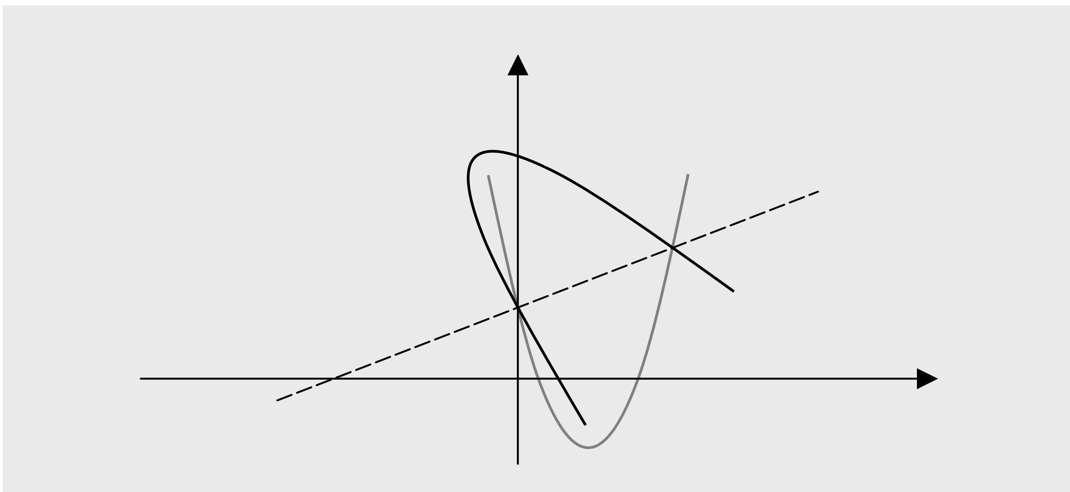
# Examples 1 in Symmetry about a line 7



Click this to start.

# Examples 1 in Symmetry about a line 7



0. Find the curve symmetric about a line $\boldsymbol{y = 2x + 1}$ to the curve of a function as follows. $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$.

1. Find the curve symmetric about a line $\boldsymbol{y = -2x + 1}$ to the curve of a function as follows. $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$.

2. Find the curve symmetric about a line $\boldsymbol{y = -2x + 2}$ to the curve of a function below. $\boldsymbol{y = f(x) = (x - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

## Suggestions or Solutions
## To the Problem in the Example 0

**Find the curve symmetric about a line $y = 2x + 1$ to the curve of a function as follows. $y = f(x) = (1 - x)^3 + 2$.**

$$\frac{4x+3y+2}{5} = (1 - \frac{-3x+4y-4}{5})^3 + 2 = (\frac{3x-4y+9}{5})^3 + 2 = \frac{(3x-4y+9)^3}{5^3} + 2 \Rightarrow \frac{4x+3y+2}{5} = \frac{(3x-4y+9)^3}{5^3} + 2$$
$$\Rightarrow (3x - 4y + 9)^3 + 250 - 25(4x + 3y + 2) = 0$$
$$\Rightarrow (3x - 4y + 9)^3 - 25(4x + 3y - 8) = 0$$, which is the curve.

*If not sure of the idea behind the solution above, follow the steps below.*

To begin with, assuming $L$ is a transformation symmetric about a line $y = ax + b$, we can put $L$ the way as follows. $L: (x, y) \longrightarrow (\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2})$.

And applying the transformation $L$ to $y = f(x)$, we get a new equation, the curve of which is symmetric about the line to the curve of $y = f(x)$. So we want to apply such a transformation, and thus, want to get the transformation first.

In this example, since $a = 2$, and $b = 1$ in the line $y = ax + b$, we get

$\frac{(1-a^2)x+2ay-2ab}{1+a^2} = \frac{(1-4)x+4y-4}{1+4} = \frac{-3x+4y-4}{5}$, and $\frac{2ax+(a^2-1)y+2b}{1+a^2} = \frac{4x+(4-1)y+2}{1+4} = \frac{4x+3y+2}{5}$.

So assuming $B$ is the transformation we want to apply, we can put $B$ the way as follows.

$B: (x, y) \longrightarrow (\frac{-3x+4y-4}{5}, \frac{4x+3y+2}{5})$. How then, do we apply it?

As explained in the section 2.6, applying the transformation $L$ to $y = f(x)$, or $f(x, y) = 0$, we can get the new equation just replacing $x$ with $\frac{(1-a^2)s+2at-2ab}{1+a^2}$, and $y$ with $\frac{2as+(a^2-1)t+2b}{1+a^2}$.

That is, the new equation is $\frac{2ax+(a^2-1)y+2b}{1+a^2} = f(\frac{(1-a^2)x+2ay-2ab}{1+a^2})$.

Or if the original equation is $f(x, y) = 0$, the new one is $f(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2}) = 0$.

So in this example, too, doing the replacements as above, we get

$\frac{4x+3y+2}{5}$ = $\boldsymbol{f}(\frac{-3x+4y-4}{5})$, which is the new equation. And we have $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$.

So putting the new equation specifically, we can put it the way below.

$$\frac{4x+3y+2}{5} = (1 - \frac{-3x+4y-4}{5})^3 + 2 = (\frac{3x-4y+9}{5})^3 + 2 = \frac{(3x-4y+9)^3}{5^3} + 2 \Rightarrow \frac{4x+3y+2}{5} = \frac{(3x-4y+9)^3}{5^3} + 2$$

$\Rightarrow \mathbf{5^2(4x + 3y + 2) = (3x - 4y + 9)^3 + 2{\cdot}5^3}$

$\Rightarrow \mathbf{(3x - 4y + 9)^3 + 250 - 25(4x + 3y + 2) = 0}$

$\Rightarrow \mathbf{(3x - 4y + 9)^3 - 25(4x + 3y - 8) = 0}$, which is the new equation, that is, the new curve.

What if however, we are not given this: $(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2})$ when we need to solve such a problem as above? Do we have to then, memorize it?

Understanding how it can be made or how it works, we can quickly get it back if we lose it. So this time, let’s get this: $(\frac{-3x+4y-4}{5}, \frac{4x+3y+2}{5})$.

To begin with, $\boldsymbol{(x, y)}$ becomes $\boldsymbol{(s, t)}$, and the two are symmetric about the line $\boldsymbol{y = 2x + 1}$, so both are the same distance away from the axis of symmetry, that is, the line $\boldsymbol{y = 2x + 1}$.

So assuming next, $\boldsymbol{(c, d)}$ is the midpoint between $\boldsymbol{(x, y)}$ and $\boldsymbol{(s, t)}$, we get $\boldsymbol{c = (x + s)/2}$ and $\boldsymbol{d = (y + t)/2}$.

And next, we know the midpoint $\boldsymbol{(c, d)}$ is in the line $\boldsymbol{y = 2x + 1}$. So we get $\boldsymbol{d = 2c + 1}$.

In other words, we get $\frac{y+t}{2} = \frac{2(x+s)}{2} + \mathbf{1}$. So we get $\boldsymbol{y + t = 2(x + s) + 2}$.

And next, we know the line segment connecting $\boldsymbol{(x, y)}$ and $\boldsymbol{(s, t)}$ is perpendicular to the line $\boldsymbol{y = 2x + 1}$, and the product of the slopes of the line segment and the line is -1.

So the slope of the line segment is -0.5, since 2 is the slope of the line $\boldsymbol{y = 2x + 1}$.

And thus, we get $\frac{y-t}{x-s} = \mathbf{-0.5} \Rightarrow \frac{2(y-t)}{x-s} = \mathbf{-1} \Rightarrow \boldsymbol{2(y - t) = -(x - s) = s - x}$.

So putting threads together now, we get a system of two equations as below.

$t - 2s = 2x - y + 2$, and $2t + s = x + 2y$.

Solving thus, the system for $s$ and $t$ beginning with $s$, we get first,
$t - 2s = 2x - y + 2 \Rightarrow 2t - 4s = 4x - 2y + 4$.

So next, we get $2t - 4s - (2t + s) = 4x - 2y + 4 - (x + 2y) \Rightarrow -5s = 3x - 4y + 4$.

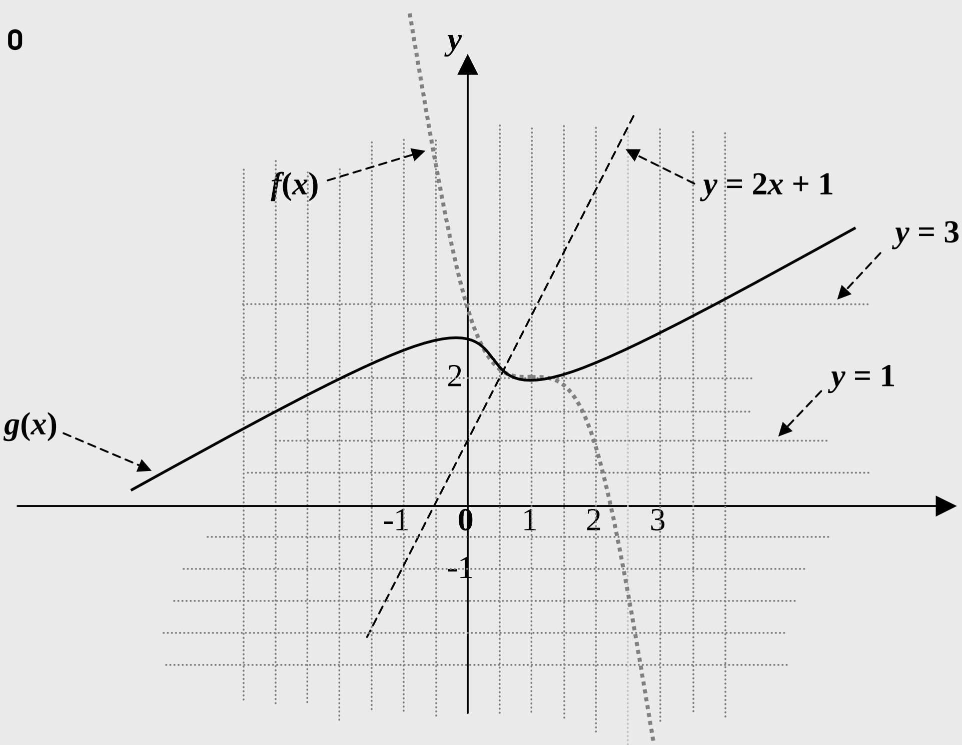
So we get $s = \frac{4y-3x-4}{5}$.

So next, putting $\frac{4y-3x-4}{5}$ into $s$ in $2t + s = x + 2y$, we get $2t + \frac{4y-3x-4}{5} = x + 2y$.

So we get $2t = \frac{-4y+3x+4}{5} + x + 2y = \frac{-4y+3x+4+5x+10y}{5} = \frac{6y+8x+4}{5}$.

And thus, we get $t = \frac{3y+4x+2}{5}$. So we now have $(s, t) = (\frac{-3x+4y-4}{5}, \frac{4x+3y+2}{5})$.

Fig. 0




$g = T \bullet f$, where $T: (x, y) \longrightarrow (\frac{-3x+4y-4}{5}, \frac{5x+3y+2}{5})$, and $y = f(x) = (1 - x)^3 + 2$.

## Suggestions or Solutions
## To the Problem in the Example 1

**Find the curve symmetric about a line $y$ = -2$x$ + 1 to the curve of a function as follows.** $y = f(x) = \sqrt[3]{1-x} + 2$.

$$\frac{-4x+3y+2}{5} = \sqrt[3]{1-\frac{-3x-4y+4}{5}} + 2 = \sqrt[3]{\frac{3x+4y+1}{5}} + 2 \Rightarrow \frac{-4x+3y+2}{5} = \sqrt[3]{\frac{3x+4y+1}{5}} + 2$$

$$\Rightarrow -4x + 3y + 2 = 5\cdot\sqrt[3]{\frac{3x+4y+1}{5}} + 10 = \sqrt[3]{25(3x+4y+1)} + 10$$

$$\Rightarrow -4x + 3y - 8 = \sqrt[3]{25(3x+4y+1)} \Rightarrow (3y - 4x - 8)^3 - 25(3x + 4y + 1) = 0, \text{ the curve.}$$

*If not sure of the idea behind the solution above, follow the steps below.*

To begin with, assuming $\boldsymbol{L}$ is a transformation symmetric about a line $\boldsymbol{y = ax + b}$, we can put $\boldsymbol{L}$ the way as follows. $\boldsymbol{L}$**:** $(x, y) \longrightarrow (\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2})$.

And applying $\boldsymbol{L}$ to $\boldsymbol{y = f(x)}$, we get a new equation, the curve of which is symmetric about the line to the curve of $\boldsymbol{y = f(x)}$. So we want to apply such a transformation, and thus, want to get the transformation first.

In this example, since $\boldsymbol{a = -2}$, and $\boldsymbol{b = 1}$ in the line $\boldsymbol{y = ax + b}$, we get

$$\frac{(1-a^2)x+2ay-2ab}{1+a^2} = \frac{(1-4)x-4y+4}{1+4} = \frac{-3x-4y+4}{5}, \quad \text{and} \quad \frac{2ax+(a^2-1)y+2b}{1+a^2} = \frac{-4x+(4-1)y+2}{1+4} = \frac{-4x+3y+2}{5}.$$

So assuming $\boldsymbol{B}$ is the transformation we want to apply, we can put $\boldsymbol{B}$ the way as follows.

$\boldsymbol{B}$**:** $(x, y) \longrightarrow (\frac{-3x-4y+4}{5}, \frac{-4x+3y+2}{5})$. How then, do we apply it?

As explained in the section 2.3, applying the transformation $\boldsymbol{L}$ to $\boldsymbol{y = f(x)}$, or $\boldsymbol{f(x, y) = 0}$, we can get the new equation just replacing $\boldsymbol{x}$ with $\frac{(1-a^2)s+2at-2ab}{1+a^2}$, and $\boldsymbol{y}$ with $\frac{2as+(a^2-1)t+2b}{1+a^2}$.

That is, the new equation is $\frac{2ax+(a^2-1)y+2b}{1+a^2} = f(\frac{(1-a^2)x+2ay-2ab}{1+a^2})$.

Or if the original equation is $\boldsymbol{f(x, y) = 0}$, the new one is $f(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2}) = 0$.

So in this example, too, doing the replacements as above, we get

$\frac{-4x+3y+2}{5}$ = $f(\frac{-3x-4y+4}{5})$, which is the new equation. And we have $y = f(x) = \sqrt[3]{1-x} + 2$.

So putting the new equation specifically, we can put it the way below.

$\frac{-4x+3y+2}{5} = \sqrt[3]{1-\frac{-3x-4y+4}{5}} + 2 = \sqrt[3]{\frac{3x+4y+1}{5}} + 2 \Rightarrow \frac{-4x+3y+2}{5} = \sqrt[3]{\frac{3x+4y+1}{5}} + 2$

$\Rightarrow -4x + 3y + 2 = 5\cdot\sqrt[3]{\frac{3x+4y+1}{5}} + 10 = \sqrt[3]{25(3x+4y+1)} + 10$

$\Rightarrow -4x + 3y - 8 = \sqrt[3]{25(3x+4y+1)} \Rightarrow (3y - 4x - 8)^3 - 25(3x + 4y + 1) = 0$, which is the new equation, that is, the new curve.

How do we get this though: $(\frac{-3x-4y+4}{5}, \frac{-4x+3y+2}{5})$ ?

To begin with, $(x, y)$ becomes $(s, t)$, and the two are symmetric about the line $y = -2x + 1$, so both are the same distance away from the axis of symmetry, that is, the line above.

So assuming next, $(c, d)$ is the midpoint between $(x, y)$ and $(s, t)$, we get
$c = (x + s)/2$ and $d = (y + t)/2$.

And next, we know the midpoint $(c, d)$ is in the line $y = -2x + 1$. So we get $d = -2c + 1$.

In other words, we get $(y + t)/2 = -2(x + s)/2 + 1$. So we get $y + t = -2(x + s) + 2$.

And next, we know the line segment connecting $(x, y)$ and $(s, t)$ is perpendicular to the line $y = -2x + 1$, and the product of the slopes of the line segment and the line is -1.

So the slope of the line segment is 0.5 since -2 is the slope of the line $y = -2x + 1$.

And thus, we get $(y - t)/(x - s) = 0.5 \Rightarrow 2(y - t) = x - s$.

So we get a system of two equations where $t + 2s = -2x - y + 2$, and $2t - s = -x + 2y$.

Solving thus, the system for $s$ first, we get first,
$t + 2s = -2x - y + 2 \Rightarrow 2t + 4s = -4x - 2y + 4$.

So next, we get $2t + 4s - (2t - s) = -4x - 2y + 4 - (-x + 2y) \Rightarrow 5s = -3x - 4y + 4$.

Thus, we get $\boldsymbol{s} = \frac{-3x-4y+4}{5}$.

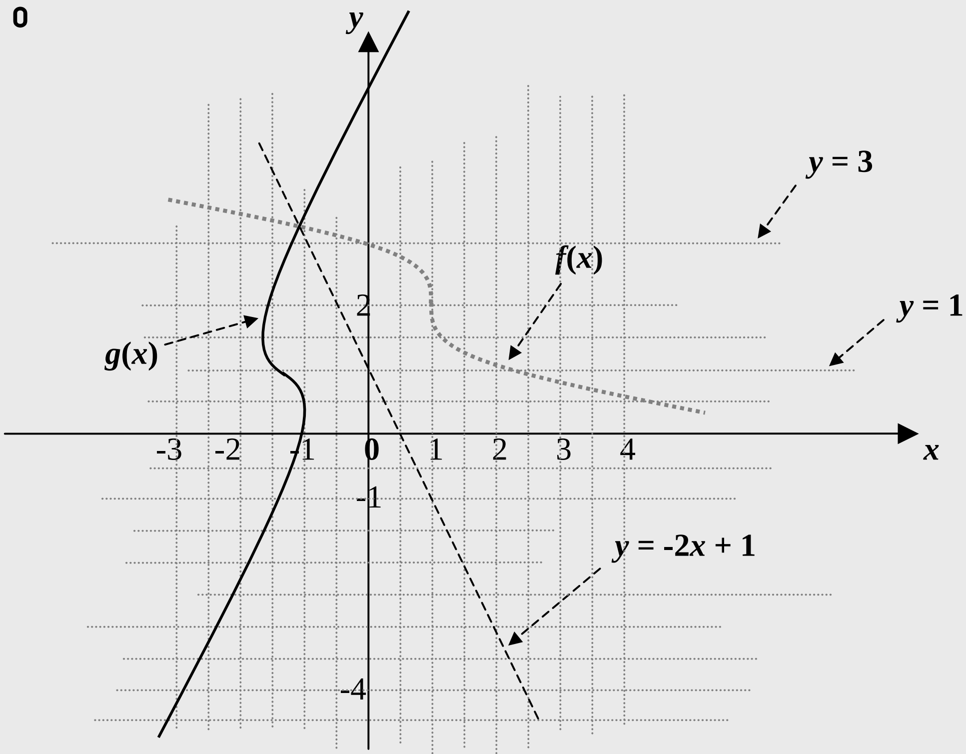
So next, putting $\frac{-3x-4y+4}{5}$ into $\boldsymbol{s}$ in $\mathbf{2}\boldsymbol{t} - \boldsymbol{s} = \mathbf{-}\boldsymbol{x} + \mathbf{2}\boldsymbol{y}$, we get $\mathbf{2}\boldsymbol{t} - \frac{-3x-4y+4}{5} = \mathbf{-}\boldsymbol{x} + \mathbf{2}\boldsymbol{y}$.

So we get $\mathbf{2}\boldsymbol{t} = \frac{-3x-4y+4}{5} - \boldsymbol{x} + \mathbf{2}\boldsymbol{y} = \frac{-3x-4y+4-5x+10y}{5} = \frac{-8x+6y+4}{5}$.

And thus, we get $\boldsymbol{t} = \frac{-4x+3y+2}{5}$.

So we now have $(\boldsymbol{s}, \boldsymbol{t}) = (\frac{-3x-4y+4}{5}, \frac{-4x+3y+2}{5})$.

**Fig. 0**




$\boldsymbol{g} = \boldsymbol{T}\bullet\boldsymbol{f}$, where $\boldsymbol{T}\mathbf{:}\ (\boldsymbol{x}, \boldsymbol{y}) \longrightarrow (\frac{-3x-4y+4}{5}, \frac{-4x+3y+2}{5})$, and $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = \sqrt[3]{1-x} + \mathbf{2}$.

## Suggestions or Solutions
## To the Problem in the Example 2

**Find the curve symmetric about a line $y = -2x + 2$ to the curve of a function below.**
**$y = f(x) = (x - 2)^2 + 1$ for $1 \le x \le 4$.**

Let's this time, get the solution getting this first: $\left(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2}\right)$.

So suppose first, $(x, y)$ is the original arbitrary point, and $(s, t)$ is the new arbitrary point.

Then, $(x, y)$ becomes $(s, t)$, and the two are symmetric about the line $y = -2x + 2$, so both are the same distance away from the axis of symmetry, that is, the line $y = -2x + 2$.

So next, assuming $(c, d)$ is the midpoint between $(x, y)$ and $(s, t)$, we get
$c = (x + s)/2$ and $d = (y + t)/2$.

And we know the midpoint $(c, d)$ is in the line $y = -2x + 2$. So next, we get $d = -2c + 2$.
In other words, we get $(y + t)/2 = -2(x + s)/2 + 2 \Rightarrow y + t = -2(x + s) + 4$.

And next, we know the line segment connecting $(x, y)$ and $(s, t)$ is perpendicular to the line $y = -2x + 2$, and the product of the slopes of the line segment and the line is -1.

So the slope of the line segment is 0.5, since -2 is the slope of the line $y = -2x + 2$.
Thus, we get $(y - t)/(x - s) = 0.5 \Rightarrow 2(y - t) = x - s$.

So we get a system of two equations where $t + 2s = -2x - y + 4$, and $2t - s = -x + 2y$.

Solving thus, the system for $s$ first, we get first,
$t + 2s = -2x - y + 4 \Rightarrow 2t + 4s = -4x - 2y + 8$.

So next, we get
$2t + 4s - (2t - s) = -4x - 2y + 8 - (-x + 2y) \Rightarrow 5s = -3x - 4y + 8 \Rightarrow s = (-3x - 4y + 8)/5$.

And next, putting the $s$ above into $2t - s = -x + 2y$, we get $2t - (-3x - 4y + 8)/5 = -x + 2y$.
So we get $2t = (-3x - 4y + 8)/5 + (-5x + 10y)/5 = (-8x + 6y + 8)/5$.

And thus, we get $\boldsymbol{t = (-4x + 3y + 4)/5}$. So we can now set $\boldsymbol{(s, t) = (\frac{-3x-4y+8}{5}, \frac{-4x+3y+4}{5})}$.

Thus, we get $\boldsymbol{s} = \frac{-3x-4y+8}{5}$, and $\boldsymbol{t} = \frac{-4x+3y+4}{5}$.

And in a transformation symmetric about a line $\boldsymbol{y = ax + b}$, the extents of the variables in the original equation can change, too. So specifying the new equation, we need to specify the new extents, that is, the new domain and range, at least the new domain if the original equation is specified with its domain.

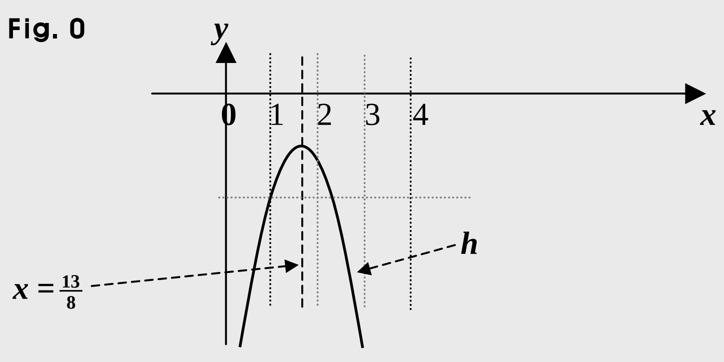
So next, moving on to the extents of the variables in the new curve, and beginning with the extent of $\boldsymbol{s}$, we can use $\boldsymbol{s} = \frac{-3x-4y+8}{5}$, and $\boldsymbol{y = f(x) = (x - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

And we can put the original equation this way, too: $\boldsymbol{y = f(x) = x^2 - 4x + 5}$ for $\boldsymbol{1 \le x \le 4}$.

Then first, we can get $\boldsymbol{-3x - 4y + 8 = -3x - 4(x^2 - 4x + 5) + 8 = -4x^2 + 13x - 12}$, which makes a parabola concave-down, where the axis of symmetry is $\boldsymbol{x = \frac{13}{8}}$, which is inside the extent, $\boldsymbol{1 \le x \le 4}$, and passes through a point closer to $\boldsymbol{(1, 0)}$, in the $\boldsymbol{x}$-axis.

So just assuming $\boldsymbol{h(x) = -4x^2 + 13x - 12}$ for $\boldsymbol{1 \le x \le 4}$, we get $\boldsymbol{h(4) \le h(x) \le h(\frac{13}{8})}$.
(We can see better how it is the case looking at the graph of $\boldsymbol{h}$.)

**Fig. 0**




Then first, we get $\boldsymbol{h(\frac{13}{8}) = -4(\frac{13}{8})^2 + 13(\frac{13}{8}) - 12 = \frac{1}{64}(-4 \cdot 13^2 + 13^2 \cdot 8 - 12 \cdot 8^2) = -\frac{23}{16}}$.

And next, we get $\boldsymbol{h(4) = -4^3 + 13 \cdot 4 - 12 = 4(-16 + 13 - 3) = -24}$.

So we get $\mathbf{-24 \le -3}\boldsymbol{x}\mathbf{ - 4}\boldsymbol{y}\mathbf{ + 8 \le -\frac{23}{16}} \Rightarrow -\frac{24}{5} \le \frac{-3x-4y+8}{5} \le -\frac{23}{80} \Rightarrow -\frac{24}{5} \le \boldsymbol{s} \le -\frac{23}{80}$.

And next, moving on to the extent of $\boldsymbol{t}$, we get to begin with this: $\boldsymbol{t} = \frac{-4x+3y+4}{5}$.
Then, first, we can get $\mathbf{-4}\boldsymbol{x}\mathbf{ + 3}\boldsymbol{y}\mathbf{ + 4 = -4}\boldsymbol{x}\mathbf{ + 3(}\boldsymbol{x}^2\mathbf{ - 4}\boldsymbol{x}\mathbf{ + 5) + 4 = 3}\boldsymbol{x}^2\mathbf{ - 16}\boldsymbol{x}\mathbf{ + 19}$, which makes a parabola concave-up, where the axis of symmetry is $\boldsymbol{x} = \frac{8}{3}$, which is inside the extent, $\mathbf{1 \le}\boldsymbol{x}\mathbf{\le 4}$, and passes through a point closer to $\mathbf{(4, 0)}$, in the $\boldsymbol{x}$-axis.

So assuming $\boldsymbol{k}(\boldsymbol{x}) = 3\boldsymbol{x}^2 - 16\boldsymbol{x} + 19$ for $1 \le \boldsymbol{x} \le 4$, we get $\boldsymbol{k}(\frac{8}{3}) \le \boldsymbol{k}(\boldsymbol{x}) \le \boldsymbol{k}(1)$.

(We can see better how it is the case looking at the graph of $\boldsymbol{k}$.)

Then first, we get $\boldsymbol{k}(\frac{8}{3}) = 3(\frac{8}{3})^2 - 16(\frac{8}{3}) + 19 = \frac{1}{9}(3 \cdot 8^2 - 3 \cdot 16 \cdot 8 + 9 \cdot 19) = -\frac{7}{3}$.

And next, we get $\boldsymbol{k}(1) = 3 \cdot 1^2 - 16 \cdot 1 + 19 = 6$.

So we get $-\frac{7}{3} \le -4\boldsymbol{x} + 3\boldsymbol{y} + 4 \le 6 \Rightarrow -\frac{7}{15} \le \frac{-4x+3y+4}{5} \le \frac{6}{5} \Rightarrow -\frac{7}{15} \le \boldsymbol{t} \le \frac{6}{5}$.

And thus, we now have $-\frac{24}{5} \le \boldsymbol{s} \le \frac{1}{5}$, and $-\frac{7}{15} \le \boldsymbol{t} \le \frac{6}{5}$.

And we know that $(\boldsymbol{s}, \boldsymbol{t})$ is the new arbitrary point, $\boldsymbol{s}$ is the $\boldsymbol{x}$-coordinate, and $\boldsymbol{t}$ is the $\boldsymbol{y}$-coordinate, so $\boldsymbol{s}$ is the variable $\boldsymbol{x}$ in the new equation $\boldsymbol{g}$, and $\boldsymbol{t}$ is the variable $\boldsymbol{y}$ in the new equation.

So the equation of $\boldsymbol{g}$ is $\frac{-4x+3y+4}{5} = \boldsymbol{f}(\frac{-3x-4y+8}{5})$ for $-\frac{24}{5} \le \boldsymbol{x} \le \frac{1}{5}$, and $-\frac{7}{15} \le \boldsymbol{y} \le \frac{6}{5}$.

And getting the expression of the new equation $\boldsymbol{g}$, we begin with the original equation, that is, $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x} - 2)^2 + 1$. Then, we get

$$\frac{-4x+3y+4}{5} = (\frac{-3x-4y+8}{5} - 2)^2 + 1 = \frac{(-3x-4y-2)^2}{25} \Rightarrow 5(-4x + 3y + 4) = (3x + 4y + 2)^2$$

$$\Rightarrow (3x + 4y + 2)^2 - 5(-4x + 3y + 4) = 0$$

$$\Rightarrow 9x^2 + 16y^2 + 4 + 2(12xy + 8y + 6x) + 20x - 15y - 20$$

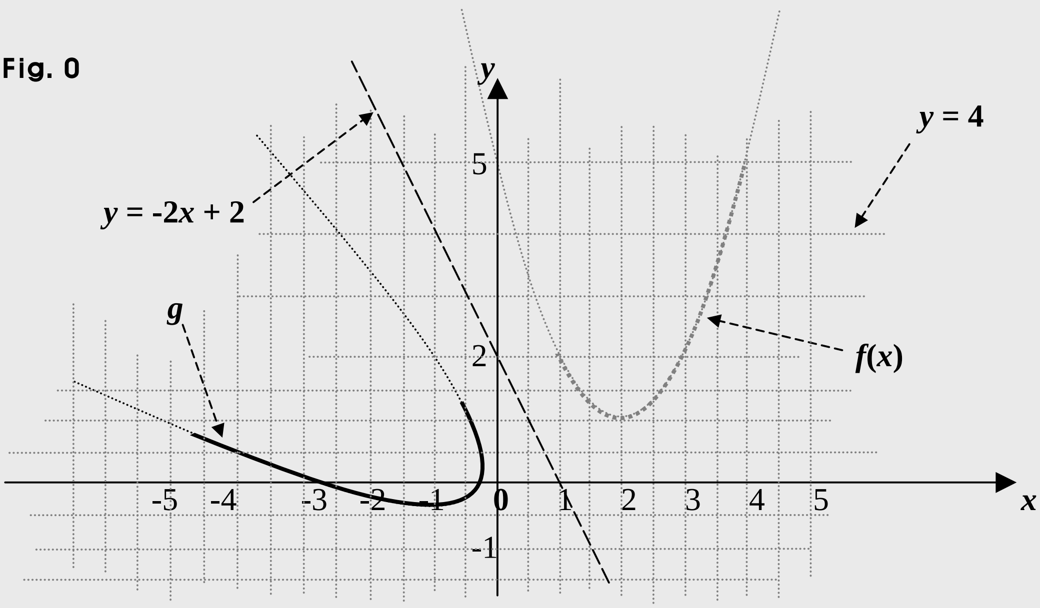
$$= 9x^2 + 16y^2 + 24xy + 32x + y - 16 = 0.$$

So the new equation $\boldsymbol{g}$ is

$\mathbf{9}\boldsymbol{x}^2 + \mathbf{16}\boldsymbol{y}^2 + \mathbf{24}\boldsymbol{xy} + \mathbf{32}\boldsymbol{x} + \boldsymbol{y} - \mathbf{16} = \mathbf{0}$ for $-\frac{24}{5} \le x \le \frac{1}{5}$, and $-\frac{7}{15} \le y \le \frac{6}{5}$.

And putting the two curves of $\boldsymbol{f}$ and $\boldsymbol{g}$ in a graph, we get

Fig. 0




$\boldsymbol{g} = \boldsymbol{T} \bullet \boldsymbol{f}$, where $\boldsymbol{T}$: $(\boldsymbol{x}, \boldsymbol{y}) \longrightarrow \left(\frac{-3x-4y+8}{5}, \frac{-4x+3y+4}{5}\right)$, and $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x}) = (\boldsymbol{x} - \mathbf{2})^2 + \mathbf{1}$ for $\mathbf{1} \le \boldsymbol{x} \le \mathbf{4}$.