Examples 1 in Transformation by turning

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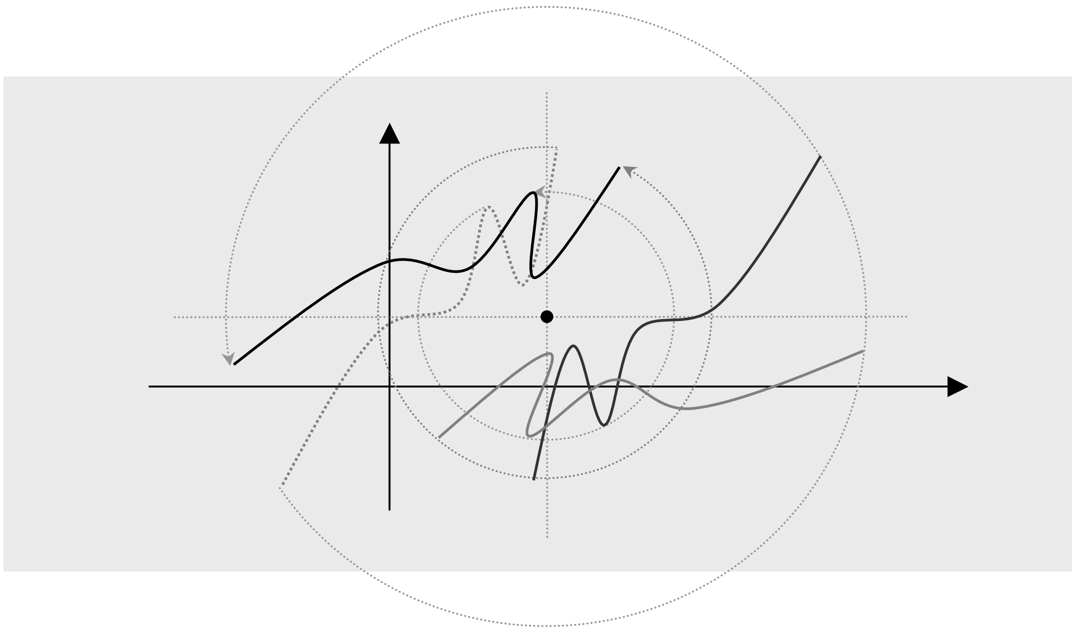


0. Find the curve we get turning the curve of the function below around a point **(1, 2)** in the amount of 90^o. $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$.

1. Find the curve we get turning the curve of the function below around the origin in the amount of 90^o. $\boldsymbol{y = f(x) = \sqrt[3]{1 - x} + 2}$.

2. Find the curve we get turning the curve of the function below around a point **(2, 1)** in the amount of 90^o. $\boldsymbol{y = f(x) = x^2 - 2x + 1}$ for $\boldsymbol{x \geq 1}$.

Suggestions or Solutions
To the Problem in the Example 0

Find the curve we get turning the curve of the function below around a point (1, 2) in the amount of 90°. $y = f(x) = (1 - x)^3 + 2$.

We have $\theta = 90^\circ$, and $(a, b) = (1, 2)$. So we get $\cos 90^\circ = 0$, $\sin 90^\circ = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = y - 1$, and $(x - a)\sin\theta + (y - b)\cos\theta + b = x + 1$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = y - 1$, and $t = x + 1$.
Then, we get $x = t - 1$, and $y = s + 1$.
So we get $y = f(x) \Rightarrow s + 1 = f(t - 1) \Rightarrow x + 1 = f(y - 1) = \{1 - (y - 1)\}^3 + 2$
$= (2 - y)^3 + 2 \Rightarrow x + 1 = (2 - y)^3 + 2 \Rightarrow x = (2 - y)^3 + 1$, which is the new curve.

If not sure of the idea behind the solution above, follow the steps below.

Assuming first, R is a transformation where the axis of turning is a point (a, b), and the amount of turning is an angle θ, and (s, t) is the arbitrary point in the new curve, we can put R the way below.

$$R: (x, y) \longrightarrow (s, t) = \{(x - a)\cos\theta - (y - b)\sin\theta + a, (x - a)\sin\theta + (y - b)\cos\theta + b\}.$$

And in this example, we have $\theta = 90^\circ$, and $(a, b) = (1, 2)$.

So we get $\cos 90^\circ = 0$, and $\sin 90^\circ = 1$. Thus, we get

$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 1)\cdot 0 - (y - 2)\cdot 1 + 1 = y - 2 + 1 = y - 1$, and

$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 1)\cdot 1 + (y - 2)\cdot 0 + 2 = x - 1 + 2 = x + 1$.

So in this example, assuming the transformation is T, we can define it the way below.

$$T: (x, y) \longrightarrow (s, t) = (y - 1, x + 1).$$

And $(y - 1, x + 1)$ is the new arbitrary point, so we set $(s, t) = (y - 1, x + 1)$, and it shows how every point in the original curve gets changed, that is, how every point in the new curve gets made. How then, can we get the new equation?

Getting the equation connecting s and t, we get the new equation. We can get it mixing (s, t) into $y = f(x)$. And we can get them mixed using the setting above.

And using it, we get first, a system of two equations from it, and solve the system for x and y, and then, put the solution into the original equation $y = f(x)$. Then, we get the mixture, which is the connective equation between s and t.

So using it now, we want to solve first for x and y the system of two equations below.
$s = y - 1$, and $t = x + 1$.

And solving the system, we get $x = t - 1$, and $y = s + 1$.

We can now thus, mix the new arbitrary point into the original curve, that is, we can mix (s, t) into $y = f(x)$. And getting the mixture, we put $x = t - 1$ and $y = s + 1$ into $y = f(x)$.

So doing the substitutions, we get $y = f(x) \Rightarrow s + 1 = f(t - 1)$, which is the equation connecting s and t. And we know s is x in the new equation, and t is y in the new.

So replacing s with x, and t with y, we get $x + 1 = f(y - 1)$, which is the new equation.

And we have $y = f(x) = (1 - x)^3 + 2$. So we get
$x + 1 = f(y - 1) = \{1 - (y - 1)\}^3 + 2 = (2 - y)^3 + 2 \Rightarrow x + 1 = (2 - y)^3 + 2$
$\Rightarrow x = (2 - y)^3 + 1$, which is the new equation, that is, the new curve.

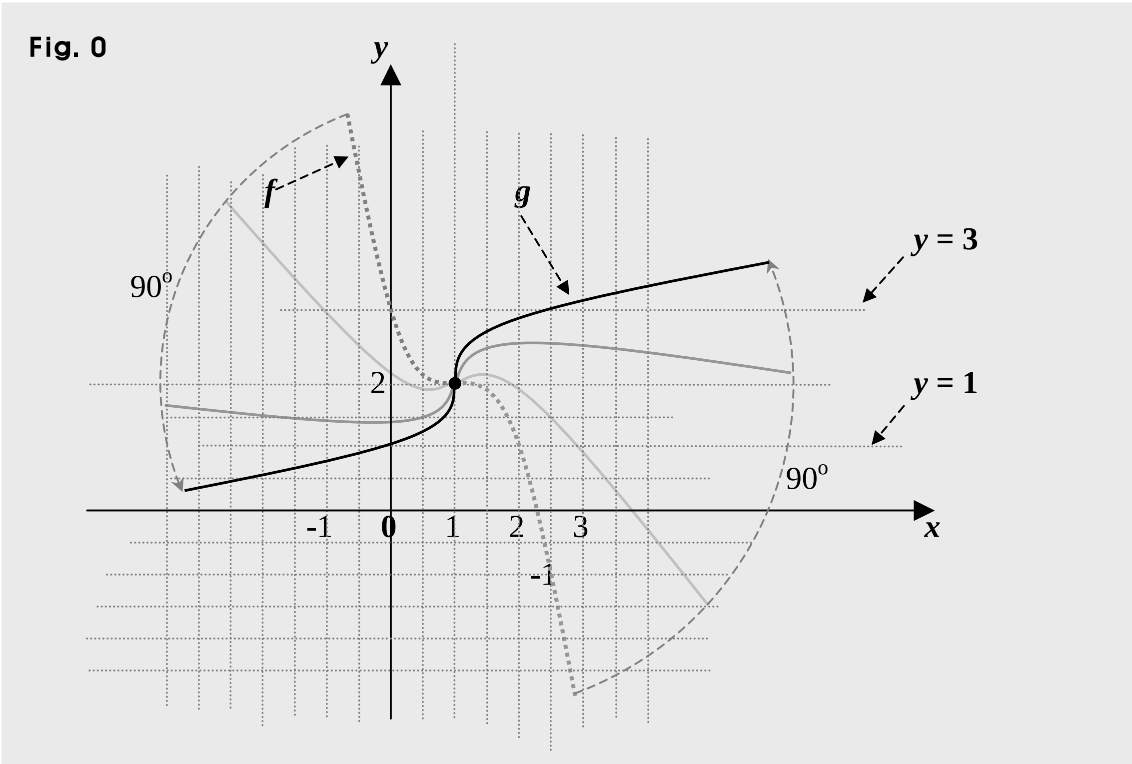
And assuming g is the new equation, we can put it this way:
$g(x, y) = x - (y - 2)^3 - 1 = 0$.

And solving it for y, we can get the new function, the curve of which is the new curve above. So solving it, we get
$x = (y - 2)^3 + 1 \Rightarrow x - 1 = (y - 2)^3 \Rightarrow y - 2 = (x - 1)^{1/3} = \sqrt[3]{x-1} \Rightarrow y = \sqrt[3]{x-1} + 2$.

So assuming g is the new function, we can put it the way below.
$y = g(x) = \sqrt[3]{x-1} + 2$.

And thus, we can express the new curve this way, too: $y = \sqrt[3]{x-1} + 2$.

And putting the two curves of f and g in a graph, we get



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (y - 1, x + 1)}$, and $\boldsymbol{y = f(x) = (1 - x)^3 + 2}$.

Suggestions or Solutions
To the Problem in the Example 1

Find the curve we get turning the curve of the function below around the origin in the amount of 90°. $y = f(x) = \sqrt[3]{1-x} + 2$.

We have $\theta = 90^o$, and $(a, b) = (0, 0)$. So we get $\cos 90^o = 0$, $\sin 90^o = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = -y$, and $(x - a)\sin\theta + (y - b)\cos\theta + b = x$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = -y$, and $t = x$.
Then, we get $x = t$, and $y = -s$.
So we get $y = f(x) \Rightarrow -s = f(t) \Rightarrow -x = f(y) = \sqrt[3]{1-y} + 2 \Rightarrow \sqrt[3]{y-1} = x + 2$
$\Rightarrow y - 1 = (x + 2)^3 \Rightarrow y = (x + 2)^3 + 1$, which is the new curve.

If not sure of the idea behind the solution above, follow the steps below.

Assuming first, R is a transformation where the axis of turning is a point (a, b), and the amount of turning is an angle θ, and (s, t) is the arbitrary point in the new curve, we can put R the way below.

$R: (x, y) \longrightarrow (s, t) = \{(x - a)\cos\theta - (y - b)\sin\theta + a, (x - a)\sin\theta + (y - b)\cos\theta + b\}$.

And in this example, we have $\theta = 90^o$, and $(a, b) = (0, 0)$.

So we get $\cos 90^o = 0$, and $\sin 90^o = 1$. Thus, we get

$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 0)\cdot 0 - (y - 0)\cdot 1 + 0 = -y$, and

$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 0)\cdot 1 + (y - 0)\cdot 0 + 0 = x$.

So in this example, assuming the transformation is T, we can define it the way below.
$T: (x, y) \longrightarrow (s, t) = (-y, x)$.

And $(-y, x)$ is the new arbitrary point, so we set $(s, t) = (-y, x)$, and it shows the mechanism where every point in the original curve gets changed, that is, the mechanism where every point in the new curve gets made. How then, can we get the new equation?

Getting the equation connecting s and t, we get the new equation. We can get it mixing (s, t) into $y = f(x)$. And we can get them mixed using the setting above.

And using it, we get first, a system of two equations from it, and solve the system for x and y, and then, put the solution into the original equation $y = f(x)$. Then, we get the mixture, which is the connective equation between s and t.

So using it now, we want to solve first for x and y the system of two equations below.
$s = -y$, and $t = x$.

And solving the system, we get $x = t$, and $y = -s$.

We can now thus, mix the new arbitrary point into the original curve, that is, we can mix (s, t) into $y = f(x)$. And getting the mixture, we put $x = t$ and $y = -s$ into $y = f(x)$.

So doing the substitutions, we get $y = f(x) \Rightarrow -s = f(t)$, which is the equation connecting s and t. And we know s is x in the new equation, and t is y in the new.

So replacing s with x, and t with y, we get $-x = f(y)$, which is the new equation.

And we have $y = f(x) = \sqrt[3]{1-x} + 2$. So we get

$$y = f(x) \Rightarrow -s = f(t) \Rightarrow -x = f(y) = \sqrt[3]{1-y} + 2 \Rightarrow \sqrt[3]{y-1} = x + 2$$

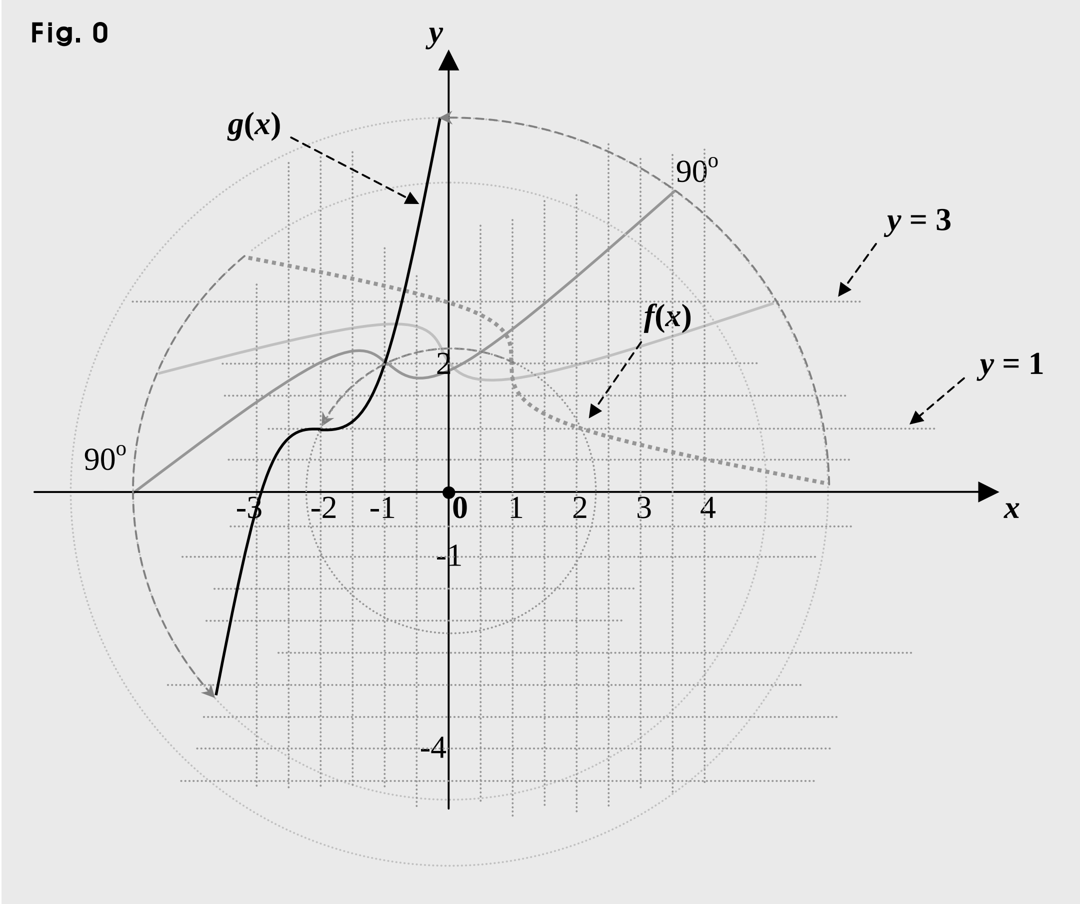
$\Rightarrow y - 1 = (x + 2)^3 \Rightarrow y = (x + 2)^3 + 1$, which is the new equation, that is, the new curve.

And assuming g is the new equation, we can put it this way.
$g(x, y) = -y + (x + 2)^3 + 1 = 0$.

And also, assuming g is the new function, we can put it the way below.
$y = g(x) = (x + 2)^3 + 1$.

And putting the two curves of f and g in a graph, we get

Fig. 0



$g = T \bullet f$, where $T: (x, y) \longrightarrow (s, t) = (-y, x)$, and $y = f(x) = \sqrt[3]{1 - x} + 2$.

Suggestions or Solutions
To the Problem in the Example 2

Find the curve we get turning the curve of the function below around a point (2, 1) in the amount of 90^o. $y = f(x) = x^2 - 2x + 1$ **for** $x \geq 1$**.**

We have $\theta = 90^o$, and $(a, b) = (2, 1)$. So we get $\cos 90^o = 0$, $\sin 90^o = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = 3 - y$, and $(x - a)\sin\theta + (y - b)\cos\theta + b = x - 1$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = 3 - y$, and $t = x - 1$.

Next, we get $x \geq 1 \Rightarrow y = x^2 - 2x + 1 = (x - 1)^2 \geq 0 \Rightarrow y \geq 0 \Rightarrow -y \leq 0 \Rightarrow 3 - y \leq 3$
$\Rightarrow s \leq 3$. And also, we get $x \geq 1 \Rightarrow x - 1 \geq 0 \Rightarrow t \geq 0$.
And next, we can get $x = t + 1$, and $y = 3 - s$. So we get
$y = f(x) \Rightarrow 3 - s = f(t + 1) \Rightarrow 3 - x = f(y + 1) = (y + 1)^2 - 2(y + 1) + 1 = y^2 \Rightarrow 3 - x = y^2$.

So the new curve is $3 - x = y^2$ for $x \leq 3$ and $y \geq 0$.

If not sure of the idea behind the solution above, follow the steps below.

Assuming first, R is a transformation where the axis of turning is a point (a, b), and the amount of turning is an angle θ, and (s, t) is the arbitrary point in the new curve, we can put R the way below.

$R: (x, y) \longrightarrow (s, t) = \{(x - a)\cos\theta - (y - b)\sin\theta + a, (x - a)\sin\theta + (y - b)\cos\theta + b\}$.

And in this example, we have $\theta = 90^o$, and $(a, b) = (2, 1)$.

So we get $\cos 90^o = 0$, and $\sin 90^o = 1$. Thus, we get

$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 2)\cdot 0 - (y - 1)\cdot 1 + 2 = -y + 1 + 2 = 3 - y$, and

$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 2)\cdot 1 + (y - 1)\cdot 0 + 1 = x - 2 + 1 = x - 1$.

So in this example, assuming the transformation is T, we can define it the way below.

$T: (x, y) \longrightarrow (s, t) = (3 - y, x - 1)$.

And $(3 - y, x - 1)$ is the new arbitrary point, so we set $(s, t) = (3 - y, x - 1)$. And it shows the mechanism where every point in the original curve gets changed, that is, the mechanism where every point in the new curve gets made. And getting the new equation, we want to get the equation connecting s and t. We can get it mixing (s, t) into $y = f(x)$, and can get them mixed using the setting above.

And using it, we get first, a system of two equations from it, and solve the system for x and y, and then, put the solution into the original equation $y = f(x)$. Then, we get the mixture, which is the connective equation between s and t.

So using it now, we want to solve first for x and y the system of two equations below.
$s = 3 - y$, and $t = x - 1$.

And solving the system, we get $x = t + 1$, and $y = 3 - s$.

We can now therefore, mix (s, t) into $y = f(x)$. And getting the mixture, we put $x = t + 1$ and $y = 3 - s$ into $y = f(x)$. Before doing the substitutions though, we want to get the new domain and range, because the original equation has a finite domain.

Finding the new domain, we want to begin with the original domain, which is $x \geq 1$.
Then, we get $x \geq 1 \Rightarrow y = x^2 - 2x + 1 = (x - 1)^2 \geq 0 \Rightarrow y \geq 0 \Rightarrow -y \leq 0 \Rightarrow 3 - y \leq 3$
$\Rightarrow s \leq 3$, which is the new domain, since s is x in the new equation.

And next, finding the new range, we can get it using $t = x - 1$, because t is y in the new equation. So finding the new range now, we get $x \geq 1 \Rightarrow x - 1 \geq 0 \Rightarrow t \geq 0$, which is the new range.

So doing now the substitutions, we get $y = f(x) \Rightarrow 3 - s = f(t + 1)$ for $s \leq 3$ and $t \geq 0$, which is the equation connecting s and t, and thus, is the new equation. And we know s is x in the new equation, and t is y in the new.

So replacing s with x, and t with y, we get $3 - x = f(y + 1)$ for $x \leq 3$ and $y \geq 0$, which is the new equation. And we have $y = f(x) = x^2 - 2x + 1 = (x - 1)^2$.

So we get $3 - x = f(y + 1) = \{(y + 1) - 1\}^2 = y^2 \Rightarrow x = 3 - y^2$ for $x \leq 3$ and $y \geq 0$, which is the new equation, that is, the new curve.

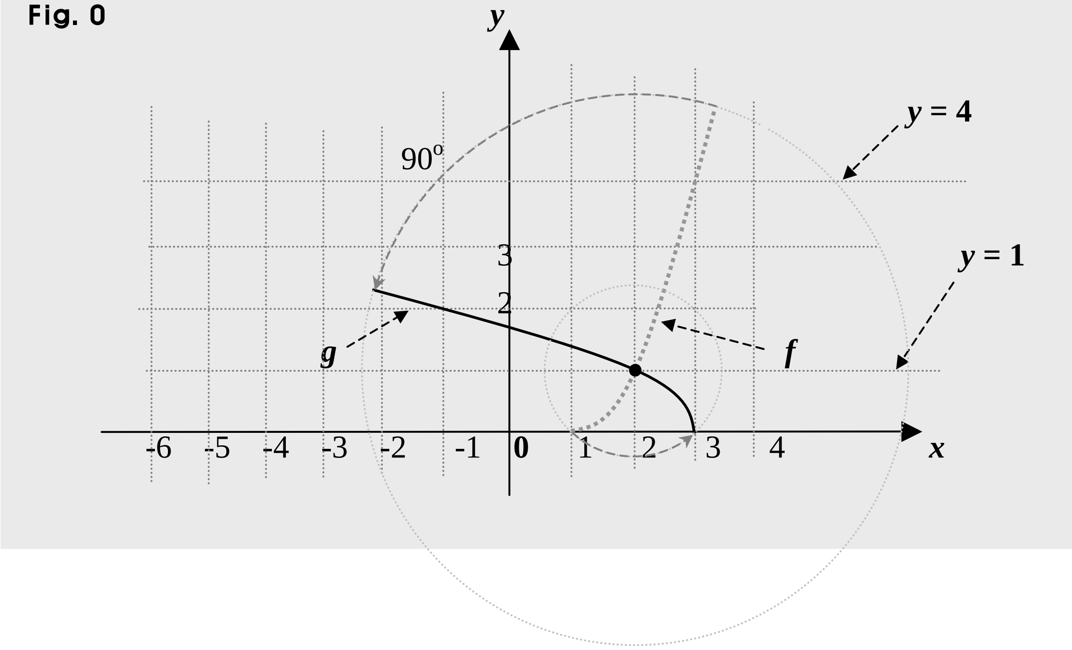
And assuming $\boldsymbol{g}$ is the new equation, we can put it the way below.
$\boldsymbol{g(x, y) = x + y^2 - 3 = 0}$ for $\boldsymbol{x \le 3}$ and $\boldsymbol{y \ge 0}$.

And solving it for $\boldsymbol{y}$, we can get the new function, the curve of which is, of course, the new curve above. So solving it, we get $\boldsymbol{x + y^2 - 3 = 0 \Rightarrow 3 - x = y^2 \Rightarrow y = \pm\sqrt{3 - x}}$.
And we know $\boldsymbol{y \ge 0}$. So we get $\boldsymbol{y = g(x) = \sqrt{3 - x}}$ for $\boldsymbol{x \le 3}$.

And thus, we can express the new curve this way, too: $\boldsymbol{y = \sqrt{3 - x}}$ for $\boldsymbol{x \le 3}$.

And putting the two curves of $\boldsymbol{f}$ and $\boldsymbol{g}$ in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (3 - y, x - 1)}$, and $\boldsymbol{y = f(x) = x^2 - 2x + 1}$ for $\boldsymbol{x \ge 1}$.