Examples 2 in GCD and LCM

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Find the GCD (Greatest Common Divisor) and LCM (Least Common Multiple) in each of the cases below.

0. $\mathbf{16}\boldsymbol{x}^{\mathbf{2}}\boldsymbol{y}^{\mathbf{4}}\boldsymbol{z}^{\mathbf{7}}$ and $\boldsymbol{x}^{\mathbf{5}}\mathbf{(}\boldsymbol{x}\mathbf{+}\boldsymbol{y}\mathbf{)}^{\mathbf{2}}\mathbf{(}\boldsymbol{x}\mathbf{+}\boldsymbol{z}\mathbf{)}^{\mathbf{8}}$

1. $\mathbf{81}$, $\mathbf{16}\boldsymbol{y}^{\mathbf{4}}\boldsymbol{z}^{\mathbf{7}}$, and $\boldsymbol{x}^{\mathbf{5}}\mathbf{(}\boldsymbol{x}\mathbf{+}\boldsymbol{y}\mathbf{)}^{\mathbf{2}}\mathbf{(}\boldsymbol{x}\mathbf{+}\boldsymbol{z}\mathbf{)}^{\mathbf{8}}$

2. $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$

3. $\boldsymbol{x}^{\mathbf{2}}\mathbf{+3}\boldsymbol{x}\mathbf{+2}$ and $\boldsymbol{x}^{\mathbf{2}}\mathbf{-1}$

4. $\mathbf{(}\boldsymbol{x}\mathbf{+1)}^{\mathbf{2}}\mathbf{(}\boldsymbol{x}\mathbf{+2)(}\boldsymbol{x}\mathbf{+3)}^{\mathbf{2}}$, $\boldsymbol{x}\mathbf{(}\boldsymbol{x}\mathbf{-1)(}\boldsymbol{x}\mathbf{+1)}^{\mathbf{4}}\mathbf{(}\boldsymbol{x}\mathbf{+3)}^{\mathbf{3}}$, and $\mathbf{(}\boldsymbol{x}\mathbf{+2)}^{\mathbf{3}}\mathbf{(}\boldsymbol{x}\mathbf{+1)}\boldsymbol{x}^{\mathbf{3}}\mathbf{(}\boldsymbol{x}\mathbf{+3)}^{\mathbf{3}}$

Suggestions or Solutions
To the Problem in the Example 0

Find the GCD and LCM of 16$x^2y^4z^7$ and $x^5(x+y)^2(x+z)^8$.

• A GCD is a divisor common and the largest.

So the GCD of $\mathbf{16x^2y^4z^7}$ and $\mathbf{x^5(x+y)^2(x+z)^8}$ is the greatest of all divisors common to $\mathbf{16x^2y^4z^7}$ and $\mathbf{x^5(x+y)^2(x+z)^8}$.

And getting the GCD, we get the product of all powers, in each of which the base is a prime factor common to both expressions above, and the exponent is the smallest of all used for the prime factor.

• So beginning with getting $\mathbf{16x^2y^4z^7}$ fully factorized, we get $\mathbf{2^4x^2y^4z^7}$.

• Next, finding all the prime factors common to $\mathbf{2^4x^2y^4z^7}$ and $\mathbf{x^5(x+y)^2(x+z)^8}$, we get $\mathbf{x}$ only.

Thus, in this case, too, we get one power only where the base is a prime factor common to $\mathbf{2^4x^2y^4z^7}$ and $\mathbf{x^5(x+y)^2(x+z)^8}$, so we don't need to take such a product as stated above.

Thus, the GCD in this case is just a power, where the base is $\mathbf{x}$, which is the prime factor, and the exponent is the smallest of all exponents used for the prime factor $\mathbf{x}$.

The smallest exponent is 2. So the GCD is $\mathbf{x^2}$.

Let's next, move on to the LCM, which is a multiple common and the smallest.

To begin with, for instance, a common multiple of a set of integers has <u>at least</u> all the prime factors all the integers have, and the exponent to be used for each prime factor is <u>at least</u> the largest of all used for the prime factor.

So the LCM, the multiple the <u>least</u> and common to all the integers has *all prime factors* all the integers have, and the *exponent* to be used for each prime factor is *the largest* of all used for the prime factor.

Now, in the case of $\boldsymbol{x^5(x+y)^2(x+z)^8}$ and $\boldsymbol{16x^2y^4z^7}$, which is factorized to $\boldsymbol{2^4x^2y^4z^7}$, all the prime factors used in both are $\mathbf{2}$, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{x+y}$, and $\boldsymbol{x+z}$.

4 is the largest exponent used for the factor 2,

5 is the largest exponent used for the factor $\boldsymbol{x}$,

4 is the largest exponent used for $\boldsymbol{y}$,

7 is the largest exponent used for $\boldsymbol{z}$,

2 is the largest exponent used for $\boldsymbol{x+y}$,

and 8 is the largest exponent used for $\boldsymbol{x+z}$.

So a multiple common to $\boldsymbol{x^5(x+y)^2(x+z)^8}$ and $\boldsymbol{2^4x^2y^4z^7}$ has as prime factors <u>at least</u> $\mathbf{2}$, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{x+y}$, and $\boldsymbol{x+z}$, and the exponents to be used for the prime factors are <u>at least</u> 4, 5, 4, 7, 2 and 8 respectively.

Therefore, the LCM is $\boldsymbol{2^4x^5y^4z^7(x+y)^2(x+z)^8}$.

Suggestions or Solutions
To the Problem in the Example 1

Find the GCD and LCM of 81, $16y^4z^7$, and $x^5(x + y)^2(x + z)^8$.

GCD is a divisor common and the largest.

So the GCD of the three, $\boldsymbol{x^5(x + y)^2(x + z)^8}$, $\boldsymbol{16y^4z^7}$, and **81** is the greatest of all the divisors common to all the three.

> And getting the GCD, we get the product of all powers, in each of which the base is a prime factor common to all the three above, and the exponent is the smallest of all used for the prime factor.

• So we want to begin with factorizing **81** and $\boldsymbol{16y^4z^7}$. Then, we get $\boldsymbol{3^4}$ and $\boldsymbol{2^4y^4z^7}$.

Thus, we are getting the GCD of $\boldsymbol{3^4}$, $\boldsymbol{2^4y^4z^7}$, and $\boldsymbol{x^5(x + y)^2(x + z)^8}$.

• Next, finding all prime factors common to all the three above, we get none this time.

Thus in this case, we get no power where the base is a prime factor common to all the three above, so we don't need to take such a product as mentioned above.

What is the GCD, though?

There is a divisor that can divide any value no matter what the value may be, and the divisor is 1, and no other can do so. Thus in this case, 1 is the GCD.

Let's next, move on to the LCM, which is a multiple common and the smallest.

A multiple common to a set of objects as integers has at least all the prime factors all the objects have, and the exponent to be used for each prime factor is at least the largest of all used for the prime factor.

So the LCM, the multiple the least and common to all the objects has *all prime factors* all the objects have, and the *exponent* to be used for each prime factor is *the largest* of all used for the prime factor.

Now, in the case of $\mathbf{3^4}$, $\mathbf{2^4}\boldsymbol{y^4z^7}$, and $\boldsymbol{x^5(x+y)^2(x+z)^8}$, all prime factors used are **2**, **3**, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{x+y}$, and $\boldsymbol{x+z}$,

4 is the largest exponent used for each of 2 and 3,

5 is the largest exponent used for $\boldsymbol{x}$,

4 is the largest exponent used for $\boldsymbol{y}$,

7 is the largest exponent used for $\boldsymbol{z}$,

2 is the largest exponent used for $\boldsymbol{x+y}$,

and 8 is the largest exponent used for $\boldsymbol{x+z}$.

So a multiple common to the three above has as prime factors at least **2**, **3**, $\boldsymbol{x}$, $\boldsymbol{y}$, $\boldsymbol{z}$, $\boldsymbol{x+y}$, and $\boldsymbol{x+z}$, and the exponents to be used for the prime factors are at least 4, 4, 5, 4, 7, 2, and 8 respectively.

Therefore, the LCM is $\boldsymbol{2^4 3^4 x^5 y^4 z^7 (x+y)^2 (x+z)^8}$.

Suggestions or Solutions
To the Problem in the Example 2

Find the GCD and LCM of *x*, *y*, and *z*.

GCD is a divisor common and the largest. So the GCD of the three monomials given in the problem is the greatest of all the divisors common to all the three.

To begin with, we want to find all the prime factors common to all the three monomials. This time however, all the three monomials themselves are all prime.
What then, is the GCD?

Every object except 0 and 1 has at least two divisors, which are 1 and itself.
In particular, 1 is a divisor common to every object.
So what are divisors common to $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$?

It is 1 only, and therefore, the GCD of $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ is 1.

Let's next, move on to the LCM, which is a multiple common and the smallest.

A common multiple of a set of monomials has <u>at least</u> all the prime factors all the monomials have, and the exponent to be used for each prime factor is <u>at least</u> the largest of all used for the prime factor.

So the LCM, the multiple the <u>least</u> and common to a set of monomials has *all prime factors* all the monomials have, and the *exponent* to be used for each prime factor is *the largest* of all used for the prime factor.

Now, in this case, the three monomials $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ themselves are all prime, 1 is the largest exponent used for each of $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$.

Therefore, a multiple common to $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ has as prime factors <u>at least</u> $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$, and the exponent to be used for each of the prime factors is <u>at least</u> 1.

Thus, the LCM of $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ is $\boldsymbol{xyz}$.

Suggestions or Solutions
To the Problem in the Example 3

Find the GCD and LCM of $x^2 + 3x + 2$ and $x^2 - 1$.

GCD is a divisor common and the largest. So the GCD of a set of polynomials is the greatest of all the divisors common to all the polynomials.

> And getting the GCD, we get the product of all powers, in each of which the base is a prime factor common to all the polynomials, and the exponent is the smallest of all used for the prime factor.

• So we want to begin with factorizing the two polynomials $\boldsymbol{x^2 + 3x + 2}$ and $\boldsymbol{x^2 - 1}$.

Then, we get $\boldsymbol{x^2 + 3x + 2 = (x + 1)(x + 2)}$, and $\boldsymbol{x^2 - 1 = (x + 1)(x - 1)}$.

• Next, finding all the prime factors common to both polynomials, we get $\boldsymbol{x + 1}$.

• Next, finding the smallest of all exponents used for the prime factor above, we get 1.

Thus, in this case, we get only one power where the base is a prime factor common to both polynomials, so we don't need to take such a product as stated above.

Thus, the GCD is just a power, where the base is $\boldsymbol{x + 1}$, which is the prime factor, and the exponent is the smallest of all exponents used for the prime factor $\boldsymbol{x + 1}$.

The smallest exponent is 1. So the GCD is $\boldsymbol{x + 1}$.

Let's next, move on to the LCM, which is a multiple common and the smallest.

A common multiple of a set of polynomials has <u>at least</u> all the prime factors all the polynomials have, and the exponent to be used for each prime factor is <u>at least</u> the largest of all used for the prime factor.

So the LCM, the multiple the <u>least</u> and common to all the polynomials has *all prime factors* all the polynomials have, and the *exponent* to be used for each prime factor is *the largest* of all used for the prime factor.

Now, in the case of two polynomials $\boldsymbol{x^2 + 3x + 2}$ and $\boldsymbol{x^2 - 1}$, factorized to $\boldsymbol{(x + 1)(x + 2)}$ and $\boldsymbol{(x + 1)(x - 1)}$, all prime factors used are $\boldsymbol{x + 1}$, $\boldsymbol{x + 2}$, and $\boldsymbol{x - 1}$.

And 1 is the largest exponent used for each of all the prime factors.

So a multiple common to both polynomials has as prime factors <u>at least</u> $\boldsymbol{x + 1}$, $\boldsymbol{x + 2}$, and $\boldsymbol{x - 1}$, and the exponent to be used for each of the prime factors is <u>at least</u> 1.

Thus, the LCM is $\boldsymbol{(x + 1)(x + 2)(x - 1)}$.

Suggestions or Solutions
To the Problem in the Example 4

Find the GCD and LCM of polynomials below.
$(x + 1)^2(x + 2)(x + 3)^2$, $x(x - 1)(x + 1)^4(x + 3)^3$, **and** $(x + 2)^3(x + 1)x^3(x + 3)^3$.

The GCD of a set of polynomials is the greatest divisor common to all the polynomials.

> And getting the GCD, we get the product of all powers, in each of which the base is a prime factor common to all the polynomials, and the exponent is the smallest of all used for the prime factor.

Now, we want to find the GCD of three polynomials, which are fully factorized already. So we are ready to get the GCD.

• First, finding all the prime factors common to the three polynomials, we get

$x + 1$ and $x + 3$.

• Next, we take the product of all the prime factors above, and apply the smallest of all the exponents used for each of the prime factors.

Then, 1 is the smallest of all the exponents used for the prime factor $x + 1$, and 2 is the smallest used for the prime factor $x + 3$, so we get $(x + 1)(x + 3)^2$, which is the GCD.

Let's next, move on to the LCM, which is a multiple common and the smallest.

A multiple common to a set of polynomials has <u>at least</u> all the prime factors all the polynomials have, and the exponent to be used for each prime factor is <u>at least</u> the largest of all used for the prime factor.

Therefore, the LCM, the <u>least</u> multiple common to a set of polynomials has *all prime factors* all the polynomials have, and the *exponent* to be used for each prime factor is *the largest* of all used for the prime factor.

Now, in the case of three polynomials $\mathbf{(x + 1)^2(x + 2)(x + 3)^2}$, $\mathbf{x(x - 1)(x + 1)^4(x + 3)^3}$, and $\mathbf{(x + 2)^3(x + 1)x^3(x + 3)^3}$, all prime factors all the three polynomials have are $\mathbf{x + 1}$, $\mathbf{x + 2}$, $\mathbf{x + 3}$, $\mathbf{x}$, and $\mathbf{x - 1}$.

4 is the largest exponent used for $\mathbf{x + 1}$,

3 is the largest exponent used for each of $\mathbf{x + 2}$ and $\mathbf{x + 3}$,

and 1 is the largest exponent used for each of $\mathbf{x}$ and $\mathbf{x - 1}$.

So a multiple common to the three polynomials has as prime factors <u>at least</u> $\mathbf{x + 1}$, $\mathbf{x + 2}$, $\mathbf{x + 3}$, $\mathbf{x}$, and $\mathbf{x - 1}$, and the exponents to be used for the prime factors are <u>at least</u> 4, 3, 3, 1, and 1 respectively.

Therefore, the LCM of the three polynomials is $\mathbf{x(x + 1)^4(x + 2)^3(x + 3)^3(x - 1)}$.