Examples K in Polynomial Factorizations

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Examples K

0. Do the arithmetic operations below.

0.0. $99 \cdot 33 + 44 \cdot 45$

0.1. $88888888 \cdot 22222222 + 44444444 \cdot 55555556$

0.2. $999999999 \cdot 999999999 + 1999999999$, and

0.3. $\dfrac{123454321}{123454322^2 - 123454321 \cdot 123454323}$

1. Suppose $\boldsymbol{A}$ is a positive integer that has 2100 digits, and every digit is 9. So all the digits in $\boldsymbol{A}$ are 9s as 9999… Suppose also, another integer $\boldsymbol{B}$ has 2101 digits, and in $\boldsymbol{B}$, the highest digit is 1, and all the other digits are all 9s. Then, find the value of $\boldsymbol{A}^2 + \boldsymbol{B}$.

2.0. Show that 1000000001 is a composite integer.

2.1. Suppose $\boldsymbol{A}$ is a positive integer as 1000…001, and has 2323 digits. So in $\boldsymbol{A}$, the highest and the lowest digits are 1, and all the other digits are all 0s. Then, show that $\boldsymbol{A}$ is a composite integer.

Suggestions or Solutions
To the Problems in the Example 0

0.0. $\mathbf{99 \cdot 33 + 44 \cdot 45}$

0.1. $\mathbf{88888888 \cdot 22222222 + 44444444 \cdot 55555556}$

0.2. $\mathbf{999999999 \cdot 999999999 + 1999999999}$**, and**

0.3. $\dfrac{123454321}{123454322^2 - 123454321 \cdot 123454323}$

Let's begin with $99 \cdot 33 + 44 \cdot 45$.
Taking the sum above, we can take many ways. How many though?

There can be infinitely many ways. Of such many, we can choose one, and it all depends on what's most convenient to us.

$99 \cdot 33 + 44 \cdot 45$

$= (90 + 9)33 + (40 + 4)45$

$= (90 + 9)(30 + 3) + (40 + 4)(40 + 5)$

$= 2700 + 270 + 270 + 27 + 1600 + 200 + 160 + 20$

$= 2700 + 270 \cdot 2 + 27 + 1800 + 180$

$= 2700 + 540 + 27 + 1980$

$= 2700 + 300 + 240 + 7 + 20 + 1980$

$= 3000 + 240 + 7 + 2000 = 5247$

$99 \cdot 33 + 44 \cdot 45$

$= (100 - 1)33 + (40 + 4)(40 + 5)$

$= 3300 - 33 + 1600 + 200 + 160 + 20$

$= 3000 + 1000 + 300 + 600 + 200 + 100 + 60 + 20 - 30 - 3$

$= 4000 + 1200 + 60 - 10 - 3$

$= 5200 + 50 - 3 = 5200 + 47 = 5247$

Of course, we can do calculation by heart in some steps presented above.

Next, we have 88888888·22222222 + 44444444·55555556.
Assuming first, ***a* = 11111111**, we get

88888888·22222222 + 44444444·55555556
$= 8a \cdot 2a + 4a(5a + 1)$
$= 4a \cdot 4a + 4a(5a + 1)$
$= 4a(4a + 5a + 1)$
$= 4a(9a + 1)$
$= 44444444 \cdot (99999999 + 1)$
$= 44444444 \cdot 10^8$, which looks better, and is more readable than 4444444400000000.

Next, we have 999999999·999999999 + 1999999999.
To begin with, assuming that ***b* = 999999999**, we get
999999999·999999999 + 1999999999
$= b^2 + 1000000000 + b$
$= b^2 + 10^9 + b$
$= b(b + 1) + 10^9$
$= b10^9 + 10^9$ because $b + 1 = 999999999 + 1 = 10^9$.
$= 10^9(b + 1)$
$= 10^9 10^9$
$= (10^9)^2$
$= 10^{18}$.

And next, we have $\frac{123454321}{123454322^2 - 123454321 \cdot 123454323}$.

Setting first, ***c* = 123454321**, we get

$$\frac{123454321}{123454322^2 - 123454321 \cdot 123454323} = \frac{a}{(a+1)^2 - a(a+2)} = \frac{a}{a^2 + 2a + 1 - a^2 - 2a} = a = 123454321.$$

Suggestions or Solutions
To the Problem in the Example 1

Suppose A is a positive integer that has 2100 digits, and every digit is 9. So its all digits are 9s as 9999… Suppose also, another integer B has 2101 digits, the highest digit in it is 1, and all the other digits are all 9s. Then, find the value of $A^2 + B$.

To begin with, $B = 10^{2100} + A$.

So we get $A^2 + B = A^2 + 10^{2100} + A = A(A + 1) + 10^{2100}$.

Meanwhile, $A + 1 = 10^{2100}$.

So we get

$$A^2 + B = A \cdot 10^{2100} + 10^{2100} = 10^{2100}(A + 1) = 10^{2100}10^{2100} = (10^{2100})^2 = 10^{4200}.$$

If not quite sure of the idea behind the processes above, follow the steps below.

Let's begin with B.

The highest digit in the integer B is 1, and B has one more digit than A, where all the digits are **9**s.

So B is 1999…9, where there are two thousand and one hundred 9s.

So we can put B in terms of A in such a way as follows. $B = 10^{2100} + A$.

Thus, we get $A^2 + B = A^2 + 10^{2100} + A = A(A + 1) + 10^{2100}$.

Meanwhile, $A + 1 = 10^{2100}$, since $A = 999…999$, which has 2100 of 9s.

How come?

We know $\boldsymbol{A}$ has 2100 of 9s, so if we add 1 to $\boldsymbol{A}$, all of 2100 of 9s will become 2100 of zeros, and the highest digit of the sum ($\boldsymbol{A} + \mathbf{1}$) is 1.

Therefore, $\boldsymbol{A} + \mathbf{1} = \mathbf{10^{2100}}$, which has 2100 zeros.

So we get $\boldsymbol{A^2 + B = A \cdot 10^{2100} + 10^{2100} = 10^{2100}(A + 1) = 10^{2100}10^{2100} = (10^{2100})^2 = 10^{4200}}$.

Note that 10^{4200} has 4200 zeros.

In short:

To begin with, $\boldsymbol{B = 10^{2100} + A}$.

So we get $\boldsymbol{A^2 + B = A^2 + 10^{2100} + A = A(A + 1) + 10^{2100}}$.

Meanwhile, $\boldsymbol{A + 1 = 10^{2100}}$.

So we get

$$\boldsymbol{A^2 + B = A \cdot 10^{2100} + 10^{2100} = 10^{2100}(A + 1) = 10^{2100}10^{2100} = (10^{2100})^2 = 10^{4200}}.$$

Suggestions or Solutions
To the **Problems** in the Example **2**

2.0. Show that 1000000001 is a composite integer.

2.1. Suppose *A* is a positive integer as 1000…001, and has 2323 digits. So in *A*, the highest and the lowest digits are 1, and all the other digits are all 0s. Then, show that *A* is a composite integer.

Saying composite numbers or prime numbers, we normally mean integers, so we mean integers composite or prime.

And just saying primes or composite numbers, they are assumed to be positive integers.

What then, about -3 and -4?

-3 is a negative prime, and -4 is a negative composite integer.

A composite integer has to be a product of two different integers that are neither 1 nor the integer itself. So composite integers are positive integers except 1 and primes.

2.0. We have $\mathbf{1000000001 = 1000000000 + 1 = 10^9 + 1 = (10^3)^3 + 1^3}$.

Also, we have $\boldsymbol{a^3 + b^3 = (a + b)(a^2 - ab + b^2)}$.

So we get $\mathbf{(10^3)^3 + 1^3 = (10^3 + 1)((10^3)^2 - 10^3 \cdot 1 + 1^2) = 1001(10^6 - 10^3 + 1)}$.

We know **1001** and $\mathbf{10^6 - 10^3 + 1}$ both are integers.

So 1000000001 is a product of two different integers that are neither 1 nor 1000000001, and thus, is a composite integer.

2.1. Suppose *A* is a positive integer as 1000…001, and has 2323 digits. So in *A*, the highest and the lowest digits are 1, and all the other digits are all 0s. Then, show that *A* is a composite integer.

To begin with, ***A*** has 2,323 digits and the highest and the lowest digit are 1s, and all of the other digits are all 0s.

So ***A*** looks like 1000000......001, and more specifically,

$\boldsymbol{A} = \mathbf{1 + 1000\ldots000}$ that has 2322 zeros.

Thus, $\boldsymbol{A} = \mathbf{1000\ldots001}$, and there are 2321 zeros between the first 1 and the last 1.

So we can set $\boldsymbol{A} = \mathbf{10^{2322} + 1}$.

Now, since $\mathbf{2322 = 3 \cdot 774}$, we get $\boldsymbol{A} = \mathbf{10^{3 \cdot 774} + 1 = (10^{774})^3 + 1}$.

So assuming that $\boldsymbol{B} = \mathbf{10^{774}}$, we get

$$\boldsymbol{A} = \boldsymbol{B}^3 + 1^3 = (\boldsymbol{B} + 1)(\boldsymbol{B}^2 - \boldsymbol{B} \cdot 1 + 1^2) = (\boldsymbol{B} + 1)(\boldsymbol{B}^2 - \boldsymbol{B} + 1)$$

$$= (10^{774} + 1)(10^{774 \cdot 2} - 10^{774} + 1) = (10^{774} + 1)(10^{1548} - 10^{774} + 1).$$

We know $\mathbf{10^{774} + 1}$ and $\mathbf{10^{1548} - 10^{774} + 1}$ are integers.

So ***A*** is a product of two different integers that are neither 1 nor itself, and therefore, is a composite integer.