Examples C in Polynomial Factorizations

Click this to start.

Examples C

Factorize the polynomials below.

0. $\mathbf{40}\boldsymbol{t}^{\mathbf{2}} + \mathbf{59}\boldsymbol{t} + \mathbf{21}$

1. $\mathbf{30}\boldsymbol{t}^{\mathbf{2}} + \mathbf{57}\boldsymbol{t} + \mathbf{18}$

2. $\mathbf{6}\boldsymbol{r}^{\mathbf{2}} - \mathbf{13}\boldsymbol{r} - \mathbf{28}$

3. $\frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{w}^{\mathbf{2}} + \mathbf{12}\boldsymbol{w} - \mathbf{30}$

4. $\mathbf{0.6}\boldsymbol{t}^{\mathbf{2}} + \mathbf{14.92}\boldsymbol{t} - \mathbf{2}$

Suggestions or Solutions
To the Problem in the Example 0

We have $\mathbf{40}\boldsymbol{t}^{2} + \mathbf{59}\boldsymbol{t} + \mathbf{21}$.

Factorizing it, we can get it the way below.

$\mathbf{40}\boldsymbol{t}^{2} + \mathbf{59}\boldsymbol{t} + \mathbf{21} = \mathbf{5\cdot 8}\boldsymbol{t}^{2} + \mathbf{(3\cdot 8 + 7\cdot 5)}\boldsymbol{t} + \mathbf{3\cdot 7} = \mathbf{5\cdot 8}\boldsymbol{t}^{2} + \mathbf{3\cdot 8}\boldsymbol{t} + \mathbf{7\cdot 5}\boldsymbol{t} + \mathbf{3\cdot 7}$

$= \mathbf{8}\boldsymbol{t}\mathbf{(5}\boldsymbol{t}\mathbf{ + 3) + 7(5}\boldsymbol{t}\mathbf{ + 3) = (5}\boldsymbol{t}\mathbf{ + 3)(8}\boldsymbol{t}\mathbf{ + 7)}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Setting first, $\boldsymbol{P} = \mathbf{40}\boldsymbol{t}^{2} + \mathbf{59}\boldsymbol{t} + \mathbf{21}$, we can say that the polynomial $\boldsymbol{P}$ takes the form of a polynomial $\boldsymbol{ut}^{2} + \boldsymbol{vt} + \boldsymbol{w}$, which is similar to $\boldsymbol{act}^{2} + (\boldsymbol{ad} + \boldsymbol{bc})\boldsymbol{t} + \boldsymbol{bd}$, which is factorize to $(\boldsymbol{at} + \boldsymbol{b})(\boldsymbol{ct} + \boldsymbol{d})$.

So taking $\boldsymbol{ac}$ as $\boldsymbol{u}$, $\boldsymbol{ad} + \boldsymbol{bc}$ as $\boldsymbol{v}$, and $\boldsymbol{bd}$ as $\boldsymbol{w}$, we get $\boldsymbol{ut}^{2} + \boldsymbol{vt} + \boldsymbol{w}$, which $\boldsymbol{P}$ is in the form of.

So $\boldsymbol{P}$ can be said to be similar to $\boldsymbol{act}^{2} + (\boldsymbol{ad} + \boldsymbol{bc})\boldsymbol{t} + \boldsymbol{bd}$, factorize to $(\boldsymbol{at} + \boldsymbol{b})(\boldsymbol{ct} + \boldsymbol{d})$.

Assuming thus, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are integers, setting $\boldsymbol{ac} = \mathbf{40}$, $\boldsymbol{ad} + \boldsymbol{bc} = \mathbf{59}$, and $\boldsymbol{bd} = \mathbf{21}$, and then, finding $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$, we can get the factorization of $\boldsymbol{P}$.

That is to say that $\boldsymbol{a}$ and $\boldsymbol{c}$ are integers the product of which is 40, $\boldsymbol{b}$ and $\boldsymbol{d}$ are integers the product of which is 21, and $\boldsymbol{ad} + \boldsymbol{bc} = \mathbf{59}$.

So $\boldsymbol{a}$ and $\boldsymbol{c}$ are divisors of 40, and $\boldsymbol{b}$ and $\boldsymbol{d}$ are divisors of 21.

And if any, we can get the integers that can be $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ via trial and error.

Suppose first, $a = 1$, $c = 40$, $b = 3$, and $d = 7$.

Then, $ad + bc = 1 \cdot 7 + 3 \cdot 40 = 127$, which is not 59, so we want to make another choice.

Suppose next, $a = 4$, $c = 10$, $b = 3$, and $d = 7$.

Then, $ad + bc = 4 \cdot 7 + 3 \cdot 10 = 58$, which is not 59, so we want to make another choice.

Suppose next, $a = 5$, $c = 8$, $b = 3$, and $d = 7$.

Then, $ad + bc = 5 \cdot 7 + 3 \cdot 8$, which is 59, so we've got the right choice.

Now, we have $P = 40t^2 + 59t + 21 = act^2 + (ad + bc)t + bd = (at + b)(ct + d)$, where $a = 5$, $c = 8$, $b = 3$, and $d = 7$.

So we get $P = 40t^2 + 59t + 21 = (at + b)(ct + d) = (5t + 3)(8t + 7)$.

And each of ($5t + 3$) and ($8t + 7$) is prime.

Therefore, $P = 40t^2 + 59t + 21$ is fully factorized to $(5t + 3)(8t + 7)$.

In short:

$$40t^2 + 59t + 21 = 5 \cdot 8t^2 + (3 \cdot 8 + 7 \cdot 5)t + 3 \cdot 7 = 5 \cdot 8t^2 + 3 \cdot 8t + 7 \cdot 5t + 3 \cdot 7$$

$$= 8t(5t + 3) + 7(5t + 3) = (5t + 3)(8t + 7).$$

Suggestions or Solutions
To the Problem in the Example 1

We have $\mathbf{30t^2 + 57t + 18}$.

Factorizing it, we can get it the way below.

To begin with, we get $\mathbf{30t^2 + 57t + 18 = 3(10t^2 + 19t + 6)}$.

And we get $\mathbf{10t^2 + 19t + 6 = 2\cdot 5t^2 + (3\cdot 5 + 2\cdot 2)t + 2\cdot 3 = 2\cdot 5t^2 + 3\cdot 5t + 2\cdot 2t + 2\cdot 3}$

$\mathbf{= 5t(2t + 3) + 2(2t + 3) = (2t + 3)(5t + 2)}$.

So we get $\mathbf{3(10t^2 + 19t + 6) = 3(2t + 3)(5t + 2)}$.

Thus, we get $\mathbf{30t^2 + 57t + 18 = 3(2t + 3)(5t + 2)}$.

If not quite sure of the idea behind the processes above, follow the steps below.

Setting first, $\mathbf{P = 30t^2 + 57t + 18}$, we can notice that 3 is a divisor common to every term in the polynomial $\mathbf{P}$.

So we get $\mathbf{P = 30t^2 + 57t + 18 = 3(10t^2 + 19t + 6)}$.

So setting next, $\mathbf{Q = 10t^2 + 19t + 6}$, we can set $\mathbf{P = 3Q}$, and $\mathbf{Q}$ is similar to a polynomial $\mathbf{act^2 + (ad + bc)t + bd}$, which is factorize to $\mathbf{(at + b)(ct + d)}$.

So putting $\mathbf{Q}$ in the form of $\mathbf{act^2 + (ad + bc)t + bd}$, we can factorize $\mathbf{Q}$ to $\mathbf{(at + b)(ct + d)}$.

Thus, finding $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, and $\mathbf{d}$, we can get the factorization of $\mathbf{Q}$, and in turn, that of $\mathbf{P}$'s.

Since $\mathbf{Q = 10t^2 + 19t + 6}$ takes the form of $\mathbf{act^2 + (ad + bc)t + bd}$, we can set up a system of equations as follows. $\mathbf{ac = 10}$, $\mathbf{ad + bc = 19}$, and $\mathbf{bd = 6}$.

Assuming thus, a, b, c, and d are integers, setting $ac = 10$, $ad + bc = 19$, and $bd = 6$, and then, finding a, b, c, and d, we can get the factorization of Q within the scope of integers.

That is to say that a and c are integers the product of which is 10, b and d are integers the product of which is 6, and $ad + bc = 19$.

So a and c are divisors of 40, and b and d are divisors of 21.

And if any, we can get the integers that can be a, b, c, and d via trial and error.

In fact, we get these: $a = 2$, $c = 5$, $b = 3$, and $d = 2$, because we can get $ac = 10$, $bd = 6$, and $ad + bc = 2\cdot2 + 3\cdot5$, which is 19.

Now, we have $Q = 10t^2 + 19t + 6 = act^2 + (ad + bc)t + bd = (at + b)(ct + d)$, where $a = 2$, $c = 5$, $b = 3$, and $d = 2$.

Thus, we get $Q = 10t^2 + 19t + 6 = (2t + 3)(5t + 2)$, so $P = 3Q = 3(2t + 3)(5t + 2)$.

And each of 3, $2t + 3$, and $5t + 2$ is prime.

Therefore, $P = 30t^2 + 57t + 18$ is (fully) factorized to $3(2t + 3)(5t + 2)$.

In short:

To begin with, we get $30t^2 + 57t + 18 = 3(10t^2 + 19t + 6)$.

And we get $10t^2 + 19t + 6 = 2\cdot5t^2 + (3\cdot5 + 2\cdot2)t + 2\cdot3 = 2\cdot5t^2 + 3\cdot5t + 2\cdot2t + 2\cdot3$

$= 5t(2t + 3) + 2(2t + 3) = (2t + 3)(5t + 2)$.

So we get $3(10t^2 + 19t + 6) = 3(2t + 3)(5t + 2)$.

Thus, we get $30t^2 + 57t + 18 = 3(2t + 3)(5t + 2)$.

Suggestions or Solutions

To the Problem in the Example 2

We have $6r^2 - 13r - 28$.

Factorizing it, we can get it the way below.

$$6r^2 - 13r - 28 = 2\cdot 3r^2 + (8 - 21)r + 4\cdot 7 = 2\cdot 3r^2 + (2\cdot 4 - 3\cdot 7)r + 4\cdot 7$$

$$= 2\cdot 3r^2 - 3\cdot 7r + 2\cdot 4r + 4\cdot 7 = 3r(2r - 7) + 4(2r - 7) = (2r - 7)(3r + 4).$$

If not quite sure of the idea behind the processes above, follow the steps below.

Setting first, $P = 6r^2 - 13r - 28$, we can say that the polynomial P is similar to a polynomial $acr^2 + (ad + bc)r + bd$, which is factorize to $(ar + b)(cr + d)$.

So putting P in the form of $acr^2 + (ad + bc)r + bd$, we can factorize P to a polynomial in the form of $(ar + b)(cr + d)$.

Thus, finding a, b, c, and d, we can get the factorization of P.

Since $P = 6r^2 - 13r - 28$ takes the form of $acr^2 + (ad + bc)r + bd$, we can set up a system of equations as follows. $ac = 6$, $ad + bc = -13$, and $bd = -28$.

Assuming thus, a, b, c, and d are integers, setting $ac = 6$, $ad + bc = -13$, and $bd = -28$, and then, finding a, b, c, and d, we can get the factorization of P within the scope of integers.

That is to say that a and c are integers the product of which is 6, b and d are integers the product of which is -28, and $ad + bc = -13$.

So $\boldsymbol{a}$ and $\boldsymbol{c}$ are divisors of 6, and $\boldsymbol{b}$ and $\boldsymbol{d}$ are divisors of -28.

And if any, we can get the integers that can be $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ via trial and error.

In fact, we get $\boldsymbol{a = 2}$, $\boldsymbol{c = 3}$, $\boldsymbol{b = -7}$, and $\boldsymbol{d = 4}$, because we can get $\boldsymbol{ac = 6}$, $\boldsymbol{bd = -28}$, and $\boldsymbol{ad + bc = 2 \cdot 4 + (-7 \cdot 3)}$, which is -13.

Now, we have $\boldsymbol{P = 6r^2 - 13r - 28 = acr^2 + (ad + bc)r + bd = (ar + b)(cr + d)}$, in which $\boldsymbol{a = 2}$, $\boldsymbol{c = 3}$, $\boldsymbol{b = -7}$, and $\boldsymbol{d = 4}$.

Thus, we get $\boldsymbol{P = 6r^2 - 13r - 28 = (2r - 7)(3r + 4)}$.

Therefore, $\boldsymbol{P = 6r^2 - 13r - 28}$ is factorized to $\boldsymbol{(2r - 7)(3r + 4)}$.

In short:

$\boldsymbol{6r^2 - 13r - 28 = 2 \cdot 3r^2 + (8 - 21)r + 4 \cdot 7 = 2 \cdot 3r^2 + (2 \cdot 4 - 3 \cdot 7)r + 4 \cdot 7}$

$\boldsymbol{= 2 \cdot 3r^2 - 3 \cdot 7r + 2 \cdot 4r + 4 \cdot 7 = 3r(2r - 7) + 4(2r - 7) = (2r - 7)(3r + 4)}$.

Suggestions or Solutions
To the Problem in the Example 3

We have $\frac{3}{2}\boldsymbol{w}^2 + \mathbf{12}\boldsymbol{w} - \mathbf{30}$.

Factorizing it, we can get it the way below.

$$2P = 2\cdot 3(\tfrac{1}{2}w^2 + 4w - 10) = 3(w^2 + 8w - 20) = 3(w-2)(w+10).$$

Thus, $P = \frac{3}{2}(w-2)(w+10)$.

If not quite sure of the idea behind the processes above, follow the steps below.

Let's set first, $P = \frac{3}{2}w^2 + 12w - 30$.

Unlike the previous ones, this one has is a fractional coefficient.
So what are we going to do about that?

Just get rid of the denominator.   How?

We can't just erase it with eraser, of course.
Multiplying by the denominator both sides of the equal sign, we can get rid of it.
Prior to the multiplication though, we want to take out 3 from the right hand side, first, since 3 is a factor.

Then, we get $2P = 2\cdot 3(\frac{1}{2}w^2 + 4w - 10) = 3(w^2 + 8w - 20) = 3(w-2)(w+10)$.

So we get $P = \frac{3}{2}(w-2)(w+10)$.   And each of ($w - 2$) and ($w + 10$) is prime.

Thus, P is factorized to $\frac{3}{2}(w-2)(w+10)$.

Suggestions or Solutions
To the Problem in the Example 4

We have $\mathbf{0.6}t^2 + \mathbf{14.92}t - \mathbf{2}$.

Factorizing it, we can get it the way below.

To begin with, $0.6t^2 + 14.92t - 2 = 0.01(60t^2 + 1492t - 200) = 0.04(15t^2 + 373t - 50)$.

And we get $15t^2 + 373t - 50 = 1\cdot 15t^2 + (375 - 2)t + 2\cdot 25 = 1\cdot 15t^2 + (15\cdot 25 - 1\cdot 2)t + 2\cdot 25$

$= 1\cdot 15t^2 + 15\cdot 25t - 1\cdot 2t + 2\cdot 25 = 15t(t + 25) - 2(t + 25) = (t + 25)(15t - 2)$.

So we get $0.04(15t^2 + 373t - 50) = 0.04(t + 25)(15t - 2)$, which can be now, taken as the factorization of $0.6t^2 + 14.92t - 2$.

And if necessary, we can factorize the coefficient **0.04**, too. We have $0.04 = 2^2 10^{-2}$.

So we can put it this way, too: $0.6t^2 + 14.92t - 2 = 2^2 10^{-2}(t + 25)(15t - 2)$.

If not quite sure of the idea behind the processes above, follow the steps below.

Let's set first, $P = 0.6t^2 + 14.92t - 2$.
This one has coefficients fractional, and this time, they are decimal fractional.
So what are we going to do about those?

Get rid of them, of course. Multiplying by 100 both sides of the equal sign, we get

$100P = 100(0.6t^2 + 14.92t - 2) = 60t^2 + 1492t - 200$, which can be reduced a bit further to $4(15t^2 + 373t - 50)$. So we now want to factorize $15t^2 + 373t - 50$.

So next, setting $\boldsymbol{Q = 15t^2 + 373t - 50}$, we get $\mathbf{100}\boldsymbol{P} \mathbf{= 4}\boldsymbol{Q}$, and $\boldsymbol{Q}$ is similar to a polynomial $\boldsymbol{act^2 + (ad + bc)t + bd}$, which is factorize to $\boldsymbol{(at + b)(ct + d)}$.

Since $\boldsymbol{Q = 15t^2 + 373t - 50}$ takes the form of $\boldsymbol{act^2 + (ad + bc)t + bd}$, we can set up a system of equations as follows. $\boldsymbol{ac = 15}$, $\boldsymbol{ad + bc = 373}$, and $\boldsymbol{bd = -50}$.

Assuming thus, $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are integers, setting $\boldsymbol{ac = 15}$, $\boldsymbol{ad + bc = 373}$, and $\boldsymbol{bd = -50}$, and then, finding $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$, we can get the factorization of $\boldsymbol{Q}$ within the scope of integers.

That is to say that $\boldsymbol{a}$ and $\boldsymbol{c}$ are integers the product of which is 15, $\boldsymbol{b}$ and $\boldsymbol{d}$ are integers the product of which is -50, and $\boldsymbol{ad + bc = 373}$.

So $\boldsymbol{a}$ and $\boldsymbol{c}$ are divisors of 15, and $\boldsymbol{b}$ and $\boldsymbol{d}$ are divisors of -50.
And if any, we can get the integers that can be $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ via trial and error.

In fact, we get $\boldsymbol{a = 1}$, $\boldsymbol{c = 15}$, $\boldsymbol{b = 25}$, and $\boldsymbol{d = -2}$, because we can get $\boldsymbol{ac = 15}$, $\boldsymbol{bd = -50}$, and $\boldsymbol{ad + bc = 1\cdot(-2) + 25\cdot 15 = -2 + 25(10 + 5) = -2 + 250 + 125 = 373}$.

Now, we have $\boldsymbol{Q = 15t^2 + 373t - 50 = act^2 + (ad + bc)t + bd = (at + b)(ct + d)}$, in which $\boldsymbol{a = 1}$, $\boldsymbol{c = 15}$, $\boldsymbol{b = 25}$, and $\boldsymbol{d = -2}$.

Thus, we get $\boldsymbol{Q = 15t^2 + 373t - 50 = (t + 25)(15t - 2)}$, so $\boldsymbol{100P = 4Q = 4(t + 25)(15t - 2)}$.

And each of 4, $\boldsymbol{t + 25}$, and $\boldsymbol{15t - 2}$ is prime.   And we have $\mathbf{0.04 = 2^2 10^{-2}}$.

Therefore, $\boldsymbol{P = 0.6t^2 + 14.92t - 2}$ is factorized to $\boldsymbol{2^2 10^{-2}(t + 25)(15t - 2)}$.

And of course, we can just put it this way, too:

$\boldsymbol{0.6t^2 + 14.92t - 2 = 0.04(t + 25)(15t - 2)}$.