Examples 2 in Transformation by turning

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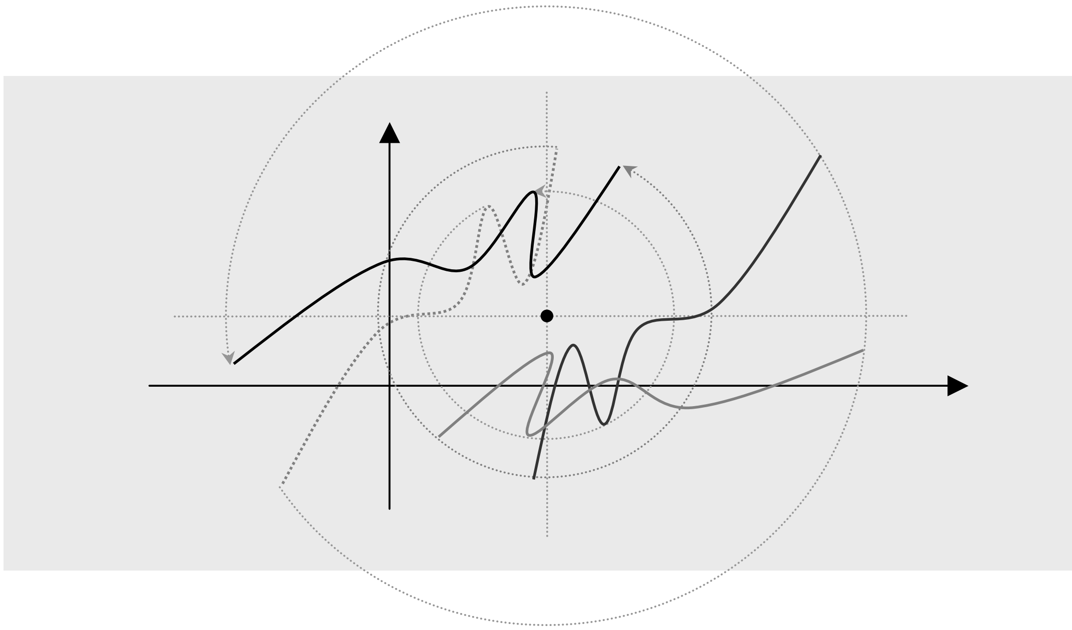


0. Find the curve we get turning the curve of the function below around a point **(0, 3)** in the amount of 90°. $\boldsymbol{y = f(x) = (1-x)^3 + 2}$ for $\boldsymbol{x \le 1}$.

1. Find the curve we get turning the curve of the function below around a point **(1, 1)** in the amount of 90°. $\boldsymbol{y = f(x) = \sqrt[3]{1-x} + 2}$ for $\boldsymbol{x \ge 1}$.

2. Find the curve we get turning the curve of the function below around the origin in the amount of 180°. $\boldsymbol{y = f(x) = 2 - \sqrt{x+1}}$ for $\boldsymbol{-1 \le x \le 3}$.

3. Find the curve we get turning the curve of the function below around a point **(3, 2)** in the amount of 90°. $\boldsymbol{y = f(x) = (x-2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

4. Find the curve we get turning the curve of the equation below around the origin in the amount of 90°. $\boldsymbol{f(x, y) = (x-2)^2 + (y-1)^2 - 2 = 0}$.

Suggestions or Solutions
To the Problem in the Example 0

Find the curve we get turning the curve of the function below around a point (0, 3) in the amount of 90°. $y = f(x) = (1 - x)^3 + 2$ **for** $x \le 1$**.**

We have $\theta = 90^o$, and $(a, b) = (0, 3)$. So we get $\cos 90^o = 0$, $\sin 90^o = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 0)\cdot 0 - (y - 3)\cdot 1 + 0 = 3 - y$, and
$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 0)\cdot 1 + (y - 3)\cdot 0 + 3 = x + 3$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = 3 - y$, and $t = x + 3$.

Next, we get $x \le 1 \Rightarrow -x \ge -1 \Rightarrow 1 - x \ge 0 \Rightarrow (1 - x)^3 \ge 0 \Rightarrow (1 - x)^3 + 2 \ge 2$
$\Rightarrow y \ge 2 \Rightarrow -y \le -2 \Rightarrow 3 - y \le 1 \Rightarrow s \le 1$, because $s = 3 - y$.

And also, we get $x \le 1 \Rightarrow x + 3 \le 4 \Rightarrow t \le 4$, because $t = x + 3$.

And next, we can get $x = t - 3$, and $y = 3 - s$. So we get
$y = f(x) \Rightarrow 3 - s = f(t - 3) \Rightarrow 3 - x = f(y - 3) = \{1 - (y - 3)\}^2 + 2 = (4 - y)^3 + 2$
$\Rightarrow 3 - x = (4 - y)^3 + 2 \Rightarrow (4 - y)^3 + x - 1 = 0$.

So the new curve is $(4 - y)^3 + x - 1 = 0$ for $x \le 1$ and $y \le 4$.

And assuming g is the new equation, that is, the new curve, we can put it this way:
$g(x, y) = (4 - y)^3 + x - 1 = 0$ for $x \le 1$ and $y \le 4$.

And solving it for y, we can get the new function, the curve of which is the new curve above. So solving it, we get
$(4 - y)^3 + x - 1 = 0 \Rightarrow x - 1 = (y - 4)^3 \Rightarrow y - 4 = (x - 1)^{1/3} = \sqrt[3]{x-1} \Rightarrow y = \sqrt[3]{x-1} + 4$.

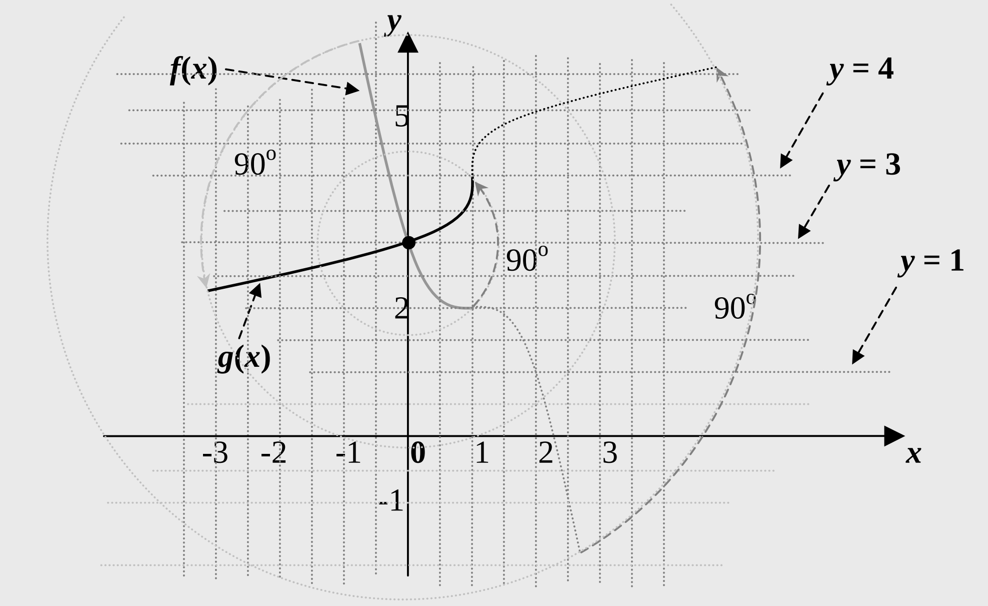
So assuming g is the new function, we can put it the way below.

$y = g(x) = \sqrt[3]{x-1} + 4$ for $x \le 1$.

And thus, we can express the new curve this way, too: $y = \sqrt[3]{x-1} + 4$ for $x \le 1$.

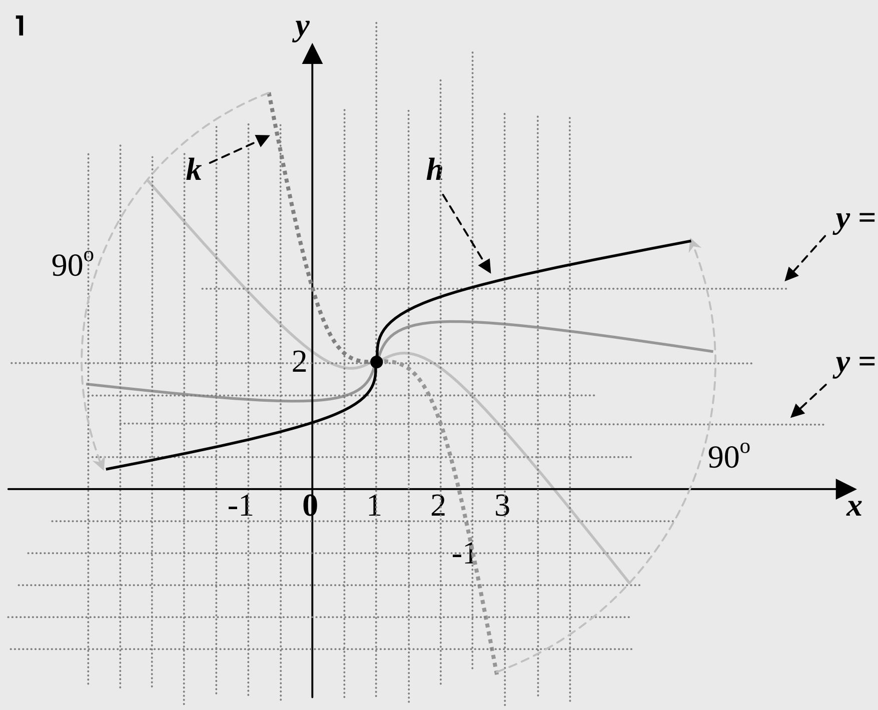
And putting the two curves of f and g in a graph, we get

Fig. 0



$g = T \bullet f$, where T: $(x, y) \longrightarrow (s, t) = (3 - y, x + 3)$, and $y = f(x) = (1 - x)^3 + 2$ for $x \le 1$.

Fig. 1



$h = T \bullet k$, where T: $(x, y) \longrightarrow (s, t) = (y - 1, x + 1)$, and $y = k(x) = (1 - x)^3 + 2$.

Suggestions or Solutions
To the Problem in the Example 1

Find the curve we get turning the curve of the function below around a point (1, 1) in the amount of 90^o. $y = f(x) = \sqrt[3]{1-x} + 2$ **for** $x \geq 1$.

We have $\theta = 90^o$, and $(a, b) = (1, 1)$. So we get $\cos 90^o = 0$, $\sin 90^o = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 1)\cdot 0 - (y - 1)\cdot 1 + 1 = 2 - y$, and
$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 1)\cdot 1 + (y - 1)\cdot 0 + 1 = x$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = 2 - y$, and $t = x$.

Next, we get $x \geq 1 \Rightarrow -x \leq -1 \Rightarrow 1 - x \leq 0 \Rightarrow \sqrt[3]{1-x} \leq 0 \Rightarrow \sqrt[3]{1-x} + 2 \leq 2$
$\Rightarrow y \leq 2 \Rightarrow -y \geq -2 \Rightarrow 2 - y \geq 0 \Rightarrow s \geq 0$, because $s = 2 - y$.

And also, we get $x \geq 1 \Rightarrow t \geq 1$, because $t = x$.

And next, we can get $x = t$, and $y = 2 - s$. So we get
$y = f(x) \Rightarrow 2 - s = f(t) \Rightarrow 2 - x = f(y) = \sqrt[3]{1-y} + 2 \Rightarrow 2 - x = \sqrt[3]{1-y} + 2 \Rightarrow x = \sqrt[3]{y-1}$.

So the new curve is $x = \sqrt[3]{y-1}$ for $x \geq 0$ and $y \geq 1$.

And assuming g is the new equation, that is, the new curve, we can put it this way:
$g(x, y) = \sqrt[3]{y-1} - x = 0$ for $x \geq 0$ and $y \geq 1$.

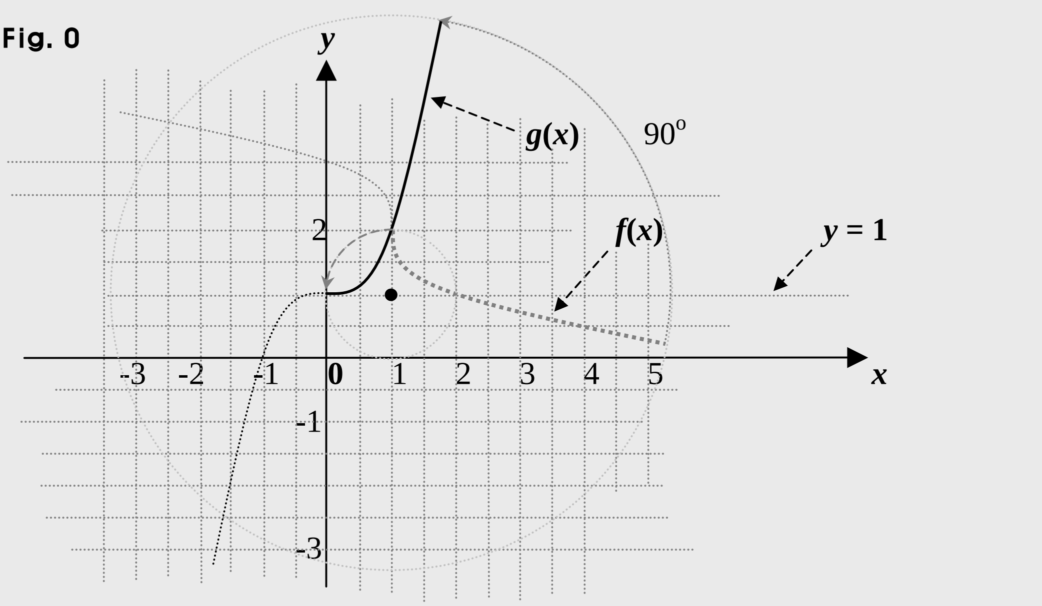
And solving it for y, we can get the new function, the curve of which is the new curve above. So solving it, we get $x = \sqrt[3]{y-1} \Rightarrow x^3 = y - 1 \Rightarrow y = x^3 + 1$.

So assuming g is the new function, we can put it the way below.
$y = g(x) = x^3 + 1$ for $x \geq 0$.

And thus, we can express the new curve this way, too: $y = x^3 + 1$ for $x \geq 0$.

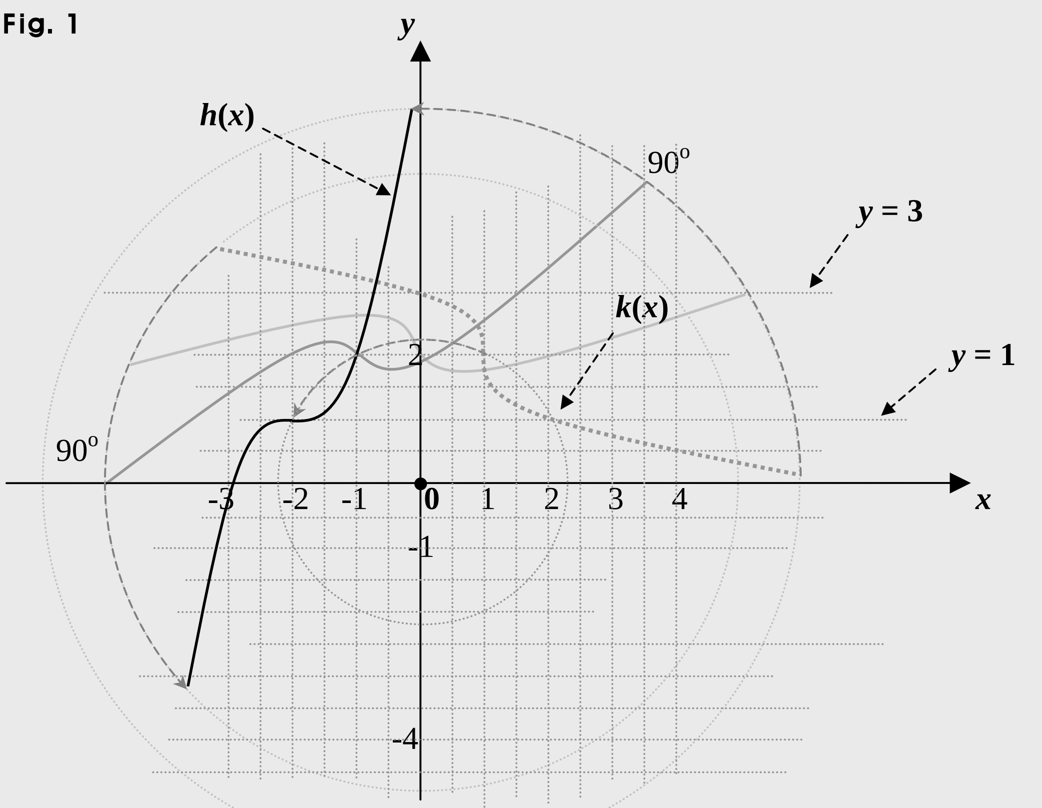
And putting the two curves of f and g in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (2 - y, x)}$, and $\boldsymbol{y = f(x) = \sqrt[3]{1-x} + 2}$ for $\boldsymbol{x \geq 1}$.

Fig. 1



$\boldsymbol{h = T \bullet k}$, where $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (-y, x)}$, and $\boldsymbol{y = k(x) = \sqrt[3]{1-x} + 2}$.

Suggestions or Solutions
To the Problem in the Example 2

Find the curve we get turning the curve of the function below around the origin in the amount of 180^o. $y = f(x) = 2 - \sqrt{x+1}$ **for** $-1 \le x \le 3$.

We have $\theta = 180^o$, and $(a, b) = (0, 0)$. So we get $\cos 180^o = -1$, $\sin 180^o = 0$, and thus,

$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 0)\cdot(-1) - (y - 0)\cdot 0 + 0 = -x$, and
$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 0)\cdot 0 + (y - 0)\cdot(-1) + 0 = -y$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = -x$, and $t = -y$.

Next, we get $-1 \le x \le 3 \Rightarrow -3 \le -x \le 1 \Rightarrow -3 \le s \le 1$, because $s = -x$.

And also, we get $-1 \le x \le 3 \Rightarrow 0 \le x + 1 \le 4 \Rightarrow 0 \le \sqrt{x+1} \le 2 \Rightarrow -2 \le -\sqrt{x+1} \le 0$
$\Rightarrow 0 \le 2 - \sqrt{x+1} \le 2 \Rightarrow 0 \le y \le 2 \Rightarrow -2 \le -y \le 0$. So we get $-2 \le t \le 0$, because $t = -y$.

And next, we can get $x = -s$, and $y = -t$. So we get
$y = f(x) \Rightarrow -t = f(-s) \Rightarrow t = -f(-s) = -(2 - \sqrt{1-s}\,) \Rightarrow t = \sqrt{1-s} - 2$.

So the new curve is $y = \sqrt{1-x} - 2$ for $-3 \le x \le 1$ and $-2 \le y \le 0$.

And assuming g is the new equation, that is, the new curve, we can put it this way:

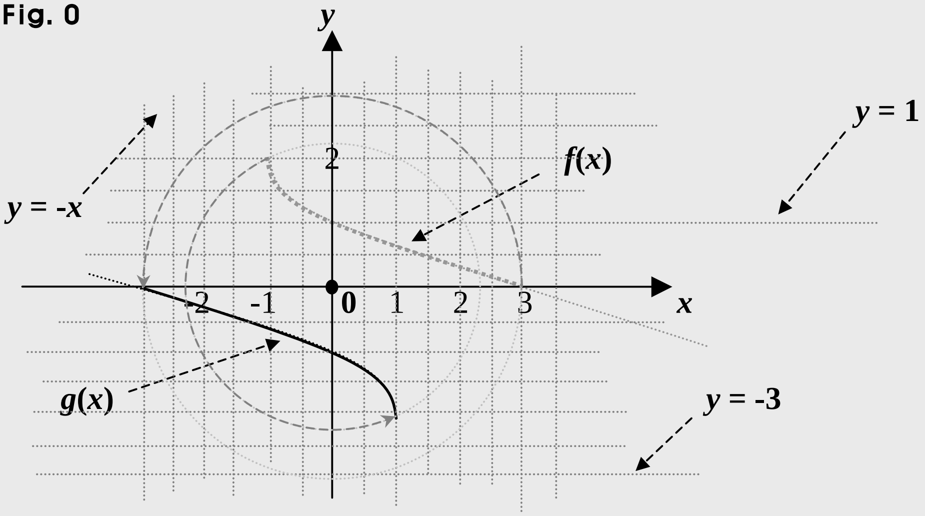
$g(x, y) = \sqrt{1-x} - 2 - y = 0$ for $-3 \le x \le 1$ and $-2 \le y \le 0$.

And also, assuming g is the new function, we can put it the way below.

$y = g(x) = \sqrt{1-x} - 2$ for $-3 \le x \le 1$.

And putting the two curves of f and g in a graph, we get

Fig. 0



g* = *T*•*f, where ***T*: (*x*, *y*)** $\longrightarrow$ **(*s*, *t*) = (-*x*, -*y*)**, and ***y* = *f*(*x*)** $= \mathbf{2 - \sqrt{x+1}}$ for $\mathbf{-1 \le x \le 3}$.

And notice that we can get ***g*** transforming ***f*** symmetrically about the origin, too.

Suggestions or Solutions
To the Problem in the Example 3

Find the curve we get turning the curve of the function below around a point (3, 2) in the amount of 90^o. $y = f(x) = (x - 2)^2 + 1$ **for** $1 \le x \le 4$.

We have $\theta = 90^o$, and $(a, b) = (3, 2)$. So we get $\cos 90^o = 0$, $\sin 90^o = 1$, and thus,
$(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 3)\cdot 0 - (y - 2)\cdot 1 + 3 = 5 - y$, and
$(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 3)\cdot 1 + (y - 2)\cdot 0 + 2 = x - 1$.

So assuming (s, t) is the arbitrary point in the new curve, we get $s = 5 - y$, and $t = x - 1$.

Next, we get $1 \le x \le 4 \Rightarrow -1 \le x - 2 \le 2 \Rightarrow 0 \le (x - 2)^2 \le 4$, because $x - 2 = 0$ for $x = 2$, since we have $1 \le x \le 4$. So we get $1 \le (x - 2)^2 + 1 \le 5 \Rightarrow 1 \le y \le 5 \Rightarrow -5 \le -y \le -1$
$\Rightarrow 0 \le 5 - y \le 4 \Rightarrow 0 \le s \le 4$, because $s = 5 - y$.

And also, we get $1 \le x \le 4 \Rightarrow 0 \le x - 1 \le 3 \Rightarrow 0 \le t \le 3$, because $t = x - 1$.

And next, we can get $x = t + 1$, and $y = 5 - s$. So we get
$y = f(x) \Rightarrow 5 - s = f(t + 1) \Rightarrow 5 - x = f(y + 1) = \{(y + 1) - 2\}^2 + 1 = (y - 1)^2 + 1$
$\Rightarrow 5 - x = (y - 1)^2 + 1 \Rightarrow x = 4 - (y - 1)^2$.

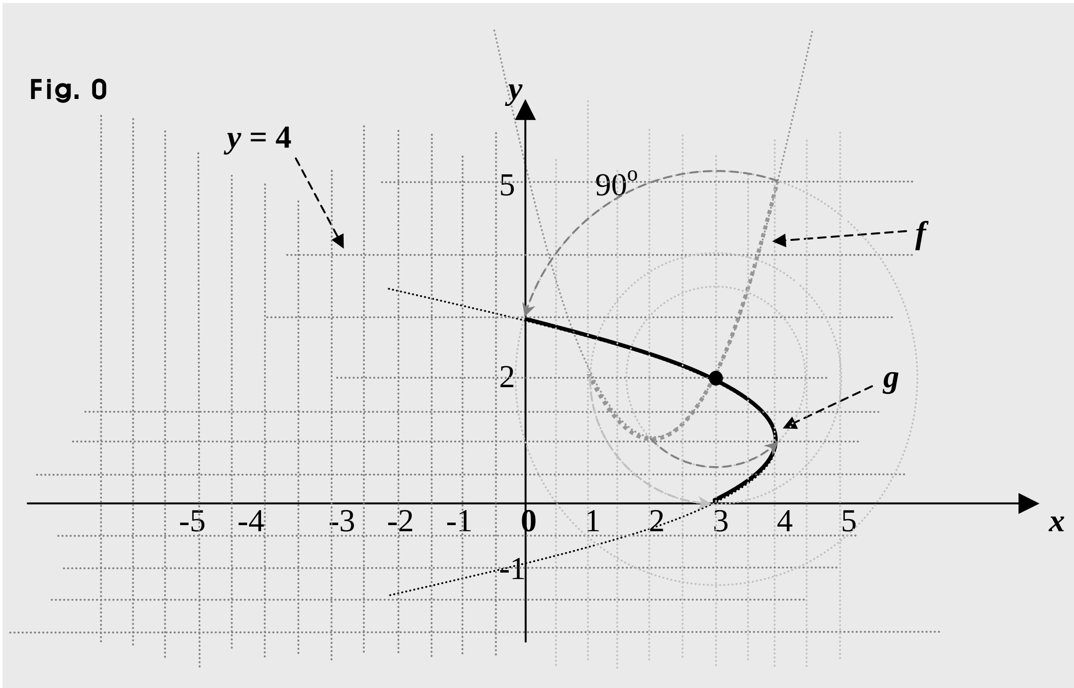
So the new curve is $x = 4 - (y - 1)^2$ for $0 \le x \le 4$ and $0 \le y \le 3$.

And assuming g is the new equation, that is, the new curve, we can put it this way:
$g(x, y) = x - 4 + (y - 1)^2 = 0$ for $0 \le x \le 4$ and $0 \le y \le 3$.

And solving it for y, we can get the new function, the curve of which is the new curve above. So solving it, we get $x - 4 + (y - 1)^2 = 0 \Rightarrow (y - 1)^2 = 4 - x \Rightarrow y - 1 = \pm\sqrt{4 - x}$
$\Rightarrow y = 1 \pm \sqrt{4 - x}$. And we know $0 \le x \le 4$ and $0 \le y \le 3$. So we get $y = 1 + \sqrt{4 - x}$.

So assuming g is the new function, we can put it the way below.
$y = g(x) = 1 + \sqrt{4 - x}$ for $0 \le x \le 4$.
And thus, we can express the new curve this way, too: $y = 1 + \sqrt{4 - x}$ for $0 \le x \le 4$.

And putting the two curves of ***f*** and ***g*** in a graph, we get



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T}$**:** $\boldsymbol{(x, y) \longrightarrow (s, t) = (5 - y, x - 1)}$, and $\boldsymbol{y = f(x) = (x - 2)^2 + 1}$ for $\boldsymbol{1 \le x \le 4}$.

Suggestions or Solutions
To the Problem in the Example 4

Find the curve we get turning the curve of the equation below around the origin in the amount of 90^o. $\boldsymbol{f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0}$**.**

We have $\boldsymbol{\theta = 90^o}$, and $\boldsymbol{(a, b) = (0, 0)}$. So we get $\boldsymbol{\cos 90^o = 0}$, $\boldsymbol{\sin 90^o = 1}$, and thus,

$\boldsymbol{(x - a)\cos\theta - (y - b)\sin\theta + a = (x - 0)\cdot 0 - (y - 0)\cdot 1 + 0 = -y}$, and

$\boldsymbol{(x - a)\sin\theta + (y - b)\cos\theta + b = (x - 0)\cdot 1 + (y - 0)\cdot 0 + 0 = x}$.

So assuming $\boldsymbol{(s, t)}$ is the new arbitrary point, we get $\boldsymbol{s = -y \Rightarrow y = -s}$, and $\boldsymbol{t = x}$.

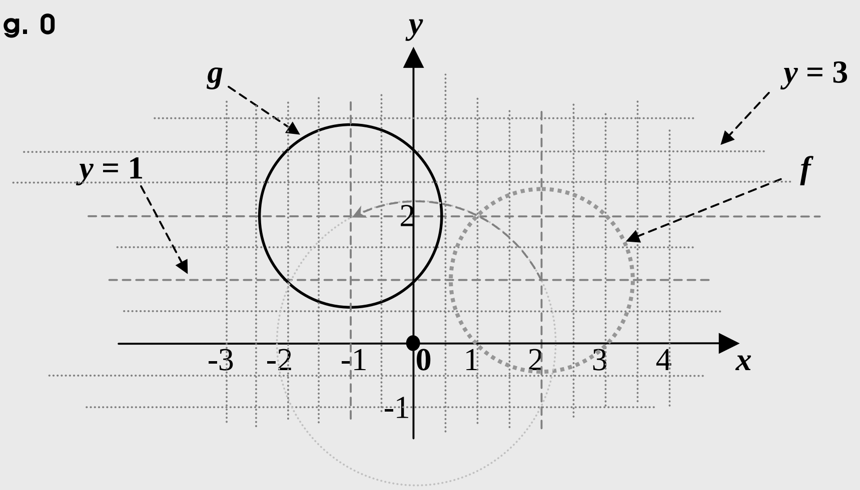
Thus, we get $\boldsymbol{x = t}$, and $\boldsymbol{y = 2 - s}$. So we get

$\boldsymbol{f(x, y) = 0 \Rightarrow f(t, 2 - s) = 0 \Rightarrow (t - 2)^2 + (2 - s - 1)^2 - 2 = 0 \Rightarrow (t - 2)^2 + (s - 1)^2 - 2 = 0}$.

So the new curve is $\boldsymbol{(y - 2)^2 + (x - 1)^2 - 2 = 0}$, which is a circle of radius $\boldsymbol{\sqrt{2}}$ centered at (1, 2). And the original curve is a circle of radius $\boldsymbol{\sqrt{2}}$ centered at (2, 1).

And putting the two curves of $\boldsymbol{f}$ and $\boldsymbol{g}$ in a graph, we get

Fig. 0



$\boldsymbol{g = T \bullet f}$, where $\boldsymbol{T: (x, y) \longrightarrow (s, t) = (-y, x)}$, and $\boldsymbol{f(x, y) = (x - 2)^2 + (y - 1)^2 - 2 = 0}$.