Examples 2 in Polynomial Factorizations

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Examples 2

0. Factorize the polynomials below.

0.0. $\boldsymbol{x^2 + 2xy + y^2}$

0.1. $\boldsymbol{x^2 - 2xy + y^2}$

0.2. $\boldsymbol{x^3 + 3x^2y + 3xy^2 + y^3}$

0.3. $\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3}$

1. Assuming that $\boldsymbol{a^2 + b^2 + 2ab = K^2}$, put $\boldsymbol{K}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Suggestions or Solutions
To the Problem 0 in the Example 0

We have $\boldsymbol{x^2 + 2xy + y^2}$.

Factorizing it, we get

$$\boldsymbol{x^2 + 2xy + y^2 = x^2 + xy + xy + y^2 = x(x + y) + y(x + y) = (x + y)(x + y) = (x + y)^2}.$$

If not quite sure of the idea behind the processes above, follow the steps below.

Doing math, we often need to do mental math, together with putting down numbers. So working in math, we don't just work with numbers doing arithmetic and algebra but keep running a stream of logic, too, along with math operations and activities.

Doing mental math, we work on particular numbers or expressions taking care of the next operations keeping in mind results of the current operations. Besides, we do other activities as substitutions.

And it is particularly the case when we do factorizations. Quite often, we need to make some predictions, too, doing factorizations. Such predictions are not random guesses but logical expectations, which are in fact, what mental math is mostly about.

Now, factorizing a polynomial, we find factors of the polynomial, and put them in a form of a product.   How then can we find the factors?

Factors are divisors. So when factorizing a polynomial, we look for divisors, each of which can divide the polynomial.   How then can we find such divisors?

Normally, when factorizing a polynomial, we try first, finding a divisor that can divide each and every term in the polynomial. Such a divisor is called a common divisor, and does divide the polynomial, since it divides every term in the polynomial.

So if an integer or an expression divides every term in a polynomial, it is said to be common to all the terms in the polynomial, and is called a common divisor.

So it divides the polynomial, and thus, is a divisor of the polynomial. And we call such a divisor a factor of the polynomial.

It's not always the case however, we find a factor the way above.
In other words, that's not the only way we can find a factor of a polynomial.
We often need to look at the same problem in a different perspective.

Suppose for instance, we can find a divisor that divides not every term but some terms.

Then, it can be the case where we get a factor, which divides the polynomial, of course.
How can we get such a factor though?

Doing a problem in math, we break it apart, and put the pieces together.
We don't just break it though.
We want to break it apart so that we can form the solution putting the pieces together.

Now, setting $\boldsymbol{P = x^2 + 2xy + y^2}$, we can say that $\boldsymbol{P}$ doesn't look factorable, because we don't see any divisor common to all its terms. What then can we do?

A polynomial can be a sum of polynomials.
So taking apart $\boldsymbol{2xy}$ into two parts, we can put $\boldsymbol{P}$ the way as follows.

$\boldsymbol{P = x^2 + xy + xy + y^2 = (x^2 + xy) + (xy + y^2)}$, which can be thus, taken as a polynomial made of two terms, one is $\boldsymbol{(x^2 + xy)}$, and the other is $\boldsymbol{(xy + y^2)}$. So what?

• Finding a factor of a polynomial, we find a divisor common to terms.

So looking closely at each of the two terms $(x^2 + xy)$ and $(xy + y^2)$, we can see x is common to all the terms in $x^2 + xy$, and y is common in $xy + y^2$.

So we get $P = x^2 + xy + xy + y^2 = x(x + y) + y(x + y)$, where $x + y$ is a divisor common to both terms, $x(x + y)$ and $y(x + y)$.

So we get $x(x + y) + y(x + y) = (x + y)(x + y)$, and thus, we get $P = (x + y)^2$.

Therefore, $x^2 + 2xy + y^2$ is factorized to $(x + y)^2$.

In short:

$$x^2 + 2xy + y^2 = x^2 + xy + xy + y^2 = x(x + y) + y(x + y) = (x + y)(x + y) = (x + y)^2.$$

Suggestions or Solutions
To the Problem 1 in the Example 0

We have $x^2 - 2xy + y^2$.

This is in fact, no other than the previous problem.
We can notice that putting $-y$ into y in $x^2 + 2xy + y^2$, we get: $x^2 - 2xy + y^2$.
And we have $x^2 + 2xy + y^2 = (x + y)^2$.

So in $(x + y)^2$, replacing y with $-y$, we can readily get $(x - y)^2$, which is P factorized.

For practice purposes though, let's get the solution the way we get the previous one.

It's a good idea to name a polynomial if it's not named. So set first $P = x^2 - 2xy + y^2$.

And next, taking apart $2xy$ into two terms, we get $P = x^2 - 2xy + y^2 = x^2 - xy - xy + y^2$.
That is, we get
$P = x^2 - 2xy + y^2 = x^2 - xy - xy + y^2 = (x^2 - xy) - (xy - y^2)$, which can be thus, taken as a polynomial made of two terms, one is $(x^2 - xy)$, and the other is $(xy - y^2)$.

Then, we can see x is common in $x^2 - xy$, and y is common in $xy - y^2$.

So we get $P = (x^2 - xy) - (xy - y^2) = x(x - y) - y(x - y)$, where $x - y$ is a divisor common to both terms, $x(x - y)$ and $y(x - y)$.

Thus, we get $P = x(x - y) - y(x - y) = (x - y)(x - y)$, which is $(x - y)^2$.
Therefore, we get $P = (x - y)^2$.

In short:

$x^2 - 2xy + y^2 = x^2 - xy - xy + y^2 = x(x - y) + y(-x + y)$

$= x(x - y) - y(x - y) = (x - y)(x - y) = (x - y)^2$.

Suggestions or Solutions

To the Problem 2 in the Example 0

We have $x^3 + 3x^2y + 3xy^2 + y^3$.

Factorizing it, we can begin with this:

$x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + 3x^2y) + (3xy^2 + y^3) = x(x^2 + 3xy) + (3xy + y^2)y$.

So first, $x^2 + 3xy = x^2 + 2xy + xy + y^2 - y^2 = x^2 + 2xy + y^2 + xy - y^2 = (x + y)^2 + xy - y^2$.

Thus, we get $x(x^2 + 3xy) = x(x + y)^2 + x^2y - xy^2$.

Next, $3xy + y^2 = x^2 - x^2 + 2xy + xy + y^2 = x^2 + 2xy + y^2 + xy - x^2 = (x + y)^2 + xy - x^2$.

So we get $(3xy + y^2)y = y(x + y)^2 + xy^2 - x^2y$.

Thus, we get

$x(x^2 + 3xy) + (3xy + y^2)y = \{x(x + y)^2 + x^2y - xy^2\} + \{y(x + y)^2 + xy^2 - x^2y\}$

$= x(x + y)^2 + y(x + y)^2 + x^2y - xy^2 + xy^2 - x^2y = x(x + y)^2 + y(x + y)^2$

$= (x + y)^2(x + y) = (x + y)^3$.

If not quite sure of the idea behind the processes above, follow the steps below.

Setting first, $P = x^3 + 3x^2y + 3xy^2 + y^3$, we can say that P has a symmetry.

Even if x and y get exchanged, we get exactly the same polynomial P. So we should be able to take advantage of it.

Thus next, we can try breaking the polynomial P into two parts the way below.

$P = x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + 3x^2y) + (3xy^2 + y^3)$.

Then, we can see a couple of cases where we can get a divisor common to all the terms in each part.

In one case, x is common in the first part, and y is common in the second part.

So we can put the polynomial P the way below.

$P = (x^3 + 3x^2y) + (3xy^2 + y^3) = x(x^2 + 3xy) + (3xy + y^2)y$.

And next, getting back to polynomial $(x^3 + 3x^2y) + (3xy^2 + y^3)$, we can see the other case, where x^2 is common in the first part, and y^2 is common in the second part.

So this time, we can put the polynomial P the way below.

$P = (x^3 + 3x^2y) + (3xy^2 + y^3) = x^2(x + 3y) + (3x + y)y^2$.

Which case then do we want to take?

We want to take the first case, where $P = x(x^2 + 3xy) + (3xy + y^2)y$.

That's because we can expect the polynomial $x^2 + 3xy$ to be converted to '$x^2 + 2xy + y^2$' seeing that $x^2 + 3xy$ has $x^2 + 2xy$, so we can get $(x + y)^2$ if we add y^2 to it.

So let's first, take care of $x(x^2 + 3xy)$ keeping in mind that we have $(3xy + y^2)y$, too.

We don't overpay, underpay, or compromise doing math. In math, we can compensate only. That is, getting one thing in math, we give the one exactly the same or exactly equivalent. So math is always a fair business, isn't it? It is fair and square, of course, and is always so.

Now, setting $3xy = 2xy + xy$, and then, adding and subtracting y^2, we can make $(x + y)^2$.

In other words, we can get

$$\boldsymbol{x^2 + 3xy = x^2 + 2xy + xy + y^2 - y^2 = \underline{x^2 + 2xy + y^2} + xy - y^2 = \underline{(x + y)^2} + xy - y^2}.$$

So we get $\boldsymbol{x(x^2 + 3xy) = x\{(x + y)^2 + xy - y^2\} = x(x + y)^2 + x^2y - xy^2}$.

Let's now, move on to the other part, which is $\boldsymbol{(3xy + y^2)y}$.
What then can we do about it?

We know the polynomial $\boldsymbol{P}$ is symmetric.   So?

So doing to the other part the same thing as the one we did to the part $\boldsymbol{x(x^2 + 3xy)}$, because of the symmetry, we get the same result. That is, we get

$$\boldsymbol{3xy + y^2 = x^2 - x^2 + 2xy + xy + y^2 = \underline{x^2 + 2xy + y^2} + xy - x^2 = \underline{(x + y)^2} + xy - x^2}.$$

So we get $\boldsymbol{(3xy + y^2)y = y(x + y)^2 + xy^2 - x^2y}$.

Thus, putting threads together, we get

$$\boldsymbol{P = x(x^2 + 3xy) + (3xy + y^2)y = \{x(x + y)^2 + x^2y - xy^2\} + \{y(x + y)^2 + xy^2 - x^2y\}}$$

$$\boldsymbol{= x(x + y)^2 + y(x + y)^2 + x^2y - xy^2 + xy^2 - x^2y = x(x + y)^2 + y(x + y)^2}.$$

Then, we can see that $\boldsymbol{(x + y)^2}$ is a divisor common to $\boldsymbol{x(x + y)^2}$ and $\boldsymbol{y(x + y)^2}$.

So $\boldsymbol{(x + y)^2}$ is a divisor of $\boldsymbol{P}$, and thus, can be a factor of $\boldsymbol{P}$.

That is to say that we get $\boldsymbol{P = x(x + y)^2 + y(x + y)^2 = (x + y)^2(x + y) = (x + y)^3}$, which is the factorization of $\boldsymbol{P}$.

In short:

$x^3 + 3x^2y + 3xy^2 + y^3 = (x^3 + 3x^2y) + (3xy^2 + y^3) = x(x^2 + 3xy) + (3xy + y^2)y$.

So first, $x^2 + 3xy = x^2 + 2xy + xy + y^2 - y^2 = x^2 + 2xy + y^2 + xy - y^2 = (x + y)^2 + xy - y^2$.

Thus, we get $x(x^2 + 3xy) = x(x + y)^2 + x^2y - xy^2$.

Next, $3xy + y^2 = x^2 - x^2 + 2xy + xy + y^2 = x^2 + 2xy + y^2 + xy - x^2 = (x + y)^2 + xy - x^2$.

So we get $(3xy + y^2)y = y(x + y)^2 + xy^2 - x^2y$.

Thus, we get

$x(x^2 + 3xy) + (3xy + y^2)y = \{x(x + y)^2 + x^2y - xy^2\} + \{y(x + y)^2 + xy^2 - x^2y\}$

$= x(x + y)^2 + y(x + y)^2 + x^2y - xy^2 + xy^2 - x^2y = x(x + y)^2 + y(x + y)^2$

$= (x + y)^2(x + y) = (x + y)^3$.

And we can have a sequence as follows.

$x + y = x + y$,

$(x + y)^2 = x^2 + 2xy + y^2$,

$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$,

$(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$,

$(x + y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$,

…

What then are the expansions of $(x + y)^{1000}$ and $(x + y)^k$, where k is a nonnegative integer?

Suggestions or Solutions
To the Problem 3 in the Example 0

We have $\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3}$.

Factorizing it, we can begin with this:
$\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3 = (x^3 - 3x^2y) + (3xy^2 - y^3) = x(x^2 - 3xy) - (-3xy + y^2)y}$.

First, $\boldsymbol{x^2 - 3xy = x^2 - 2xy - xy + y^2 - y^2 = x^2 - 2xy + y^2 - xy - y^2 = (x - y)^2 - xy - y^2}$.

So we get $\boldsymbol{x(x^2 - 3xy) = x(x - y)^2 - x^2y - xy^2}$.

Next, $\boldsymbol{-3xy + y^2 = x^2 - x^2 - 2xy - xy + y^2 = x^2 - 2xy + y^2 - xy - x^2 = (x - y)^2 - xy - x^2}$.

So we get $\boldsymbol{-y(-3xy + y^2) = -y(x - y)^2 + xy^2 + x^2y}$.

Thus, we get this:

$\boldsymbol{x(x^2 - 3xy) - y(-3xy + y^2) = x(x^2 - 3xy) + \{-y(-3xy + y^2)\}}$
$\boldsymbol{= \{x(x - y)^2 - x^2y - xy^2\} + \{-y(x - y)^2 + xy^2 + x^2y\}}$
$\boldsymbol{= x(x - y)^2 - y(x - y)^2 - x^2y - xy^2 + xy^2 + x^2y = x(x - y)^2 - y(x - y)^2}$
$\boldsymbol{= (x - y)^2(x - y) = (x - y)^3}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Suppose first, $\boldsymbol{P = x^3 - 3x^2y + 3xy^2 - y^3}$.
The polynomial $\boldsymbol{P}$ is quite similar to the one in the problem 2 above.
In fact, $\boldsymbol{P}$ is as good as the one in the problem 2.   How?

It's just a matter of changing $\boldsymbol{y}$ with $\boldsymbol{-y}$.
Substituting $\boldsymbol{y}$ with $\boldsymbol{-y}$ in $\boldsymbol{P}$, we get the one in the problem 2.
So putting $\boldsymbol{-y}$ into $\boldsymbol{y}$ in $\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3}$, we get this:

$\boldsymbol{x^3 - 3x^2(-y) + 3xy^2 - (-y)^3 = x^3 + 3x^2y + 3xy^2 + y^3}$.

And we know $\boldsymbol{x^3 + 3x^2y + 3xy^2 + y^3 = (x + y)^3}$.

So changing $\boldsymbol{y}$ with $\boldsymbol{-y}$ in the polynomial in the problem 2, we get this:

$\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3 = (x - y)^3}$.

Let's see though, how the factorization of $\boldsymbol{P}$ can actually proceed in a usual manner.

There is a symmetry in $\boldsymbol{P}$: changing $\boldsymbol{x}$ with $\boldsymbol{-y}$, and $\boldsymbol{y}$ with $\boldsymbol{-x}$. we get the same $\boldsymbol{P}$.
So we should take advantage of it.

And the factorization method or procedure will be just about the same as the one we used for the previous example. So first, we break $\boldsymbol{P}$ into two parts.
And next, we take one part at a time keeping in mind the other part to be taken care of.

So to begin with, $\boldsymbol{x}$ is common in $\boldsymbol{x^3 - 3x^2y}$, so we get $\boldsymbol{x^3 - 3x^2y = x(x^2 - 3xy)}$.

Next, $\boldsymbol{-y}$ is common in $\boldsymbol{3xy^2 - y^3}$, so we get $\boldsymbol{3xy^2 - y^3 = -y(-3xy + y^2)}$.
Why is $\boldsymbol{-y}$ common though?

We can get $\boldsymbol{3xy^2 - y^3 = 3x(-y)(-y) + (-y^3) = 3x(-y)(-y) + (-y)y^2}$, where $\boldsymbol{-y}$ is common.

Now, beginning with $\boldsymbol{x^2 - 3xy}$, we can first, put it the way below.

$\boldsymbol{x^2 - 3xy = x^2 - 2xy - xy + y^2 - y^2 = \underline{x^2 - 2xy + y^2} - xy - y^2 = \underline{(x - y)^2} - xy - y^2}$.

So next, we can get $\boldsymbol{x(x^2 - 3xy) = x\{(x - y)^2 - xy - y^2\} = x(x - y)^2 - x^2y - xy^2}$.

And next, moving on to the other part, $\boldsymbol{-3xy + y^2}$, we can first, put it the way below.

$\boldsymbol{-3xy + y^2 = x^2 - x^2 - 2xy - xy + y^2 = \underline{x^2 - 2xy + y^2} - xy - x^2 = \underline{(x - y)^2} - xy - x^2}$.

Thus next, we get $\boldsymbol{3xy^2 - y^3 = -y(-3xy + y^2) = -y(x - y)^2 + xy^2 + x^2y}$.

Now, putting threads together, we get

$\boldsymbol{P = x^3 - 3x^2y + 3xy^2 - y^3 = x(x^2 - 3xy) - y(-3xy + y^2) = x(x^2 - 3xy) + \{-y(-3xy + y^2)\}}$

$\boldsymbol{= \{x(x - y)^2 - x^2y - xy^2\} + \{-y(x - y)^2 + xy^2 + x^2y\}}$

$\boldsymbol{= x(x - y)^2 - y(x - y)^2 - x^2y - xy^2 + xy^2 + x^2y = x(x - y)^2 - y(x - y)^2}$, where $\boldsymbol{(x - y)^2}$ is common to $\boldsymbol{x(x - y)^2}$ and $\boldsymbol{- y(x - y)^2}$, and therefore, is a divisor, that is, a factor of $\boldsymbol{P}$.

So we get $\boldsymbol{P = x(x - y)^2 - y(x - y)^2 = (x - y)^2(x - y) = (x - y)^3}$, which is $(\boldsymbol{x - y})$ to the third power, and thus, is $\boldsymbol{(x - y)(x - y)(x - y)}$.

And we can say that $\boldsymbol{P}$ is a product of three prime factors, which are three of $(\boldsymbol{x - y})$s.

In short:

$\boldsymbol{x^3 - 3x^2y + 3xy^2 - y^3 = (x^3 - 3x^2y) + (3xy^2 - y^3) = x(x^2 - 3xy) - (-3xy + y^2)y}$.

First, $\boldsymbol{x^2 - 3xy = x^2 - 2xy - xy + y^2 - y^2 = x^2 - 2xy + y^2 - xy - y^2 = (x - y)^2 - xy - y^2}$.

So we get $\boldsymbol{x(x^2 - 3xy) = x(x - y)^2 - x^2y - xy^2}$.

Next, $\boldsymbol{-3xy + y^2 = x^2 - x^2 - 2xy - xy + y^2 = x^2 - 2xy + y^2 - xy - x^2 = (x - y)^2 - xy - x^2}$.

So we get $\boldsymbol{-y(-3xy + y^2) = -y(x - y)^2 + xy^2 + x^2y}$.

Thus, we get

$\boldsymbol{x(x^2 - 3xy) - y(-3xy + y^2) = x(x^2 - 3xy) + \{-y(-3xy + y^2)\}}$

$\boldsymbol{= \{x(x - y)^2 - x^2y - xy^2\} + \{-y(x - y)^2 + xy^2 + x^2y\}}$

$\boldsymbol{= x(x - y)^2 - y(x - y)^2 - x^2y - xy^2 + xy^2 + x^2y = x(x - y)^2 - y(x - y)^2}$

$\boldsymbol{= (x - y)^2(x - y) = (x - y)^3}$.

And we can have a sequence as follows.

$x - y = x - y$,

$(x - y)^2 = x^2 - 2xy + y^2$,

$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$,

$(x - y)^4 = x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$,

$(x - y)^5 = x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$,

…

What then are the expansions of $(x - y)^{1000}$ and $(x - y)^k$ where k is a nonnegative integer?

Suggestions or Solutions
To the Problem in the Example 1

Assuming that $a^2 + b^2 + 2ab = K^2$, put K in terms of a and b.

$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$.

So $K^2 = (a + b)^2$. Thus, $K = \pm(a + b)$.

If not quite sure of the idea behind the processes above, follow the steps below.

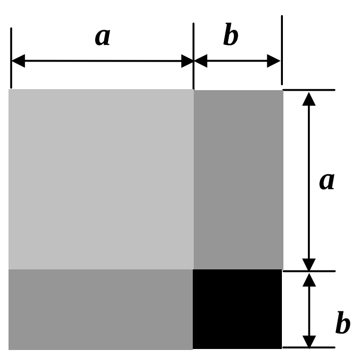
Suppose first, a and b are lengths of two line segments.

Then, a^2 and b^2 indicate areas of two different squares, and $2ab = ab + ab$ indicates the sum of areas of two same rectangles.

And suppose now, $a > b$.

Then, we can put together the two squares and two rectangles the way below.

Fig. 1.0



Then, we can see that the sum of all the areas of the rectangles and squares is equal to an area of one large square, which is $(a + b)$ by $(a + b)$.

In other words, we get $(a + b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$. So we get

$K^2 = (a + b)^2 \Rightarrow K = a + b$ or $-(a + b)$, since we have $\{-(a + b)\}^2 = (a + b)^2$, too.

Thus, we can put K this way, too: $K = \pm(a + b)$.

Though it sounds quite natural, we want to keep in mind that if for instance, $x^2 = y^2$, we get $x = y$ or $-y$, and not just $x = y$ only. In short, we have $x^2 = y^2 \Rightarrow x = \pm y$.

What if we set $a^2 + b^2 - 2ab = K^2$, and want to put K in terms of a and b?

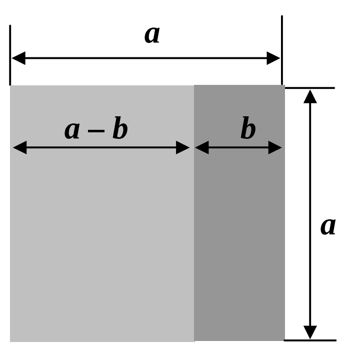
Let's suppose again, a and b are lengths of two line segments.

Then, a^2 and b^2 indicate areas of two different squares, and $2ab = ab + ab$ indicates the sum of areas of two same rectangles.

Suppose now again, $a > b$.

Suppose also, one rectangle a by b is put on top of the square a by a the way below.

Fig. 1.1



Suppose next, the other rectangle a by b is put on top of the two quadrangles as shown in the figure below on the right.

Fig. 1.2

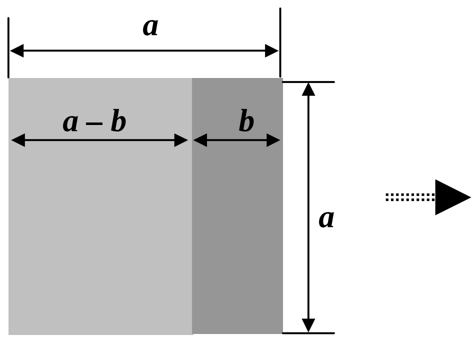
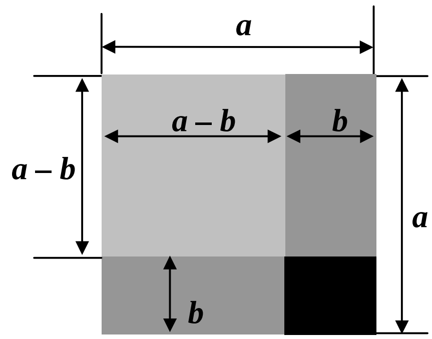


Fig. 1.3



Then, we can see that the area the two same rectangles overlap is $\boldsymbol{b^2}$.

That is, the area of the small square in black is $\boldsymbol{b^2}$.

Also, we can see another square, which is $\boldsymbol{(a - b)}$ by $\boldsymbol{(a - b)}$, so its area is $\boldsymbol{(a - b)^2}$.

Thus, we now have three different squares: small, medium, and large.

The large is $\boldsymbol{a}$ by $\boldsymbol{a}$, the medium is $\boldsymbol{(a - b)}$ by $\boldsymbol{(a - b)}$, and the small is $\boldsymbol{b}$ by $\boldsymbol{b}$.

Let's now, express the medium square in terms of the two rectangles $\boldsymbol{a}$ by $\boldsymbol{b}$, together with the other two squares. That is, we want to put $\boldsymbol{(a - b)^2}$ in terms of $\boldsymbol{ab}$, $\boldsymbol{a^2}$, and $\boldsymbol{b^2}$.

Then, we get $\boldsymbol{(a - b)^2 = a^2 - ab - ab + b^2}$. How do we have to add $\boldsymbol{b^2}$, though?

Subtracting twice the rectangle $\boldsymbol{a}$ by $\boldsymbol{b}$, from the large square $\boldsymbol{a}$ by $\boldsymbol{a}$, we get the medium square less the small square, which is $\boldsymbol{b^2}$. So we need to add it back.

And in the figure on the right hand side above, a rectangle in dark gray is $\boldsymbol{(a - b)}$ by $\boldsymbol{b}$.

So the area of the medium square $\boldsymbol{(a - b)}$ by $\boldsymbol{(a - b)}$ is as follows.

$\boldsymbol{a^2 - (a - b)b - (a - b)b - b^2 = a^2 - ab + b^2 - ab + b^2 - b^2 = a^2 - ab - ab + b^2}$.

Thus, we get $\boldsymbol{(a - b)^2 = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2 \Rightarrow (a - b)^2 = a^2 - 2ab + b^2}$.

So we get $\boldsymbol{K^2 = (a - b)^2 \Rightarrow K = a - b}$ or $\boldsymbol{-(a - b)}$, because $\boldsymbol{\{-(a - b)\}^2 = (a - b)^2}$, too.

Thus, we can put $\boldsymbol{K}$ this way, too: $\boldsymbol{K = \pm(a - b)}$.

Why do we do this example though?

• First, practicing polynomial factorizations, we can develop and improve algebra skills.

• And next, doing algebra, or setting up equations, we don't want to just put expressions and equal signs. We want to know the meanings of those expressions.

So for instance, using factorizations, we can show how an area of a square changes as we increase or decrease each side by the same amount.

And the factorizations are $\boldsymbol{(a + b)^2 = a^2 + 2ab + b^2}$, and $\boldsymbol{(a - b)^2 = a^2 - 2ab + b^2}$.

So if each side of the square $\boldsymbol{a}$ by $\boldsymbol{a}$ is increased by the amount of $\boldsymbol{b}$, the amount increased is $\boldsymbol{2ab + b^2}$.

And if each side of the square $\boldsymbol{a}$ by $\boldsymbol{a}$ is decreased by the amount of $\boldsymbol{b}$, the amount decreased is $\boldsymbol{2ab - b^2}$, because we have $\boldsymbol{a^2 - 2ab + b^2 = a^2 - (2ab - b^2)}$.

In short:

$\boldsymbol{(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2}$.

So $\boldsymbol{K^2 = (a + b)^2}$. Thus, $\boldsymbol{K = \pm(a + b)}$.

$\boldsymbol{(a - b)^2 = (a - b)(a - b) = a^2 - ab - ab + b^2 = a^2 - 2ab + b^2}$.

So $\boldsymbol{K^2 = (a - b)^2}$. Therefore, $\boldsymbol{K = \pm(a - b)}$.