Examples in Composite Transformations

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0. Assuming U is a transformation where a point at $(0, 0)$ moves to $(3, 8)$, and V is a transformation where the point at $(3, 8)$ moves to $(8, 3)$, find $U \bullet V$.

1. Assuming $U: (x, y) \longrightarrow (y/2, 3x)$, $V: (x, y) \longrightarrow (x + 1, 4y - 2)$, and although we apply $U \bullet V$ to a point P, the point P still remains at the same position, find the point P.

2. Find $S \bullet T \bullet f$, and $T \bullet S \bullet f$, where $y = f(x) = 3x + 5$,
$S: (x, y) \longrightarrow (2x, y - 1)$, and $T: (x, y) \longrightarrow (2y, x - 1)$.

3. Assuming $S: (x, y) \longrightarrow (x - 2, 2 - y)$, $T: (x, y) \longrightarrow (x + 1, -y/2)$,
$U: (x, y) \longrightarrow (x + a, 2y + b)$, $y = f(x) = 3x^2 + 5x + 1$, and $U \bullet T \bullet S \bullet f = f$, find a and b.

4. Assuming $S: (x, y) \longrightarrow (ay, x)$ for $a \neq 0$, $T: (x, y) \longrightarrow (x + b, y + c)$,
$f(x, y) = y^2 + 2y - 1 - x = 0$, $y = g(x) = x^2$, and $g = T \bullet S \bullet f$, find a, b, and c.

5. Assuming $S: (x, y) \longrightarrow (x - 1, y + 1)$, $T: (x, y) \longrightarrow (y, x)$, and $y = f(x) = 5^x$, find $S \bullet f$, and $S \bullet T \bullet f$.

Suggestions or Solutions
To the Problem in the Example 0

Assuming U is a transformation where a point at (0, 0) moves to (3, 8), and V is a transformation where the point at (3, 8) moves to (8, 3), find $U \bullet V$.

U: $(x, y) \longrightarrow \{x + (3 - 0), y + (8 - 0)\} = (x + 3, y + 8)$.
V: $(x, y) \longrightarrow \{x + (8 - 3), y + (3 - 8)\} = (x + 5, y - 5)$.
So $U \bullet V$: $(x + 5, y - 5) \longrightarrow \{(x + 5) + 3, (y - 5) + 8\} = (x + 8, y + 3)$.
And thus, $U \bullet V$: $(x, y) \longrightarrow (x + 8, y + 3)$.

If not sure of the ideas behind the processes about, follow the steps below.

To begin with, what transformations are U and V?

They can be called parallel transformations.

So assuming first, T is a parallel transformation, we can put T the way below.
T: $(x, y) \longrightarrow (x + \Delta x, y + \Delta y)$, which can be the definition for parallel transformations.

So in the definition above, Δx is the amount of change in the x-coordinate, and Δy is the amount of change in the y-coordinate.

Thus, for U, we get $\Delta x = 3 - 0 = 3$, and $\Delta y = 8 - 0 = 8$.

And for V, we get $\Delta x = 8 - 3 = 5$, and $\Delta y = 3 - 8 = -5$.

So we can put U and V the way below.
U: $(x, y) \longrightarrow (x + 3, y + 8)$, and V: $(x, y) \longrightarrow (x + 5, y - 5)$.

And applying $U \bullet V$, we apply V first, and then, U.

So applying V first, and assuming (s, t) is the new arbitrary point, we get

$(s, t) = (x + 5, y - 5)$.   That is, we get $s = x + 5$, and $t = y - 5$.

And next, applying $\boldsymbol{U}$ to the curve where $\mathbf{(}\boldsymbol{s, t}\mathbf{)}$ is the arbitrary point, and assuming $\mathbf{(}\boldsymbol{u, v}\mathbf{)}$ is the new arbitrary point, we get $\mathbf{(}\boldsymbol{s, t}\mathbf{)} \longrightarrow \mathbf{(}\boldsymbol{u, v}\mathbf{)} = \mathbf{(}\boldsymbol{s}\mathbf{ + 3, }\boldsymbol{t}\mathbf{ + 8)}$.

And we have $\boldsymbol{s} = \boldsymbol{x} + \mathbf{5}$, and $\boldsymbol{t} = \boldsymbol{y} - \mathbf{5}$. So we get $\mathbf{(}\boldsymbol{u, v}\mathbf{)} = \mathbf{(}\boldsymbol{x}\mathbf{ + 5 + 3, }\boldsymbol{y}\mathbf{ - 5 + 8)}$.

And thus, we can put $\boldsymbol{U \bullet V}$ the way as follows. $\boldsymbol{U \bullet V}$: $(x, y) \longrightarrow (x + 8, y + 3)$.

And putting it more specifically, we get

$$\boldsymbol{U \bullet V}: (x, y) \xrightarrow{\boldsymbol{V}} (x + 5, y - 5) \xrightarrow{\boldsymbol{U}} \{(x + 5) + 3, (y - 5) + 8\} = (x + 8, y + 3).$$

And we can put the ideas above the way as follows, too.

Transforming the transformation $\boldsymbol{V}$ by $\boldsymbol{U}$, we get the composite transformation $\boldsymbol{U \bullet V}$.
So applying $\boldsymbol{U}$ to $\boldsymbol{V}$, we get $\boldsymbol{U \bullet V}$.
And applying $\boldsymbol{U}$ to $\boldsymbol{V}$, we apply $\boldsymbol{U}$ to the new point $\boldsymbol{V}$ makes.
Then, we get the new point made in $\boldsymbol{U \bullet V}$, that is, we get the new point the composite transformation ($\boldsymbol{U \bullet V}$) makes.

The new point made in $\boldsymbol{V}$ is $(x + 5, y - 5)$.
The new point made in $\boldsymbol{U}$ is $(x + 3, y + 8)$.

So assuming (s, t) is the new point made in $\boldsymbol{B \bullet C}$, we get
$(s, t) = \{(x + 5) + 3, (y - 5) + 8\} = (x + 8, y + 3)$.

So we get $\boldsymbol{B \bullet C}$: $(x, y) \longrightarrow (x + 8, y + 3)$.

In short:

$\boldsymbol{U}$: $(x, y) \longrightarrow \{x + (3 - 0), y + (8 - 0)\} = (x + 3, y + 8)$.

$\boldsymbol{V}$: $(x, y) \longrightarrow \{x + (8 - 3), y + (3 - 8)\} = (x + 5, y - 5)$.

So $\boldsymbol{U \bullet V}$: $(x + 5, y - 5) \longrightarrow \{(x + 5) + 3, (y - 5) + 8\} = (x + 8, y + 3)$.

And thus, $\boldsymbol{U \bullet V}$: $(x, y) \longrightarrow (x + 8, y + 3)$.

Suggestions or Solutions
To the Problem in the Example 1

Assuming U: $(x, y) \longrightarrow (y/2, 3x)$, V: $(x, y) \longrightarrow (x + 1, 4y - 2)$, and although we apply $U \bullet V$ to a point P, the point P still remains at the same position, find P.

First, $\{(4y - 2)/2, 3(x + 1)\} = (2y - 1, 3x - 3)$.
Next, assuming P is (s, t), we get $(2t - 1, 3s - 3) = (s, t)$.
So we get $s = 2t - 1 \Rightarrow 2t = s + 1$, and $t = 3s + 3 \Rightarrow 2t = 6s + 6$.
Thus, we get $6s + 6 = s + 1 \Rightarrow 5s = -5 \Rightarrow s = -1 \Rightarrow t = 3s + 3 = 3(-1) + 3 = 0$.
So P is $(-1, 0)$.

If not sure of the ideas behind the processes above, follow the steps below.

Finding first, the new point $U \bullet V$ makes, in the new point U makes, replace x with the x-coordinate in the new point V makes, and replace y with the y-coordinate in the new point V makes.

That is to say that in $(y/2, 3x)$, replace x with $x + 1$, and replace y with $4y - 2$.

Then, we get $\{(4y - 2)/2, 3(x + 1)\} = (2y - 1, 3x - 3)$, which is the new point $U \bullet V$ makes

And thus, assuming P is (s, t), and applying $U \bullet V$ to P assuming (u, v) is the new point, we get $(s, t) \longrightarrow (u, v) = (2t - 1, 3s - 3)$.

We know however, $(2t - 1, 3s - 3) = (s, t)$.

So we get $s = 2t - 1$, and $t = 3s - 3$, which is a system of two equations for s and t. And thus, solving it, we can get first, $s = 2t - 1 \Rightarrow 2t = s + 1$, and $t = 3s + 3 \Rightarrow 2t = 6s + 6$.

So we get $6s + 6 = s + 1 \Rightarrow 5s = -5 \Rightarrow s = -1 \Rightarrow t = 3s + 3 = 3(-1) + 3 = 0$.

And thus, P is $(-1, 0)$.

Suggestions or Solutions

To the Problem in the Example 2

Find $S \bullet T \bullet f$, and $T \bullet S \bullet f$, where $y = f(x) = 3x + 5$,
$S: (x, y) \longrightarrow (2x, y - 1)$, and $T: (x, y) \longrightarrow (2y, x - 1)$.

$S \bullet T: (x, y) \longrightarrow (4y, x - 2)$.
$T \bullet S: (x, y) \longrightarrow (2y - 2, 2x - 1)$.

So first, setting $(s, t) = (4y, x - 2)$, we get $s = 4y \Rightarrow y = s/4$, and $t = x - 2 \Rightarrow x = t + 2$.

Thus, we get $y = f(x) \Rightarrow y = f(x) \Rightarrow s/4 = f(t + 2) = 3(t + 2) + 5$. So we get

$x/4 = f(y + 2) = 3(y + 2) + 5 \Rightarrow x = 12(y + 2) + 20 \Rightarrow 12y - x + 44 = 0 \Rightarrow y = x/12 - 11/3$.

So we get $S \bullet T \bullet f = x/12 - 11/3$.

And next, setting $(s, t) = (2y - 2, 2x - 1)$, we get

$s = 2y - 2 \Rightarrow y = (s + 2)/2$, and $t = 2x - 1 \Rightarrow x = (t + 1)/2$.

Thus, we get $y = f(x) \Rightarrow y = f(x) \Rightarrow (s + 2)/2 = f\{(t + 1)/2\} = 3(t + 1)/2 + 5$. So we get

$(x + 2)/2 = f\{(y + 1)/2\} = 3(y + 1)/2 + 5 \Rightarrow x + 2 = 3y + 3 + 10 \Rightarrow 3y - x + 11 = 0$

$\Rightarrow y = x/3 - 11/3 \Rightarrow T \bullet S \bullet f = x/3 - 11/3$.

If not sure of the ideas behind the processes above, follow the steps below.

Finding first, the new point $S \bullet T$ makes, in the new point S makes, replace x with the x-coordinate in the new point T makes, and replace y with the y-coordinate in the new point T makes.

That is to say that in $(2x, y - 1)$, replace x with $2y$, and replace y with $x - 1$.
Then, we get $\{2(2y), (x - 1) - 1\} = (4y, x - 2)$, which is the new point $S \bullet T$ makes

And we can get $S \bullet T$ the way below, too.

$S \bullet T: (x, y) \xrightarrow{T} (2y, x - 1) \xrightarrow{S} \{2(2y), (x - 1) - 1\} = (4y, x - 2)$. So we get

$S \bullet T: (x, y) \longrightarrow (4y, x - 2)$.

Finding next, the new point $\boldsymbol{T \bullet S}$ makes, in the new point $\boldsymbol{T}$ makes, replace $\boldsymbol{x}$ with the $\boldsymbol{x}$-coordinate in the new point $\boldsymbol{S}$ makes, and replace $\boldsymbol{y}$ with the $\boldsymbol{y}$-coordinate in the new point $\boldsymbol{S}$ makes.

That is to say that in $\mathbf{(2}\boldsymbol{y}\mathbf{,}\ \boldsymbol{x} \mathbf{- 1)}$, replace $\boldsymbol{x}$ with $\mathbf{2}\boldsymbol{x}$, and replace $\boldsymbol{y}$ with $\boldsymbol{y} \mathbf{- 1}$.

Then, we get $\mathbf{\{2(}\boldsymbol{y} \mathbf{- 1), 2}\boldsymbol{x} \mathbf{- 1\} = (2}\boldsymbol{y} \mathbf{- 2, 2}\boldsymbol{x} \mathbf{- 1)}$, which is the new point $\boldsymbol{T \bullet S}$ makes

And we can get $\boldsymbol{T \bullet S}$ the way below, too.

$\boldsymbol{T \bullet S}\mathbf{:}\ \mathbf{(}\boldsymbol{x, y}\mathbf{)} \xrightarrow{\boldsymbol{S}} \mathbf{(2}\boldsymbol{x}\mathbf{,}\ \boldsymbol{y} \mathbf{- 1)} \xrightarrow{\boldsymbol{T}} \mathbf{\{2(}\boldsymbol{y} \mathbf{- 1), 2}\boldsymbol{x} \mathbf{- 1\} = (2}\boldsymbol{y} \mathbf{- 2, 2}\boldsymbol{x} \mathbf{- 1)}$. So we get

$\boldsymbol{T \bullet S}\mathbf{:}\ \mathbf{(}\boldsymbol{x, y}\mathbf{)} \longrightarrow \mathbf{(2}\boldsymbol{y} \mathbf{- 2, 2}\boldsymbol{x} \mathbf{- 1)}$.

So next, applying first, $\boldsymbol{S \bullet T}$ to $\boldsymbol{f}$, and assuming $\boldsymbol{(s, t)}$ is the new point, we get

$\mathbf{(}\boldsymbol{s, t}\mathbf{) = (4}\boldsymbol{y}\mathbf{,}\ \boldsymbol{x} \mathbf{- 2)}$. Thus, we get $\boldsymbol{s} \mathbf{= 4}\boldsymbol{y} \Rightarrow \boldsymbol{y = s}\mathbf{/4}$, and $\boldsymbol{t = x} \mathbf{- 2} \Rightarrow \boldsymbol{x = t} \mathbf{+ 2}$.

So next, getting the connective expression between $\boldsymbol{s}$ and $\boldsymbol{t}$, we get

$\boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{)} \Rightarrow \boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{)} \Rightarrow \boldsymbol{s}\mathbf{/4} = \boldsymbol{f}\mathbf{(}\boldsymbol{t} \mathbf{+ 2) = 3(}\boldsymbol{t} \mathbf{+ 2) + 5}$, since we have $\boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{) = 3}\boldsymbol{x} \mathbf{+ 5}$.

And we know $\boldsymbol{s}$ is $\boldsymbol{x}$ and, $\boldsymbol{t}$ is $\boldsymbol{y}$, so we get

$\boldsymbol{x}\mathbf{/4} = \boldsymbol{f}\mathbf{(}\boldsymbol{y} \mathbf{+ 2) = 3(}\boldsymbol{y} \mathbf{+ 2) + 5} \Rightarrow \boldsymbol{x} \mathbf{= 12(}\boldsymbol{y} \mathbf{+ 2) + 20} \Rightarrow \mathbf{12}\boldsymbol{y} - \boldsymbol{x} \mathbf{+ 44 = 0} \Rightarrow \boldsymbol{y = x}\mathbf{/12 - 11/3}$.

And $\boldsymbol{S \bullet T \bullet f}$ is the new function we get, transforming $\boldsymbol{f}$ by $\boldsymbol{S \bullet T}$.

So we get $\boldsymbol{S \bullet T \bullet f = x}\mathbf{/12 - 11/3}$.

And assuming $\boldsymbol{g}$ is the new function, we can put it this way: $\boldsymbol{y = g}\mathbf{(}\boldsymbol{x}\mathbf{)} = \boldsymbol{x}\mathbf{/12 - 11/3}$.

And applying next, $\boldsymbol{T \bullet S}$ to $\boldsymbol{f}$, and assuming $\boldsymbol{(s, t)}$ is the new point, we get

$\mathbf{(}\boldsymbol{s, t}\mathbf{) = (2}\boldsymbol{y} \mathbf{- 2, 2}\boldsymbol{x} \mathbf{- 1)} \Rightarrow \boldsymbol{s} \mathbf{= 2}\boldsymbol{y} \mathbf{- 2} \Rightarrow \boldsymbol{y} \mathbf{= (}\boldsymbol{s} \mathbf{+ 2)/2}$, and $\boldsymbol{t} \mathbf{= 2}\boldsymbol{x} \mathbf{- 1} \Rightarrow \boldsymbol{x} \mathbf{= (}\boldsymbol{t} \mathbf{+ 1)/2}$.

So next, getting the connective expression between $\boldsymbol{s}$ and $\boldsymbol{t}$, we get

$\boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{)} \Rightarrow \boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{)} \Rightarrow \mathbf{(}\boldsymbol{s} \mathbf{+ 2)/2} = \boldsymbol{f}\mathbf{\{(}\boldsymbol{t} \mathbf{+ 1)/2\} = 3(}\boldsymbol{t} \mathbf{+ 1)/2 + 5}$, since $\boldsymbol{y = f}\mathbf{(}\boldsymbol{x}\mathbf{) = 3}\boldsymbol{x} \mathbf{+ 5}$.

And we know $\boldsymbol{s}$ is $\boldsymbol{x}$ and, $\boldsymbol{t}$ is $\boldsymbol{y}$, so we get $\mathbf{(}\boldsymbol{x} \mathbf{+ 2)/2} = \boldsymbol{f}\mathbf{\{(}\boldsymbol{y} \mathbf{+ 1)/2\} = 3(}\boldsymbol{y} \mathbf{+ 1)/2 + 5}$

$\Rightarrow \boldsymbol{x} \mathbf{+ 2 = 3}\boldsymbol{y} \mathbf{+ 3 + 10} \Rightarrow \mathbf{3}\boldsymbol{y} - \boldsymbol{x} \mathbf{+ 11 = 0} \Rightarrow \boldsymbol{y = x}\mathbf{/3 - 11/3}$.

And $\boldsymbol{T \bullet S \bullet f}$ is the new function we get, transforming $\boldsymbol{f}$ by $\boldsymbol{T \bullet S}$.

So we get $\boldsymbol{T \bullet S \bullet f = x}\mathbf{/3 - 11/3}$.

And assuming $\boldsymbol{h}$ is the new function, we can put it this way: $\boldsymbol{y = h}\mathbf{(}\boldsymbol{x}\mathbf{)} = \boldsymbol{x}\mathbf{/3 - 11/3}$.

Suggestions or Solutions
To the Problem in the Example 3

Assuming S: $(x, y) \longrightarrow (x - 2, 2 - y)$, T: $(x, y) \longrightarrow (x + 1, -y/2)$,
U: $(x, y) \longrightarrow (x + a, 2y + b)$, $y = f(x) = 3x^2 + 5x + 1$, and $U\bullet T\bullet S\bullet f = f$, find a and b.

Finding first $T\bullet S$, we get

$T\bullet S$: $(x, y) \xrightarrow{S} (x - 2, 2 - y) \xrightarrow{T} \{(x - 2) + 1, -(2 - y)/2\} = \{x - 1, (y - 2)/2\}$.

So we get $T\bullet S$: $(x, y) \longrightarrow \{x - 1, (y - 2)/2\}$.

And finding next $U\bullet T\bullet S$, we get

$T\bullet S$: $(x, y) \xrightarrow{T\bullet S} \{x - 1, (y - 2)/2\} \xrightarrow{U} [(x - 1) + a, 2\{(y - 2)/2\} + b]$.

So we get $U\bullet T\bullet S$: $(x, y) \longrightarrow (x + a - 1, y + b - 2)$.

Now, if $A\bullet f = f$, where A is a transformation, and f is a curve, what transformation is A?

Since the new curve is the same as the original curve, we can put A the way below.
A: $(x, y) \longrightarrow (x, y)$. That's because we get no change in the original curve.

And such a transformation is called an identity transformation. So A is an identity transformation. What transformation then, is $U\bullet T\bullet S$?

Since we have $U\bullet T\bullet S\bullet f = f$, $U\bullet T\bullet S$ has to be an identity transformation.

But we have $U\bullet T\bullet S$: $(x, y) \longrightarrow (x + a - 1, y + b - 2)$.

So we need $(x + a - 1, y + b - 2) = (x, y)$.

That is to say these: $x = x + a - 1$, and $y = y + b - 2$. And thus, we get $a = 1$, and $b = 2$.

In short:

To begin with, $T\bullet S$: $(x, y) \longrightarrow \{(x - 2) + 1, -(2 - y)/2\} = \{x - 1, (y - 2)/2\}$.

Next, $U\bullet T\bullet S$: $(x, y) \longrightarrow [(x - 1) + a, 2\{(y - 2)/2\} + b] = (x + a - 1, y + b - 2)$.

And next, since we have $U\bullet T\bullet S\bullet f = f$, $U\bullet T\bullet S$ is an identity transformation.

So we get $x = x + a - 1 \Rightarrow a = 1$, and $y = y + b - 2 \Rightarrow b = 2$.

Suggestions or Solutions
To the Problem in the Example 4

Assuming S: $(x, y) \longrightarrow (ay, x)$ for $a \neq 0$, T: $(x, y) \longrightarrow (x + b, y + c)$,
$f(x, y) = y^2 + 2y - 1 - x = 0$, $y = g(x) = x^2$, and $g = T \bullet S \bullet f$, find a, b, and c.

First, $T \bullet S$: $(x, y) \longrightarrow (ay + b, x + c)$. So next, setting $(s, t) = (ay + b, x + c)$, we get $s = ay + b \Rightarrow y = (s - b)/a$, and $t = x + c \Rightarrow x = t - c$. Thus, we get

$f(x, y) = y^2 + 2y - 1 - x = 0 \Rightarrow f\{t - c, (s - b)/a\} = \{(s - b)/a\}^2 + 2\{(s - b)/a\} - 1 - t - c$

$= (s - b)^2/a^2 + 2(s - b)/a - 1 - t - c = 0 \Rightarrow (x - b)^2/a^2 + 2(x - b)/a - 1 - y - c = 0$

$\Rightarrow y = g(x) = (x - b)^2/a^2 + 2(x - b)/a - 1 - c$.

And we have this, too: $y = g(x) = x^2$. So we get $(x - b)^2/a^2 + 2(x - b)/a - 1 - c = x^2$.
Then, we get first, $a^2 = 1 \Rightarrow a = \pm 1$.

Then, $a = 1 \Rightarrow (x - b)^2 + 2(x - b) - 1 - c = x^2 - 2bx + b^2 + 2x - 2b - 1 - c = x^2$.
So we get $2 - 2b = 0 \Rightarrow b = 1$, and $b^2 - 2b - 1 - c = 0 \Rightarrow 1 - 2 - 1 - c = 0 \Rightarrow c = -2$.
And thus, we get $a = 1$, $b = 1$, and $c = -2$.

And next, $a = -1 \Rightarrow (x - b)^2 - 2(x - b) - 1 - c = x^2 - 2bx + b^2 - 2x + 2b - 1 - c = x^2$.
So we get $2 + 2b = 0 \Rightarrow b = -1$, and $b^2 + 2b - 1 - c = 0 \Rightarrow 1 - 2 - 1 - c = 0 \Rightarrow c = -2$.
And thus, we get $a = -1$, $b = -1$, and $c = -2$.

If not sure of the ideas behind the processes above, follow the steps below.

Finding first $T \bullet S$, we get

$T \bullet S$: $(x, y) \xrightarrow{S} (ay, x) \xrightarrow{T} (ay + b, x + c)$.

So we get $T \bullet S$: $(x, y) \longrightarrow (ay + b, x + c)$.

So next, applying $T \bullet S$ to f assuming (s, t) is the new arbitrary point, we get $(s, t) = (ay + b, x + c)$. So we get $s = ay + b$, and $t = x + c$. And thus, solving for x and y the system above, we get $y = (s - b)/a$, and $x = t - c$.
So next, finding the connective expression between s and t, we get

$f(x, y) = y^2 + 2y - 1 - x = 0 \Rightarrow f\{t - c, (s - b)/a\} = \{(s - b)/a\}^2 + 2\{(s - b)/a\} - 1 - t - c$

$= (s - b)^2/a^2 + 2(s - b)/a - 1 - t - c = 0$.

And we know the curve of g is the new curve, and is in the x-y system.

So the equation of g is $(x - b)^2/a^2 + 2(x - b)/a - 1 - y - c = 0$.

And we can put it this way: $y = (x - b)^2/a^2 + 2(x - b)/a - 1 - c$.

And we have this, too: $y = g(x) = x^2$, that is, $y = x^2$.

So we get $(x - b)^2/a^2 + 2(x - b)/a - 1 - c = x^2$.

And thus, comparing the coefficients in $(x - b)^2/a^2 + 2(x - b)/a - 1 - c = x^2$, we get first, $a^2 = 1$, since the coefficient of x^2 is 1, and expanding (simplifying?) this: $(x - b)^2$, we get this: $x^2 - 2bx + b^2$.

So we get $a = \pm 1$.

Then first, when $a = 1$, we get $(x - b)^2 + 2(x - b) - 1 - c = x^2$.

And simplifying the left hand side, we get $x^2 - 2bx + b^2 + 2x - 2b - 1 - c = x^2$.

More specifically, we have $x^2 - (2b + 2)x + b^2 - 2b - 1 - c = x^2 + 0 \cdot x + 0$.

Then again, comparing the coefficients, we get

$2 - 2b = 0 \Rightarrow b = 1$, and $b^2 - 2b - 1 - c = 0 \Rightarrow 1 - 2 - 1 - c = 0 \Rightarrow c = -2$.

So we get $a = 1$, $b = 1$, and $c = -2$.

And next, moving on to the case where $a = -1$, we get $(x - b)^2 - 2(x - b) - 1 - c = x^2$.

And simplifying the left hand side, we get $x^2 - 2bx + b^2 - 2x + 2b - 1 - c = x^2$.

More specifically, we have $x^2 - (2b + 2)x + b^2 + 2b - 1 - c = x^2 + 0 \cdot x + 0$.

Then again, comparing the coefficients, we get

$2 + 2b = 0 \Rightarrow b = -1$, and $b^2 + 2b - 1 - c = 0 \Rightarrow 1 - 2 - 1 - c = 0 \Rightarrow c = -2$.

So we get $a = -1$, $b = -1$, and $c = -2$.

And putting threads together, we get $a = 1$, $b = 1$, and $c = -2$ or $a = -1$, $b = -1$, and $c = -2$.

Suggestions or Solutions
To the Problem in the Example 5

Assuming S: $(x, y) \longrightarrow (x - 1, y + 1)$, T: $(x, y) \longrightarrow (y, x)$, and $y = f(x) = 5^x$, find $S \bullet f$, and $S \bullet T \bullet f$.

Setting $(s, t) = (x - 1, y + 1)$, we get $s = x - 1 \Rightarrow x = s + 1$, and $t = y + 1 \Rightarrow y = t - 1$.
Then, we get $y = f(x) = 5^x \Rightarrow t - 1 = f(s + 1) = 5^{(s+1)} \Rightarrow t - 1 = 5^{(s+1)} \Rightarrow t = 5^{(s+1)} + 1$.
So we get $S \bullet f = 5^{(x+1)} + 1$.

And next, we get $S \bullet T$: $(x, y) \longrightarrow (y - 1, x + 1)$.
Setting $(s, t) = (y - 1, x + 1)$, we get $s = y - 1 \Rightarrow y = s + 1$, and $t = x + 1 \Rightarrow x = t - 1$.
Then, we get $y = f(x) = 5^x \Rightarrow s + 1 = f(t - 1) = 5^{(t-1)} \Rightarrow s + 1 = 5^{(t-1)}$.
Then, we get $x + 1 = 5^{(y-1)} \Rightarrow y - 1 = \log_5(x + 1)$, and $x + 1 > 0 \Rightarrow x > -1$.
And thus, $S \bullet T \bullet f = \log_5(x + 1) + 1$ for $x > -1$.

If not sure of the ideas behind the processes above, follow the steps below.

Applying first S to f assuming (s, t) is the new point, we get
$(s, t) = (x - 1, y + 1)$. So we get $s = x - 1 \Rightarrow x = s + 1$, and $t = y + 1 \Rightarrow y = t - 1$.

Thus next, finding the connective expression between s and t, we get
$y = f(x) = 5^x \Rightarrow t - 1 = f(s + 1) = 5^{(s+1)} \Rightarrow t - 1 = 5^{(s+1)} \Rightarrow t = 5^{(s+1)} + 1$.

And we know s is x, and t is y, so we get $y = 5^{(x+1)} + 1$.

And $S \bullet f$ is the new function we get, transforming f by S. So we get $S \bullet f = 5^{(x+1)} + 1$.

And also, assuming g is the new function, we can put it this way: $y = g(x) = 5^{(x+1)} + 1$.

And next, finding $S \bullet T$, we get
$T \bullet S$: $(x, y) \xrightarrow{T} (y, x) \xrightarrow{S} (y - 1, x + 1)$.
So we get $T \bullet S$: $(x, y) \longrightarrow (y - 1, x + 1)$.

Thus next, applying $\boldsymbol{S \bullet T}$ to $\boldsymbol{f}$, and assuming $\mathbf{(s, t)}$ is the new point, we get $\mathbf{(s, t) = (y - 1, x + 1)}$. So we get $\mathbf{s = y - 1 \Rightarrow y = s + 1}$, and $\mathbf{t = x + 1 \Rightarrow x = t - 1}$.

Thus next, finding the connective expression between $\boldsymbol{s}$ and $\boldsymbol{t}$, we get $\mathbf{y = f(x) = 5^x \Rightarrow s + 1 = f(t - 1) = 5^{(t-1)} \Rightarrow s + 1 = 5^{(t-1)}}$.

And we know $\boldsymbol{s}$ is $\boldsymbol{x}$, and $\boldsymbol{t}$ is $\boldsymbol{y}$, so we get $\mathbf{x + 1 = 5^{(y-1)}}$.

And using the definition for logs, we can put $\boldsymbol{y}$ in terms of $\boldsymbol{x}$.

The definition is $\mathbf{y = a^x \Leftrightarrow x = \log_a y}$ where $\mathbf{y > 0}$, $\boldsymbol{x}$ is real, $\mathbf{a > 0}$, and $\mathbf{a \neq 1}$.

So by the definition above, we get $\mathbf{x + 1 = 5^{(y-1)} \Rightarrow y - 1 = \log_5 (x + 1) \Rightarrow y = \log_5 (x + 1) + 1}$.

And by the definition again, we get $\mathbf{x + 1 > 0 \Rightarrow x > -1}$.

And $\boldsymbol{S \bullet T \bullet f}$ is the new function we get, transforming $\boldsymbol{f}$ by $\boldsymbol{S \bullet T}$.

So we get $\mathbf{S \bullet T \bullet f = \log_5 (x + 1) + 1}$ for $\mathbf{x > -1}$.

If not sure of logs, log functions, or exponential functions, refer to the sections in the book, **Powers and Logarithms**.