Examples 2 on Definitions for Logarithms

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Find the value of x in each case below.

0. $x = \log_3 81$

1. $x = \log_{3\sqrt{2}} 324$

2. $x = \log_3 9\sqrt{3}$

3. $x = \log_{0.2} 125$

4. $x = \log_{0.4} 15.625$

5. $x = \log_{0.3} 0.027\sqrt{0.3}$

6. $x = \log_{16}(\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}})$

7. $x = \log_{12}(\sqrt{5+2\sqrt{6}} + \sqrt{5-2\sqrt{6}})$

8. $x = \log_4(\sqrt{3+\sqrt{5}} - \sqrt{3-\sqrt{5}})$

9. $\log_x 3\sqrt{3} = \frac{2}{3}$

A. $\log_{\sqrt{x}} 3\sqrt{3} = \frac{5}{2}$

B. $\log_{\sqrt{x}} 2\sqrt{3} = -\frac{1}{2}$

C. $\log_x 125 = 5$

D. $\log_{\sqrt{x}} 15.625 = -3$

E. $x = \log_{0.5} 8$

F. $\log_2 3x = 2$

G. $\log_3 3x = 9$

H. $\log_2 (\log_3 x) = 1$

I. $\log_{0.2} (\log_{0.3} x) = 2$

J. $\log_{12} (\log_3 x^2) = 3$

K. $\log_3 (\log_{0.2} x^2) = -2$

L. $\log_{0.2} (\log_{12} x^{-2}) = 3$

M. $\log_2 (\log_x 16) = 3$

N. $\log_2 (\log_{\sqrt{x}} 8) = -3$

O. $\log_2 (\log_{x^2} 3\sqrt{3}) = -2$

P. $\log_2 (\log_{x^{0.3}} 4) = -1$

Q. $\log_2 \{\log_3 (\log_5 x)\} = 2$

Suggestions or Solutions
To the Examples 2 on Definitions for Logarithms

The definition for logs is $\boldsymbol{A = b^x \Leftrightarrow x = \log_b A}$.

So using the definition, we can get the solutions the way below.

0. $\boldsymbol{x = \log_3 81 \Leftrightarrow 81 = 3^x}$. And we have $81 = 3^4$. So we get $\boldsymbol{x = 4}$.

1. $\boldsymbol{x = \log_{3\sqrt{2}} 324 \Leftrightarrow 324 = (3\sqrt{2})^x}$.

And we have $324 = 2 \cdot 162 = 2 \cdot 2 \cdot 81 = 2^2 3^4 = (\sqrt{2})^4 3^4 = (3\sqrt{2})^4$. So we get $\boldsymbol{x = 4}$.

2. $\boldsymbol{x = \log_3 9\sqrt{3} \Leftrightarrow 9\sqrt{3} = 3^x}$. And we have $\boldsymbol{9\sqrt{3} = 3^2 3^{0.5} = 3^{2.5}}$. So we get $\boldsymbol{x = 2.5}$.

3. $\boldsymbol{x = \log_{0.2} 125 \Leftrightarrow 125 = 0.2^x}$. And we have $0.2 = 1/5 = 5^{-1}$, and $125 = 5 \cdot 25 = 5^3$.

So we get $\boldsymbol{125 = 0.2^x \Rightarrow 5^3 = (5^{-1})^x = 5^{-x} \Rightarrow 5^3 = 5^{-x} \Rightarrow x = -3}$.

4. $\boldsymbol{x} = \mathbf{log_{0.4}\,15.625} \Leftrightarrow \mathbf{15.625 = 0.4}^{\boldsymbol{x}}$.

And we have $0.4 = 2/5 = 2{\cdot}5^{-1}$, and $15.625 = 15625{\cdot}10^{-3}$.

So we get $\mathbf{0.4}^{\boldsymbol{x}} = \mathbf{(2{\cdot}5^{-1})}^{\boldsymbol{x}}$.

And we have $15625 = 5{\cdot}3125 = 5^2{\cdot}625 = 5^2{\cdot}25^2 = 5^6$, and $10^{-3} = 2^{-3}5^{-3}$.

So we get $15.625 = 15625{\cdot}10^{-3} = 5^6 2^{-3}5^{-3} = 2^{-3}5^3 = (2^3 5^{-3})^{-1} = (2{\cdot}5^{-1})^{-3}$.

Thus, we get $\mathbf{15.625 = 0.4}^{\boldsymbol{x}} \Rightarrow \mathbf{(2{\cdot}5^{-1})^{-3}} = \mathbf{(2{\cdot}5^{-1})}^{\boldsymbol{x}} \Rightarrow \boldsymbol{x} = \mathbf{-3}$.

5. $\boldsymbol{x} = \mathbf{log_{0.3}\,0.027\sqrt{0.3}} \Leftrightarrow \mathbf{0.027\sqrt{0.3} = 0.3}^{\boldsymbol{x}}$.

And we have $\mathbf{0.027\sqrt{0.3} = 0.3^3 0.3^{0.5} = 0.3^{3.5}}$. So we get $\boldsymbol{x} = \mathbf{3.5}$.

6. $\boldsymbol{x} = \mathbf{log_{16}(\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}})} \Leftrightarrow \mathbf{\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}} = 16}^{\boldsymbol{x}}$.

And we have $7+4\sqrt{3} = 7+2\sqrt{4\cdot 3} = 4+3+2\sqrt{4\cdot 3} = (\sqrt{4}+\sqrt{3})^2 = (2+\sqrt{3})^2$.

So by the same token, we get $7-4\sqrt{3} = 7-2\sqrt{4\cdot 3} = 4+3-2\sqrt{4\cdot 3} = (2-\sqrt{3})^2$.

Thus, we get $\sqrt{7+4\sqrt{3}} + \sqrt{7-4\sqrt{3}} = 2+\sqrt{3}+2-\sqrt{3} = 4$.

So we get $4 = 16^x = 4^{2x} \Rightarrow 2x = 1 \Rightarrow x = 1/2$.

7. $\boldsymbol{x = \log_{12}(\sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}}) \Leftrightarrow \sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}} = 12^x}$.

And we have $5+2\sqrt{6} = 5+2\sqrt{3\cdot 2} = 3+2+2\sqrt{3\cdot 2} = (\sqrt{3}+\sqrt{2})^2$.

By the same token, we get $5-2\sqrt{6} = 5-2\sqrt{3\cdot 2} = 3+2-2\sqrt{3\cdot 2} = (\sqrt{3}-\sqrt{2})^2$.

So we get $\sqrt{5+2\sqrt{6}}+\sqrt{5-2\sqrt{6}} = \sqrt{3}+\sqrt{2}+\sqrt{3}-\sqrt{2} = 2\sqrt{3}$.

Thus, we get $\boldsymbol{2\sqrt{3} = 12^x}$. And we have $2\sqrt{3} = \sqrt{12} = 12^{0.5}$.

So we get $\boldsymbol{12^{0.5} = 12^x \Rightarrow x = 0.5}$.

8. $\boldsymbol{x = \log_{4}(\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}}) \Leftrightarrow \sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}} = 4^x}$.

And we have $3+\sqrt{5} = \frac{1}{2}(6+2\sqrt{5}) = \frac{1}{2}(5+1+2\sqrt{5\cdot 1}) = \frac{1}{2}(\sqrt{5}+1)^2$.

By the same token, we get $3-\sqrt{5} = \frac{1}{2}(6-2\sqrt{5}) = \frac{1}{2}(5+1-2\sqrt{5\cdot 1}) = \frac{1}{2}(\sqrt{5}-1)^2$.

So we get $\sqrt{3+\sqrt{5}}-\sqrt{3-\sqrt{5}} = \frac{1}{\sqrt{2}}\{(\sqrt{5}+1)-(\sqrt{5}-1)\} = \frac{2}{\sqrt{2}} = \sqrt{2} = 2^{0.5}$.

Thus, we get $\boldsymbol{2^{0.5} = 4^x}$, which is $\boldsymbol{2^{2x}}$. So we get $\boldsymbol{2x = 0.5 \Rightarrow x = 1/4}$.

9. $\boldsymbol{\log_x 3\sqrt{3} = \frac{2}{3} \Leftrightarrow 3\sqrt{3} = x^{\frac{2}{3}}}$.

And we have $\boldsymbol{3\sqrt{3} = 3^{\frac{3}{2}} = 3^{\frac{3}{2}\cdot 1} = 3^{\frac{3}{2}\cdot\frac{3}{2}\cdot\frac{2}{3}} = 3^{\frac{9}{4}\cdot\frac{2}{3}} = (3^{\frac{9}{4}})^{\frac{2}{3}}}$.

So we get $\boldsymbol{3\sqrt{3} = x^{\frac{2}{3}} \Rightarrow (3^{\frac{9}{4}})^{\frac{2}{3}} = x^{\frac{2}{3}} \Rightarrow x = 3^{\frac{9}{4}}}$.

A. $\mathbf{\log_{\sqrt{x}} 3\sqrt{3} = \frac{5}{2} \Leftrightarrow 3\sqrt{3} = (\sqrt{x})^{\frac{5}{2}}}$.

And we have $\mathbf{(\sqrt{x})^{\frac{5}{2}} = (x^{\frac{1}{2}})^{\frac{5}{2}} = x^{\frac{5}{4}}}$. Also, we get $\mathbf{3\sqrt{3} = 3^{\frac{3}{2}} = 3^{\frac{3}{2}\cdot 1} = 3^{\frac{3}{2}\cdot\frac{4}{5}\cdot\frac{5}{4}} = (3^{\frac{6}{5}})^{\frac{5}{4}}}$.

So we get $\mathbf{(3^{\frac{6}{5}})^{\frac{5}{4}} = x^{\frac{5}{4}} \Rightarrow x = 3^{\frac{6}{5}}}$.

B. $\mathbf{\log_{\sqrt{x}} 2\sqrt{3} = -\frac{1}{2} \Leftrightarrow 2\sqrt{3} = (\sqrt{x})^{-\frac{1}{2}} = x^{-\frac{1}{4}}}$.

And we have $\mathbf{2\sqrt{3} = \sqrt{12} = 12^{\frac{1}{2}} = (12^{2})^{\frac{1}{4}} = (12^{-2})^{-\frac{1}{4}}}$.

So we get $\mathbf{(12^{-2})^{-\frac{1}{4}} = x^{-\frac{1}{4}} \Rightarrow x = 12^{-2}}$.

C. $\mathbf{\log_{x} 125 = 5 \Leftrightarrow 125 = x^{5}}$.

And we have $125 = 5^{3} = 5^{3\cdot 1} = 5^{3\cdot\frac{1}{5}\cdot 5} = 5^{\frac{3}{5}\cdot 5} = (5^{\frac{3}{5}})^{5}$.

So we get $(5^{\frac{3}{5}})^{5} = x^{5} \Rightarrow x = 5^{\frac{3}{5}}$.

D. $\mathbf{\log_{\sqrt{x}} 15.625 = -3 \Leftrightarrow 15.625 = (\sqrt{x})^{-3} = x^{-\frac{3}{2}}}$. And we have

$15.625 = 15625\cdot 10^{-3} = 5^{6}10^{-3} = 5^{6}(5\cdot 2)^{-3} = 5^{3}2^{-3}$

$\mathbf{= (\frac{5}{2})^{3} = (\frac{5}{2})^{3\cdot 1} = (\frac{5}{2})^{3\cdot(-\frac{1}{2})(-2)} = (\frac{5}{2})^{(-\frac{3}{2})(-2)} = (\frac{5}{2})^{(-2)(-\frac{3}{2})} = \{(\frac{5}{2})^{(-2)}\}^{-\frac{3}{2}} = (\frac{4}{25})^{-\frac{3}{2}}}$.

So we get $\mathbf{(\frac{4}{25})^{-\frac{3}{2}} = x^{-\frac{3}{2}} \Rightarrow x = \frac{4}{25}}$.

E. $\mathbf{x = \log_{0.5} 8 \Leftrightarrow 8 = x^{0.5}}$. And we have $8 = 2^{3} = 2^{6\cdot 0.5} = (2^{6})^{0.5}$.

So we get $\mathbf{8 = x^{0.5} \Rightarrow (2^{6})^{0.5} = x^{0.5} \Rightarrow x = 2^{6}}$.

F. $\mathbf{log_2 3}\boldsymbol{x}\mathbf{ = 2 \Leftrightarrow 3}\boldsymbol{x}\mathbf{ = 2^2 = 4}$. So we get $\mathbf{3}\boldsymbol{x}\mathbf{ = 4 \Rightarrow }\boldsymbol{x}\mathbf{ = 4/3}$.

G. $\mathbf{log_3 3}\boldsymbol{x}\mathbf{ = 9 \Leftrightarrow 3}\boldsymbol{x}\mathbf{ = 3^9}$. So we get $\boldsymbol{x}\mathbf{ = 3^9/3 = 3^{9-1} = 3^8}$.

H. $\mathbf{log_2 (log_3}\,\boldsymbol{x}\mathbf{) = 1 \Leftrightarrow log_3}\,\boldsymbol{x}\mathbf{ = 2^1 = 2}$.

And by the definition again, we can get $\mathbf{log_3}\,\boldsymbol{x}\mathbf{ = 2 \Leftrightarrow }\boldsymbol{x}\mathbf{ = 3^2}$.

I. $\mathbf{log_{0.2} (log_{0.3}}\,\boldsymbol{x}\mathbf{) = 2 \Leftrightarrow log_{0.3}}\,\boldsymbol{x}\mathbf{ = 0.2^2 = 0.04 \Leftrightarrow }\boldsymbol{x}\mathbf{ = 0.3^{0.04}}$.

J. $\mathbf{log_{12} (log_3}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{) = 3 \Leftrightarrow log_3}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 12^3}$.

And by the definition again, we can get $\mathbf{log_3}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 12^3 \Leftrightarrow }\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 3^{12^3}}$.

And we have $\mathbf{3^{12^3} = 3^{2\cdot\frac{1}{2}\cdot 12^3} = (3^{\frac{12^3}{2}})^2}$. Thus, we get $\boldsymbol{x}\mathbf{ = 3^{\frac{12^3}{2}}}$.

K. $\mathbf{log_3 (log_{0.2}}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{) = -2 \Leftrightarrow log_{0.2}}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 3^{-2} = 1/9}$.

And by the definition again, we can get $\mathbf{log_{0.2}}\,\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 1/9 \Leftrightarrow }\boldsymbol{x}^{\mathbf{2}}\mathbf{ = 0.2^{\frac{1}{9}} = 0.2^{\frac{2}{18}} = (0.2^{\frac{1}{18}})^2}$.

Thus, we get $\boldsymbol{x}\mathbf{ = 0.2^{\frac{1}{18}}}$.

L. $\mathbf{log_{0.2} (log_{12}}\,\boldsymbol{x}^{\mathbf{-2}}\mathbf{) = 3 \Leftrightarrow log_{12}}\,\boldsymbol{x}^{\mathbf{-2}}\mathbf{ = 0.2^3 = 0.008}$. So we get $\mathbf{log_{12}}\,\boldsymbol{x}^{\mathbf{-2}}\mathbf{ = 0.008}$.

And by the definition, we get $\mathbf{log_{12}}\,\boldsymbol{x}^{\mathbf{-2}}\mathbf{ = 0.008 \Leftrightarrow }\boldsymbol{x}^{\mathbf{-2}}\mathbf{ = 12^{0.008} = 12^{\frac{0.008}{-2}\cdot(-2)} = (12^{\frac{0.008}{-2}})^{-2}}$.

Thus, we get $\boldsymbol{x}\mathbf{ = 12^{-0.004}}$.

M. $\log_2(\log_x 16) = 3 \Leftrightarrow \log_x 16 = 2^3 = 8$.

And by the definition again, we can get $\log_x 16 = 8 \Leftrightarrow 16 = x^8$.

And we have $16 = 2^4 = (2^{0.5})^8$. So we get $x = 2^{0.5} = \sqrt{2}$.

N. $\log_2(\log_{\sqrt{x}} 8) = -3 \Leftrightarrow \log_{\sqrt{x}} 8 = 2^{-3} = \frac{1}{8}$.

And by the definition again, we can get $\log_{\sqrt{x}} 8 = \frac{1}{8} \Leftrightarrow 8 = (\sqrt{x})^{\frac{1}{8}} = x^{\frac{1}{16}}$.

And we have $8 = 2^3 = 2^{3 \cdot 16 \cdot \frac{1}{16}} = 2^{48 \cdot \frac{1}{16}} = (2^{48})^{\frac{1}{16}}$. So we get $x = 2^{48}$.

O. $\log_2(\log_{x^2} 3\sqrt{3}) = -2 \Leftrightarrow \log_{x^2} 3\sqrt{3} = 2^{-2} = \frac{1}{4}$.

And by the definition again, we can get $\log_{x^2} 3\sqrt{3} = \frac{1}{4} \Leftrightarrow 3\sqrt{3} = (x^2)^{\frac{1}{4}} = x^{\frac{1}{2}}$.

And we have $3\sqrt{3} = 3^{\frac{3}{2}} = (3^3)^{\frac{1}{2}}$. So we get $x = 3^3$.

P. $\log_2(\log_{x^{0.3}} 4) = -1 \Leftrightarrow \log_{x^{0.3}} 4 = 2^{-1} = \frac{1}{2} \Leftrightarrow 4 = (x^{0.3})^{\frac{1}{2}} = x^{\frac{0.3}{2}} = x^{\frac{3}{20}}$.

And we have $4 = 2^2 = 2^{2 \cdot \frac{20}{3} \cdot \frac{3}{20}} = 2^{\frac{40}{3} \cdot \frac{3}{20}} = (2^{\frac{40}{3}})^{\frac{3}{20}}$. So we get $x = 2^{\frac{40}{3}}$.

Q. $\log_2\{\log_3(\log_5 x)\} = 2 \Leftrightarrow \log_3(\log_5 x) = 2^2 = 4 \Leftrightarrow \log_5 x = 3^4 = 81 \Leftrightarrow x = 5^{81}$.