(x) = (x - 2)^2 + 1$ for $1 \le x \le 4$, find $N \bullet f$.

Assuming first, g is the new function $N \bullet f$, and (s, t) is the new point, we can set $(s, t) = (-y, -x)$. Then, we get $y = f(x) \Rightarrow -s = f(-t) = (-t - 2)^2 + 1 \Rightarrow -(s + 1) = (t + 2)^2$.

Next, $1 \le x \le 4 \Rightarrow -1 \le x - 2 \le 2 \Rightarrow 0 \le (x - 2)^2 \le 4 \Rightarrow 1 \le (x - 2)^2 + 1 \le 5 \Rightarrow 1 \le y \le 5$ $\Rightarrow -5 \le -y \le -1 \Rightarrow -5 \le s \le -1$. And next, $1 \le x \le 4 \Rightarrow -4 \le -x \le -1 \Rightarrow -4 \le t \le -1$.

So the new equation is $-(x + 1) = (y + 2)^2$ for $-5 \le x \le -1$, and $-4 \le y \le -1$.

And next, getting the new function g, we get first
$-(x + 1) = (y + 2)^2 \Rightarrow y + 2 = \pm\sqrt{-x-1} \Rightarrow y = -2 \pm \sqrt{-x-1}$.

Then, first, $-5 \le x \le -1 \Rightarrow 1 \le -x \le 5 \Rightarrow 0 \le -x - 1 \le 4 \Rightarrow 0 \le \sqrt{-x-1} \le 2$
$\Rightarrow -2 \le \sqrt{-x-1} - 2 \le 0 \Rightarrow -2 \le y \le 0$. We have this though: $-4 \le y \le -1$.
So we get $-2 \le y \le -1$.
Then first, we get $-2 = -2 + \sqrt{-x-1} \Rightarrow x = -1$.
And next, we get $-1 = -2 + \sqrt{-x-1} \Rightarrow \sqrt{-x-1} = 1 \Rightarrow -x - 1 = 1 \Rightarrow x = -2$.
Thus, we get $y = \sqrt{x-1} - 2$ for $-2 \le x \le -1$.

And next, $0 \le \sqrt{-x-1} \le 2 \Rightarrow -2 \le -\sqrt{-x-1} \le 0 \Rightarrow -4 \le -2 - \sqrt{x-1} \le -2 \Rightarrow -4 \le y \le -2$.
So we get $y = -\sqrt{-x-1} - 2$ for $-5 \le x \le -1$.

So the new function g is $y = g(x) = \begin{cases} \sqrt{-x-1} - 2 \text{ for } -2 \le x \le -1. \\ -\sqrt{-x-1} - 2 \text{ for } -5 \le x \le -1. \end{cases}$

If not quite sure of the processes above, follow the steps below.

Assuming first, g is the new function $R \bullet f$, and (s, t) is the new point, we can set $(s, t) = (-y, -x)$. That is, we get $x = -t$, and $y = -s$.
So next, we get $y = f(x) \Rightarrow -s = f(-t) = (-t - 2)^2 + 1 \Rightarrow -s - 1 = (-t - 2)^2 = (t + 2)^2$.
$\Rightarrow -(s + 1) = (t + 2)^2$. And next, we want to get the new domain and range.

To begin with, we have $\boldsymbol{y = -s \Rightarrow s = -y}$, and $\boldsymbol{y = (x-2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$. So we get $\boldsymbol{1 \le x \le 4 \Rightarrow -1 \le x - 2 \le 2 \Rightarrow 0 \le (x-2)^2 \le 4}$, because the value of $\boldsymbol{(x-2)^2}$ is minimum when $\boldsymbol{x = 2}$, and the minimum is 0.

So we get $\boldsymbol{1 \le (x-2)^2 + 1 \le 5 \Rightarrow 1 \le y \le 5 \Rightarrow -5 \le -y \le -1 \Rightarrow -5 \le s \le -1}$, since $\boldsymbol{s = -y}$.

And next, we have $\boldsymbol{t = -x}$, and we know $\boldsymbol{1 \le x \le 4}$. So we get $\boldsymbol{-4 \le -x \le -1 \Rightarrow -4 \le t \le -1}$.

So the new equation is $\boldsymbol{-(x+1) = (y+2)^2}$ for $\boldsymbol{-5 \le x \le -1}$, and $\boldsymbol{-4 \le y \le -1}$, which indicates the curve of $\boldsymbol{g}$.

And expressing $\boldsymbol{y}$ in terms of $\boldsymbol{x}$, we want to solve for $\boldsymbol{y}$ the new equation above.
Then, we get the new function $\boldsymbol{g}$. So solving it, we get
$\boldsymbol{-(x+1) = (y+2)^2 \Rightarrow y + 2 = \pm\sqrt{-x-1} \Rightarrow y = -2 \pm \sqrt{-x-1}}$.

Then, we have two cases. One is $\boldsymbol{y = \sqrt{-x-1} - 2}$, and the other is $\boldsymbol{y = -\sqrt{-x-1} - 2}$.
So we need to determine the extent of $\boldsymbol{x}$ in each so that each behaves within $\boldsymbol{-5 \le x \le -1}$, and $\boldsymbol{-4 \le y \le -1}$.

Beginning with $\boldsymbol{y = \sqrt{-x-1} - 2}$, we get
$\boldsymbol{-5 \le x \le -1 \Rightarrow 1 \le -x \le 5 \Rightarrow 0 \le -x - 1 \le 4 \Rightarrow 0 \le \sqrt{-x-1} \le 2 \Rightarrow -2 \le \sqrt{-x-1} - 2 \le 0}$
$\boldsymbol{\Rightarrow -2 \le y \le 0}$, which is not OK, since we have $\boldsymbol{-4 \le y \le -1}$, which is the new range.
The $\boldsymbol{y}$-value has to be in the range. So it has to be the case where $\boldsymbol{-2 \le y \le -1}$, because it satisfies both $\boldsymbol{-2 \le y \le 0}$ and $\boldsymbol{-4 \le y \le -1}$.
What then, is the extent of $\boldsymbol{x}$, that is, the domain of $\boldsymbol{y = \sqrt{-x-1} - 2}$ for $\boldsymbol{-2 \le y \le -1}$?

We know that the value of ($\boldsymbol{\sqrt{-x-1} - 2}$), that is, $\boldsymbol{y}$ keeps decreasing as $\boldsymbol{x}$ keeps increasing between –5 and –1.
And we know $\boldsymbol{-2 \le y \le -1}$. So we can say that $\boldsymbol{y}$ keeps decreasing from –2 to –1.

And we know since we have $\boldsymbol{-5 \le x \le -1}$, $\boldsymbol{x}$ begins with -5, and then, increases up to -1.

And we have $\boldsymbol{y = \sqrt{-x-1} - 2}$, so when $\boldsymbol{x = -1}$, $\boldsymbol{y = -2}$. So we now want to find $\boldsymbol{x}$ for which we get $\boldsymbol{y = -1}$. So finding it, we get $\boldsymbol{-1 = \sqrt{-x-1} - 2 \Rightarrow \sqrt{-x-1} = 1 \Rightarrow -x - 1 = 1}$
$\boldsymbol{\Rightarrow x = -2}$. Thus, we get $\boldsymbol{y = \sqrt{-x-1} - 2}$ for $\boldsymbol{-2 \le x \le -1}$.

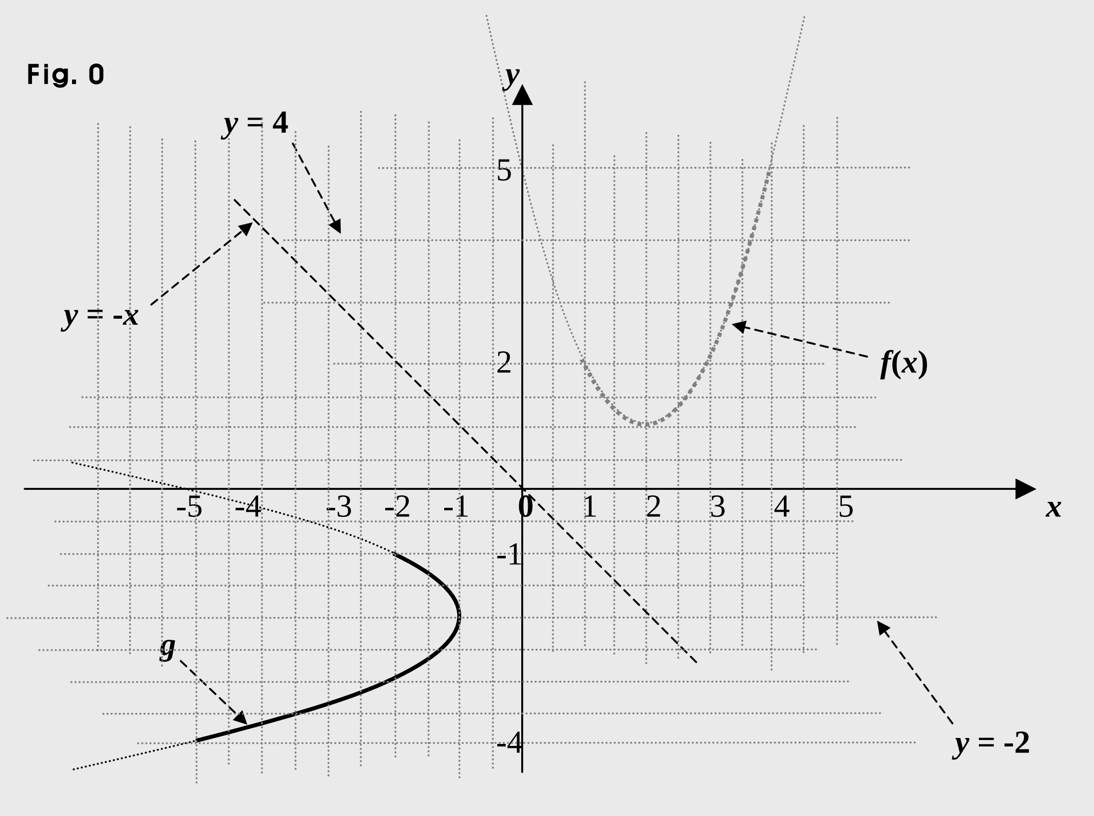
And next, moving on to $y = -\sqrt{-x-1} - 2$, since we already have $0 \le \sqrt{-x-1} \le 2$, we get $0 \le \sqrt{-x-1} \le 2 \Rightarrow -2 \le -\sqrt{-x-1} \le 0 \Rightarrow -4 \le -\sqrt{-x-1} - 2 \le -2 \Rightarrow -4 \le y \le -2$, which is OK, since we have $-4 \le y \le -1$, which is the new range.

Thus, we get $y = -\sqrt{-x-1} - 2$ for $-5 \le x \le -1$.

So the new function g is $y = g(x) = \begin{cases} \sqrt{-x-1} - 2 & \text{for } -2 \le x \le -1. \\ -\sqrt{-x-1} - 2 & \text{for } -5 \le x \le -1. \end{cases}$

And of course, we can put it the way below, too.
$y = g(x) = \sqrt{-x-1} - 2$ for $-2 \le x \le -1$, and $y = g(x) = -\sqrt{-x-1} - 2$ for $-5 \le x \le -1$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 2

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0$, find $N \bullet f$.

Assuming first, g is the new equation, and (x_g, y_g) is the new point, and (x_f, y_f) is the original point, we can set $(x_g, y_g) = (-y_f, -x_f)$. Then, we get $x_f = -y_g$, and $y_f = -x_g$.

So next, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) = 0 \Rightarrow f(-y_g, -x_g) = (-y_g - 2)^2 + (-x_g - 1)^2 - 2 = 0$.
And thus, the new equation g is $g(x, y) = (x + 1)^2 + (y + 2)^2 - 2 = 0$.

The transformation N is symmetric about the line $y = -x$. So the new curve is the curve symmetric to the original curve about the line $y = -x$. In this example the curve we apply the transformation N to is a circle, where the center is $(2, 1)$, and the radius is $\sqrt{2}$.

Thus, f is the equation of the circle above, and assuming g is the new equation, we can say that the two equations f and g are symmetric about the line $y = -x$.

And since N is a symmetric transformation, the curve itself of g, that is, the new circle itself is the same as the original circle itself. The center of the circle g can be however, different from the center of the circle f.

If a circle gets transformed symmetrically, its center gets transformed the same manner, too. So the center of g is symmetric to the center of f about the line $y = -x$.

And knowing the center and the radius of a circle, we can readily get the circle. That is, using the standard form for the equation for circles, we can easily get the equation of the circle. The equation of the circle f, that is, the original equation is given in fact, in the standard form. So let's first, get the new circle finding the center of the new circle.

We can get the new center applying the transformation N to the original center, that is, the center of the circle f. So let's now apply N to the original center $(2, 1)$.

Assuming first, (x, y) is the original point, (s, t) is the new, we get $(s, t) = (-y, -x)$.
So since the original center is $(2, 1)$, the new center is $(-1, -2)$.

And thus, the new circle is simply $\boldsymbol{(x + 1)^2 + (y + 2)^2 - 2 = 0}$.

So the new equation $\boldsymbol{g}$ can be put this way: $\boldsymbol{g(x, y) = (x + 1)^2 + (y + 2)^2 - 2 = 0}$.

And let's next, get the new circle using the method we used for the previous examples.

Assuming again, $\boldsymbol{(s, t)}$ is the new point, and $\boldsymbol{(x, y)}$ is the original point, we can set $\boldsymbol{(s, t) = (-y, -x)}$. Thus, we get $\boldsymbol{x = -t}$, and $\boldsymbol{y = -s}$.

So next, connecting $\boldsymbol{s}$ and $\boldsymbol{t}$, we put $\boldsymbol{x = -t}$ and $\boldsymbol{y = -s}$ into the original equation.

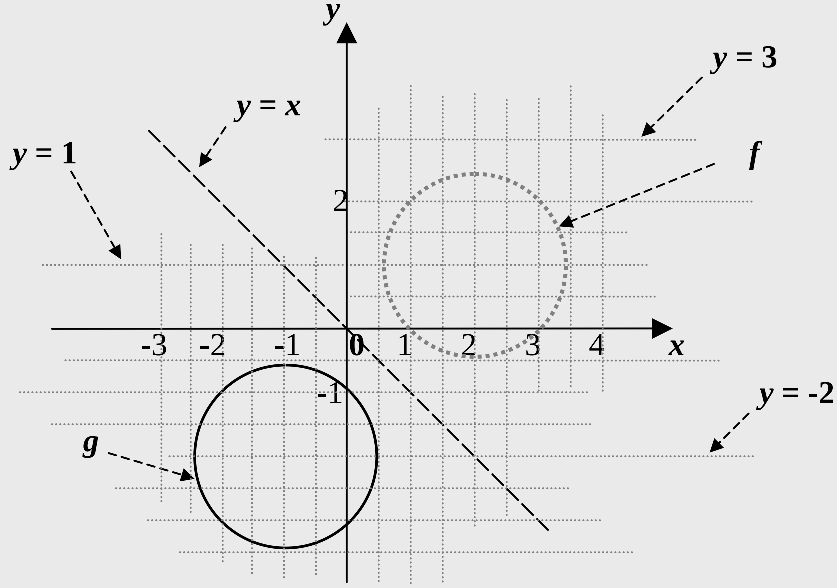
In other words, we get $\boldsymbol{f(x, y) = 0 \Rightarrow f(-t, -s) = 0}$, which is the new equation $\boldsymbol{g}$.
That is, we get $\boldsymbol{g(x, y) = f(-y, -x) = 0}$.

And we have $\boldsymbol{f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0}$.

So we get $\boldsymbol{g(x, y) = f(-y, -x) = (-x - 2)^2 + (-y - 1)^2 - 2 = 0}$.

And thus, the new equation $\boldsymbol{g}$ is $\boldsymbol{g(x, y) = (x + 2)^2 + (y + 1)^2 - 2 = 0}$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 3

Assuming N: $(x, y) \longrightarrow (-y, -x)$, and $f(x, y) = \frac{y^2}{1-x} + 2 = 0$, find $N \bullet f$.

Assuming first, g is the new equation $N \bullet f$, (x_g, y_g) is the new point, and (x_f, y_f) is the original point, we can set $(x_g, y_g) = (-y_f, -x_f)$. And we get $x_f = -y_g$, and $y_f = -x_g$.

So next, we get $f(x, y) = 0 \Rightarrow f(x_f, y_f) = 0 \Rightarrow f(-y_g, -x_g) = 0 \Rightarrow g(x_g, y_g) = f(-y_g, -x_g) = 0$.

And we have $f(x, y) = \frac{y^2}{1-x} + 2 = 0$. So we get $g(x_g, y_g) = f(-y_g, -x_g) = \frac{(-x_g)^2}{1-(-y_g)} + 2 = 0$.

So the new equation g is $g(x, y) = \frac{x^2}{1+y} + 2 = 0$. And let's now put f and g in a graph.

To begin with, converting the two equations, we get

$$f(x, y) = \frac{y^2}{1-x} + 2 = 0 \Rightarrow y^2 + 2(1-x) = 0 \Rightarrow x = \frac{y^2}{2} + 1.$$

$$g(x, y) = \frac{x^2}{y+1} + 2 = 0 \Rightarrow x^2 + 2(y+1) = 0 \Rightarrow y = -\frac{x^2}{2} - 1.$$

We want to note however, the actual equations are $\frac{y^2}{1-x} + 2 = 0$ and $\frac{x^2}{y+1} + 2 = 0$.

So the curve of f cannot have **(1, 0)**, and the curve of g cannot have **(0, -1)**, since a denominator cannot be 0.

Fig. 0