Examples 2 in Symmetry about a line 7

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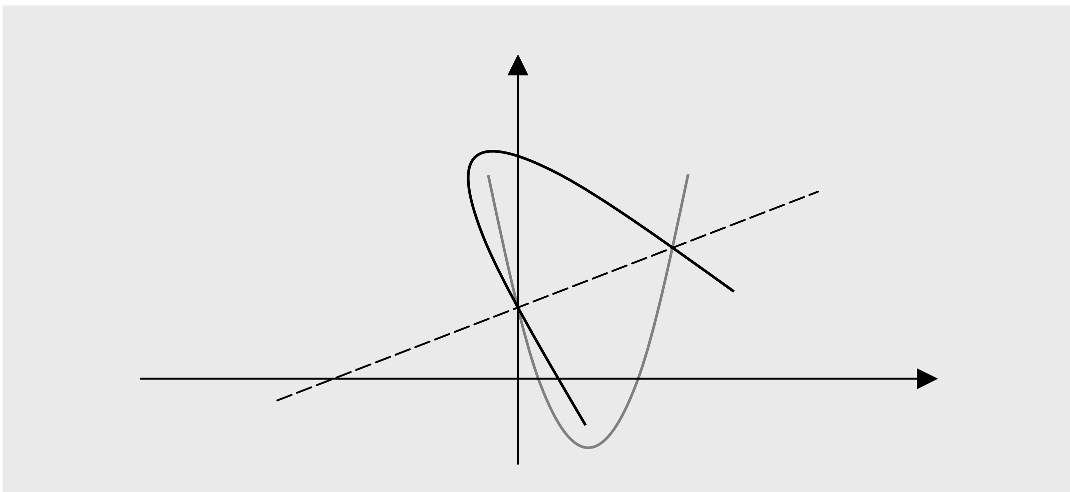


0. Find the function or equation symmetric about a line $\boldsymbol{y = -x + 5}$ to the function as follows. $\boldsymbol{y = f(x) = x^2 - 2x + 1}$ for $\boldsymbol{x \geq 1}$.

1. Find the function or equation symmetric about a line $\boldsymbol{y = x - 1}$ to the function below. $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \leq 1}$.

2. Find the function or equation symmetric about a line $\boldsymbol{y = x + 3}$ to the function as follows. $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x \geq 1}$.

Suggestions or Solutions
To the Problem in the Example 0

Find the function or equation symmetric about a line $y = -x + 5$ to the function as follows. $y = f(x) = x^2 - 2x + 1$ for $x \geq 1$.

Assuming $\boldsymbol{g}$ is the function we want, we get $\boldsymbol{y = g(x) = 4 - \sqrt{5 - x}}$ for $\boldsymbol{x \leq 5}$.

If not sure of the idea behind the solution above, follow the steps below.

To begin with, assuming $\boldsymbol{L}$ is a transformation symmetric about a line $\boldsymbol{y = ax + b}$, we can put $\boldsymbol{L}$ the way as follows. $\boldsymbol{L}$**:** $\boldsymbol{(x, y) \longrightarrow \left(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2}\right)}$.

And applying $\boldsymbol{L}$ to $\boldsymbol{y = f(x)}$, we get a new function or equation, which is symmetric about the line to $\boldsymbol{y = f(x)}$. So we want to apply such a transformation, and thus, want to get the transformation first. So let's get it now.

Suppose first, $\boldsymbol{(x, y)}$ is the original arbitrary point, and $\boldsymbol{(s, t)}$ is the new arbitrary point. Then, $\boldsymbol{(x, y)}$ becomes $\boldsymbol{(s, t)}$, and the two are symmetric about the line $\boldsymbol{y = -x + 5}$, so both are the same distance away from the axis of symmetry, that is, the line $\boldsymbol{y = -x + 5}$.

So assuming $\boldsymbol{(c, d)}$ is the midpoint between $\boldsymbol{(x, y)}$ and $\boldsymbol{(s, t)}$, we get $\boldsymbol{c = \frac{x+s}{2}}$, and $\boldsymbol{d = \frac{y+t}{2}}$.

Next, we know the midpoint $\boldsymbol{(c, d)}$ is in the line $\boldsymbol{y = -x + 5}$. So we get $\boldsymbol{d = -c + 5}$.
In other words, we get $\boldsymbol{(y + t)/2 = -(x + s)/2 + 5 \Rightarrow y + t = -(x + s) + 10}$.

And next, we know the line segment connecting $\boldsymbol{(x, y)}$ and $\boldsymbol{(s, t)}$ is perpendicular to the line $\boldsymbol{y = -x + 5}$, and the product of the slopes of the line segment and the line is -1.
So the slope of the line segment is 1 since -1 is the slope of the line $\boldsymbol{y = -x + 5}$.
And thus, we get $\boldsymbol{(y - t)/(x - s) = 1 \Rightarrow y - t = x - s}$.

So we get a system of two equations where $\boldsymbol{t + s = -x - y + 10}$, and $\boldsymbol{t - s = -x + y}$.
Solving thus, the system for $\boldsymbol{t}$ first, we get
$\boldsymbol{t + s + (t - s) = -x - y + 10 + (-x + y) \Rightarrow 2t = -2x + 10 \Rightarrow t = -x + 5}$. And next, putting the $\boldsymbol{t}$ above into $\boldsymbol{t + s = -x - y + 10}$, we get $\boldsymbol{-x + 5 + s = -x - y + 10 \Rightarrow s = -y + 5}$.

So we can set $(s, t) = (-y + 5, -x + 5)$.

And next, moving on to the extents of the variables in the new curve, and beginning with s, we can work with $s = -y + 5$, along with $y = f(x) = x^2 - 2x + 1 = (x - 1)^2$ for $x \geq 1$.

Then, we get $y \geq 0 \Rightarrow -y \leq 0 \Rightarrow -y + 5 \leq 5$. Thus, we get $s \leq 5$.

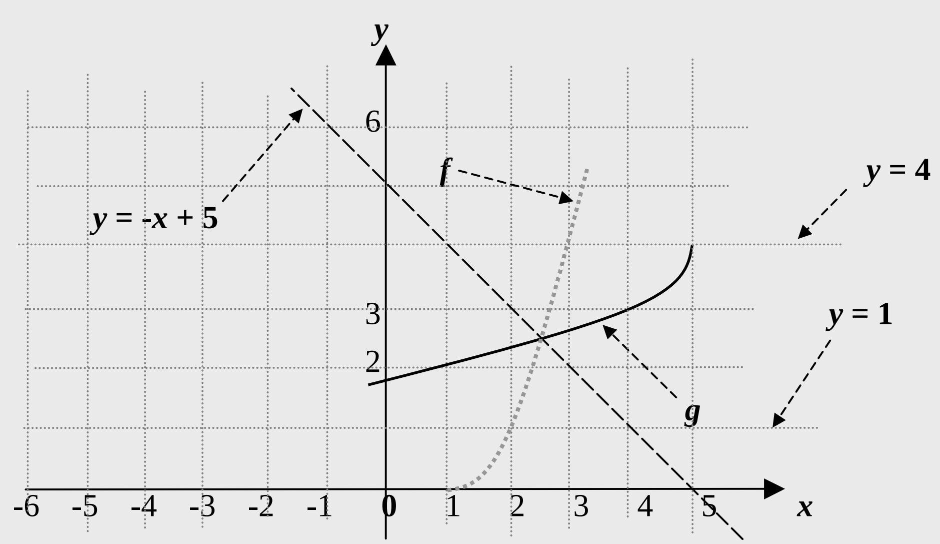
And next, moving on to the extent of t, we can work with $t = -x + 5$.
And we have $x \geq 1$. So we get $-x \leq -1 \Rightarrow -x + 5 \leq 4$. Thus, we get $t \leq 4$.

So the new equation, that is, the curve we want is $5 - x = f(5 - y)$ for $x \leq 5$, and $y \leq 4$.
And we have $y = f(x) = (x - 1)^2$ for $x \geq 1$.

So we get $5 - x = (5 - y - 1)^2 \Rightarrow 5 - x = (4 - y)^2$.
And thus, the new equation is $x = 5 - (4 - y)^2$ for $x \leq 5$, and $y \leq 4$.

And expressing y in terms of x, we get
$x = 5 - (4 - y)^2 \Rightarrow (4 - y)^2 = 5 - x \Rightarrow 4 - y = \pm\sqrt{5 - x} \Rightarrow y = 4 \pm \sqrt{5 - x}$.
And we know $y \leq 4$, and $\sqrt{5 - x}$ is positive or 0 if $x \leq 5$.
So we need this: $-\sqrt{5 - x}$, and not this: $\sqrt{5 - x}$.
So the new equation can be put this way, too: $y = 4 - \sqrt{5 - x}$ for $x \leq 5$.
And assuming g the new function, we get $y = g(x) = 4 - \sqrt{5 - x}$ for $x \leq 5$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 1

Find the function or equation symmetric about a line $y = x - 1$ to the function as follows. $y = f(x) = (1 - x)^3 + 2$ for $x \leq 1$.

Assuming $\boldsymbol{g}$ is the function we want, we get $\boldsymbol{y = g(x) = \sqrt[3]{3 - x}}$ for $\boldsymbol{x \geq 3}$.

If not sure of the idea behind the solution above, follow the steps below.

To begin with, assuming $\boldsymbol{L}$ is a transformation symmetric about a line $\boldsymbol{y = ax + b}$, we can put $\boldsymbol{L}$ the way as follows. $\boldsymbol{L: (x, y) \longrightarrow \left(\frac{(1-a^2)x+2ay-2ab}{1+a^2}, \frac{2ax+(a^2-1)y+2b}{1+a^2}\right)}$.

And applying $\boldsymbol{L}$ to $\boldsymbol{y = f(x)}$, we get a new function or equation, symmetric about the line to $\boldsymbol{y = f(x)}$. So we want to apply such a transformation, and thus, want to get the transformation first. So let's get it now.

Suppose first, $\boldsymbol{(s, t)}$ is the new arbitrary point. Then, $\boldsymbol{(x, y)}$ becomes $\boldsymbol{(s, t)}$, and the two are symmetric about the line $\boldsymbol{y = x - 1}$, so both are equal distance away from the line.
So assuming next, $\boldsymbol{(c, d)}$ is the midpoint, we get $\boldsymbol{c = (x + s)/2}$ and $\boldsymbol{d = (y + t)/2}$.

And next, the midpoint $\boldsymbol{(c, d)}$ is in the line $\boldsymbol{y = x - 1}$. So we get $\boldsymbol{d = c - 1}$.
In other words, we get $\boldsymbol{(y + t)/2 = (x + s)/2 - 1 \Rightarrow y + t = x + s - 2}$.

And next, we know the line segment connecting $\boldsymbol{(x, y)}$ and $\boldsymbol{(s, t)}$ is perpendicular to the line $\boldsymbol{y = x - 1}$, and the product of the slopes of the line segment and the line is -1.
So the slope of the line segment is -1 since 1 is the slope of the line $\boldsymbol{y = x - 1}$.
And thus, we get $\boldsymbol{(y - t)/(x - s) = -1 \Rightarrow y - t = s - x}$.

So we get a system of two equations where $\boldsymbol{t - s = x - y - 2}$, and $\boldsymbol{t + s = x + y}$.
Solving thus, the system for $\boldsymbol{t}$ first, we get

$\boldsymbol{t + s + (t - s) = x + y + (x - y - 2) \Rightarrow 2t = 2x - 2 \Rightarrow t = x - 1}$. And next, putting the $\boldsymbol{t}$ above into $\boldsymbol{t + s = x + y}$, we get $\boldsymbol{x - 1 + s = x + y \Rightarrow s = y + 1}$.

So we can set $\boldsymbol{(s, t) = (y + 1, x - 1)}$.

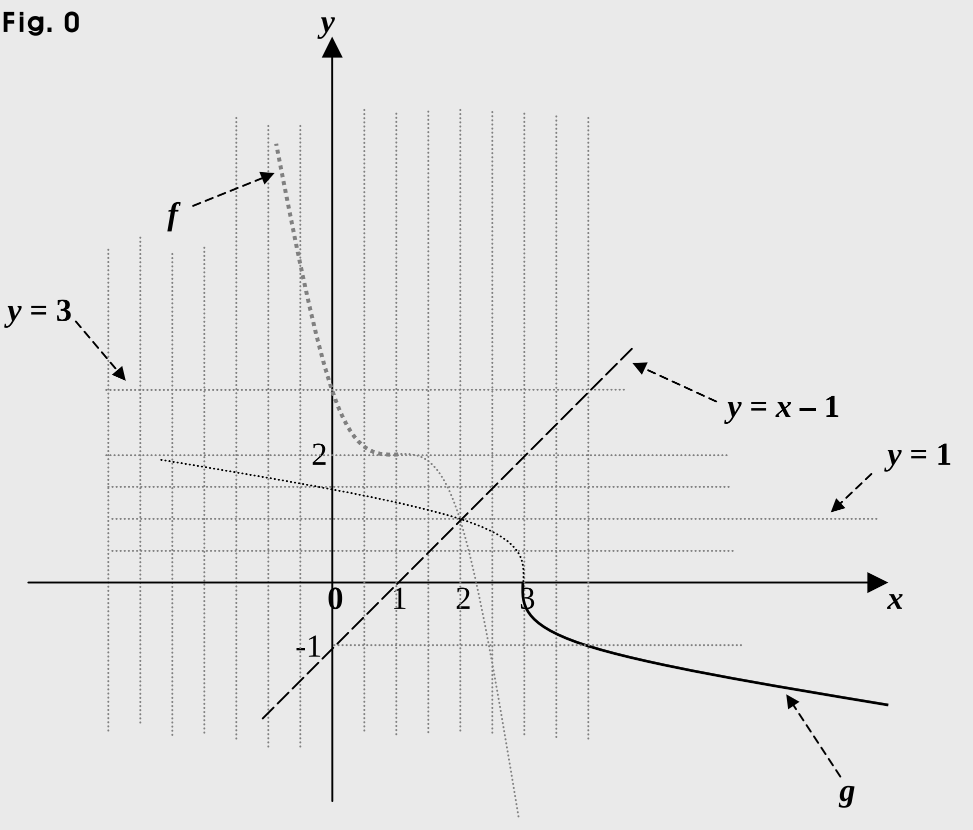
And next, moving on to the extents of the variables in the new curve, and beginning with $\boldsymbol{s}$, we can work with $\boldsymbol{s = y + 1}$, along with $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \le 1}$.

Then, we get $\boldsymbol{y \ge 2 \Rightarrow y + 1 \ge 3}$. Thus, we get $\boldsymbol{s \ge 3}$.

And next, moving on to the extent of $\boldsymbol{t}$, we can work with $\boldsymbol{t = x - 1}$.
And we have $\boldsymbol{x \le 1}$. So we get $\boldsymbol{x - 1 \le 0}$. Thus, we get $\boldsymbol{t \le 0}$.

So the new equation is $\boldsymbol{x - 1 = f(y + 1)}$ for $\boldsymbol{x \ge 3}$, and $\boldsymbol{y \le 0}$.
And we have $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$ for $\boldsymbol{x \le 1}$.
So we get $\boldsymbol{x - 1 = \{1 - (y + 1)\}^3 + 2 \Rightarrow x = 3 - y^3}$.
And thus, the curve of $\boldsymbol{g}$ is $\boldsymbol{x = 3 - y^3}$ for $\boldsymbol{x \ge 3}$, and $\boldsymbol{y \le 0}$.
And expressing $\boldsymbol{y}$ in terms of $\boldsymbol{x}$, we get $\boldsymbol{x = 3 - y^3 \Rightarrow y^3 = 3 - x \Rightarrow y = \sqrt[3]{3 - x}}$.
So the new equation can be put this way, too: $\boldsymbol{y = \sqrt[3]{3 - x}}$ for $\boldsymbol{x \ge 3}$.
And assuming $\boldsymbol{g}$ is the new function, we can put it this way: $\boldsymbol{y = g(x) = \sqrt[3]{3 - x}}$ for $\boldsymbol{x \ge 3}$.

Fig. 0



Suggestions or Solutions
To the Problem in the Example 2

Find the function or equation symmetric about a line $y = x + 3$ to the function as follows. $y = f(x) = \sqrt[3]{1-x} + 2$ for $x \geq 1$.

Assuming g is the function we want, we get $y = g(x) = (2 - x)^3 + 2$ for $x \leq -1$.

If not sure of the idea behind the solution above, follow the steps below.

Suppose first, (x, y) is the original arbitrary point, and (s, t) is the new arbitrary point. Then, (x, y) becomes (s, t), and the two are symmetric about the line $y = x + 3$, so both are the same distance away from the line.

So assuming (c, d) is the midpoint between (x, y) and (s, t), we get $c = \frac{x+s}{2}$, and $d = \frac{y+t}{2}$.

And next, the midpoint (c, d) is in the line $y = x + 3$. So we get $d = c + 3$.
In other words, we get $(y + t)/2 = (x + s)/2 + 3 \Rightarrow y + t = x + s + 6$.

And next, we know the line segment connecting (x, y) and (s, t) is perpendicular to the line $y = x + 3$, and the product of the slopes of the line segment and the line is -1.

So the slope of the line segment is -1 since 1 is the slope of the line $y = x - 1$.
And thus, we get $(y - t)/(x - s) = -1 \Rightarrow y - t = s - x$.

So we get a system of two equations where $t - s = x - y + 6$, and $t + s = x + y$.

Solving thus, the system for t first, we get
$t + s + (t - s) = x + y + (x - y + 6) \Rightarrow 2t = 2x + 6 \Rightarrow t = x + 3$. And next, putting the t above into $t + s = x + y$, we get $x + 3 + s = x + y \Rightarrow s = y - 3$.

So we can set $(s, t) = (y - 3, x + 3)$.

And next, moving on to the extents of the variables in the new curve, and beginning with s, we can work with $s = y - 3$, along with $y = f(x) = \sqrt[3]{1-x} + 2$ for $x \geq 1$.

Then, we get $\boldsymbol{y \le 2 \Rightarrow y - 3 \le -1}$. Thus, we get $\boldsymbol{s \le -1}$.

And next, moving on to the extent of $\boldsymbol{t}$, we can work with $\boldsymbol{t = x + 3}$.
And we have $\boldsymbol{x \ge 1}$. So we get $\boldsymbol{x + 3 \ge 4}$. Thus, we get $\boldsymbol{t \ge 4}$.

And thus, the new equation is $\boldsymbol{x + 3 = f(y - 3)}$ for $\boldsymbol{x \le -1}$, and $\boldsymbol{y \ge 4}$.

And we have $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$ for $\boldsymbol{x \ge 1}$.

So we get $\boldsymbol{x + 3 = \sqrt[3]{1 - (y - 3)} + 2 \Rightarrow x = \sqrt[3]{2 - y} + 2}$.

Thus, the new equation can be put this way, too: $\boldsymbol{x = \sqrt[3]{2 - y} + 2}$ for $\boldsymbol{x \le -1}$, and $\boldsymbol{y \ge 4}$.

And expressing $\boldsymbol{y}$ in terms of $\boldsymbol{x}$, we get
$\boldsymbol{x = \sqrt[3]{2 - y} + 2 \Rightarrow 2 - y = (x - 2)^3 \Rightarrow y = (2 - x)^3 + 2}$.

So the new equation can be put this way, too: $\boldsymbol{y = (2 - x)^3 + 2}$ for $\boldsymbol{x \le -1}$.

And assuming $\boldsymbol{g}$ is the new function, we get $\boldsymbol{y = g(x) = (2 - x)^3 + 2}$ for $\boldsymbol{x \le -1}$.

Fig. 0