Examples 1 on Log Identities

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We have some log identities as follows.

$\boldsymbol{y \log_b A = \log_b A^y}$.   So for instance, $2 \log_3 9 = \log_3 9^2$.

$\boldsymbol{\log_b M + \log_b N = \log_b MN}$.   So for instance, $\log_3 9 + \log_3 27 = \log_3 9{\cdot}27 = \log_3 243$.

$\boldsymbol{\log_b M - \log_b N = \log_b \frac{M}{N}}$.   So for instance, $\log_3 27 - \log_3 9 = \log_3 27/9 = \log_3 3$.

$\boldsymbol{\log_a C = \log_b D = \log_{ab} CD}$.   So for instance, $\log_3 9 = \log_2 4 = \log_6 36$.

$\boldsymbol{\log_a C = \log_b D = \log_{\frac{a}{b}} \frac{C}{D} = \log_{\frac{b}{a}} \frac{D}{C}}$.   So for instance, $\log_4 16 = \log_2 4 = \log_2 4 = \log_{\frac{1}{2}} \frac{1}{4}$.

$\boldsymbol{\log_b b = 1}$, and $\boldsymbol{\log_b 1 = 0}$.   So for instance, $\log_{10} 10 = \log_2 2 = 1$, and $\log_7 1 = \log_3 1 = 0$.

$\boldsymbol{\log_b A = \dfrac{\log_c A}{\log_c b}}$.   So for instnace, $\log_4 16 = \dfrac{\log_2 16}{\log_2 4} = \dfrac{\log_3 16}{\log_3 4} = \dfrac{\log_4 16}{\log_4 4} = \dfrac{\log_5 16}{\log_5 4} = \ldots$

$\boldsymbol{\log_b A = \dfrac{1}{\log_A b}}$.   So for instance, $\log_4 16 = \dfrac{1}{\log_{16} 4}$.

Now, do the examples as below.

0. Assuming $\mathbf{\log_{10} 2 = 0.301}$, and $\mathbf{\log_{10} 3 = 0.4771}$, find the value of $\mathbf{\log_2 54}$.

1. Assuming $\mathbf{\log_2 12 = 3.587}$, find the value of $\mathbf{\log_2 54}$.

2. Assuming $\mathbf{\log_2 12} = \boldsymbol{x}$, express $\mathbf{\log_2 54}$ in terms of $\boldsymbol{x}$.

3. Assuming $\boldsymbol{x} = \mathbf{\log}_{\boldsymbol{b}} \frac{\mathbf{16}}{\mathbf{9}}$, and $\boldsymbol{y} = \mathbf{\log}_{\boldsymbol{b}} \frac{\mathbf{27}}{\mathbf{8}}$, express $\mathbf{\log}_{\boldsymbol{b}} \mathbf{24}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

4. Assuming $\boldsymbol{a^x b^y = b}$ where $\boldsymbol{a}$ and $\boldsymbol{b} > \mathbf{0}$ but $\boldsymbol{a} \neq \mathbf{1}$, express $\mathbf{\log}_{\boldsymbol{a}} \boldsymbol{a^y b^x}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

5. Assuming $\boldsymbol{a^3 b^5 = b}$ where $\boldsymbol{a}$ and $\boldsymbol{b} > \mathbf{0}$ but $\boldsymbol{a} \neq \mathbf{1}$, find the value of $\mathbf{\log}_{\boldsymbol{a}} \boldsymbol{a^5 b^3}$.

6. Assuming $\frac{\boldsymbol{a^x}}{\boldsymbol{b^y}} = \boldsymbol{a}$ where $\boldsymbol{a}$ and $\boldsymbol{b} > \mathbf{0}$ but $\boldsymbol{b} \neq \mathbf{1}$, express $\mathbf{\log}_{\boldsymbol{b}} \frac{\boldsymbol{a^y}}{\boldsymbol{b^x}}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

7. Assuming $\frac{\boldsymbol{a^3}}{\boldsymbol{b^5}} = \boldsymbol{a}$ where $\boldsymbol{a}$ and $\boldsymbol{b} > \mathbf{0}$ but $\boldsymbol{b} \neq \mathbf{1}$, find the value of $\mathbf{\log}_{\boldsymbol{b}} \frac{\boldsymbol{a^5}}{\boldsymbol{b^3}}$.

8. Find the value of $\mathbf{\log_2 2\sqrt{6} - \frac{1}{2}\log_2 \frac{1}{7} - \frac{5}{2}\log_2 \sqrt[5]{21}}$.

Suggestions or Solutions
To the Examples 1 on Log Identities

0. Assuming $\mathbf{\log_{10} 2 = 0.301}$, and $\mathbf{\log_{10} 3 = 0.4771}$, find the value of $\mathbf{\log_2 54}$.

First, we have a log identity where $\boldsymbol{\log_b A = \dfrac{\log_c A}{\log_c b}}$. So we can set $\mathbf{\log_2 54 = \dfrac{\log_{10} 54}{\log_{10} 2}}$.

Next, we have another log identity where $\boldsymbol{\log_b M + \log_b N = \log_b MN}$.

So we can get $\mathbf{\log_{10} 54 = \log_{10} 2 \cdot 27 = \log_{10} 2 + \log_{10} 27}$, where $\mathbf{\log_{10} 2 = 0.301}$.

And next, we have another log identity where $\boldsymbol{y \log_b A = \log_b A^y}$.

So we can get $\mathbf{\log_{10} 27 = \log_2 3^3 = 3 \log_2 3}$, where $\mathbf{\log_{10} 3 = 0.4771}$.

Thus, we get $\mathbf{\log_{10} 2 + \log_{10} 27 = \log_{10} 2 + 3 \log_2 3 = 0.301 + 3 \cdot 0.4771 = 1.7323}$.

In sum, we get

$\mathbf{\log_{10} 54 = \log_{10} 2 \cdot 27 = \log_{10} 2 + \log_{10} 27 = \log_{10} 2 + \log_2 3^3 = \log_{10} 2 + 3 \log_2 3}$

$\mathbf{= 0.301 + 3 \cdot 0.4771 = 1.7323}$.

So we get $\mathbf{\log_2 54 = \frac{\log_{10} 54}{\log_{10} 2} = \frac{1.7323}{0.301} = 5.7551}$.

1. Assuming $\mathbf{\log_2 12 = 3.587}$, find the value of $\mathbf{\log_2 54}$.

$\mathbf{\log_2 12 = \log_2 3 \cdot 4 = \log_2 3 + \log_2 4 = \log_2 3 + \log_2 2^2 = \log_2 3 + 2 \log_2 2 = \log_2 3 + 2 \cdot 1}$

$\mathbf{= 3.587 \Rightarrow \log_2 3 = 3.587 - 2 = 1.587 \Rightarrow \log_2 3 = 1.587}$.

$\mathbf{\log_2 54 = \log_2 2 \cdot 27 = \log_2 2 + \log_2 27 = 1 + \log_2 3^3 = 1 + 3 \log_2 3 = 1 + 3 \cdot 1.587 = 5.761}$.

2. Assuming $\log_2 12 = x$, express $\log_2 54$ in terms of x.

$\log_2 12 = \log_2 3 \cdot 4 = \log_2 3 + \log_2 4 = \log_2 3 + 2 = x \Rightarrow \log_2 3 = x - 2$.

$\log_2 54 = \log_2 2 \cdot 27 = \log_2 2 + \log_2 27 = 1 + 3 \log_2 3 = 1 + 3x - 6 = 3x - 5$.

3. Assuming $x = \log_b \frac{16}{9}$, and $y = \log_b \frac{27}{8}$, express $\log_b 24$ in terms of x and y.

First, $\log_b 24 = \log_b 3 \cdot 8 = \log_b 3 + \log_b 8 = \log_b 3 + \log_b 2^3 = \log_b 3 + 3 \log_b 2$.

Next, we have a log identity where $\log_b M - \log_b N = \log_b \frac{M}{N}$.

So we can get $x = \log_b \frac{16}{9} = \log_b 16 - \log_b 9 = \log_b 2^4 - \log_b 3^2 = 4 \log_b 2 - 2 \log_b 3$, and

$y = \log_b \frac{27}{8} = \log_b 27 - \log_b 8 = \log_b 3^3 - \log_b 2^3 = 3 \log_b 3 - 3 \log_b 2$.

So next, setting $u = \log_b 2$, and $v = \log_b 3$, we have
$\log_b 24 = v + 3u$, $x = 4u - 2v$, and $y = 3v - 3u$.

How then can we put $\log_b 24$ in terms of x and y?

We can take the last two equations as a system for u and v.
So solving it, and then, putting the solution into the first equation, we can put $\log_b 24$ in terms of x and y.

So to begin with, solving the system where $x = 4u - 2v$ and $y = 3v - 3u$, we can put them this way first: $3x = 12u - 6v$ and $2y = 6v - 6u$.

Thus next, we can get $3x + 2y = 12u - 6v + 6v - 6u = 6u \Rightarrow u = \frac{1}{6}(3x + 2y)$.

So next, putting $\boldsymbol{u}$ into either of the two equations in the system, we can get $\boldsymbol{v}$.

Using $\boldsymbol{x = 4u - 2v}$, we get $\mathbf{2}\boldsymbol{v} = \mathbf{4}\boldsymbol{u} - \boldsymbol{x} = \frac{4}{6}(3x + 2y) - x = 2x + \frac{4}{3}y - x = x + \frac{4}{3}y$.

So we get $\boldsymbol{v} = \frac{1}{2}x + \frac{2}{3}y$.

Thus, we get $\log_b 24 = v + 3u = \frac{1}{2}x + \frac{2}{3}y + \frac{1}{2}(3x + 2y) = 2x + \frac{5}{3}y$.

4. Assuming $a^x b^y = b$ where a and $b > 0$ but $a \neq 1$, express $\log_a a^y b^x$ in terms of x and y.

To begin with, we can get $\log_a a^y b^x = \log_a a^y + \log_a b^x = y \log_a a + x \log_a b = y + x \log_a b$.

So we get $\log_a a^y b^x = y + x \log_a b$. What then do we want to get?

We want to put $\log_a b$ in terms of x and y. How though?

We have $a^x b^y = b$ where a and $b > 0$ but $a \neq 1$,

So taking first, the log sub a of both sides in $a^x b^y = b$, we get $\log_a a^x b^y = \log_a b$.

Next, we can get $\log_a a^x b^y = \log_a a^x + \log_a b^y = x \log_a a + y \log_a b = x + y \log_a b$.

So we get $x + y \log_a b = \log_a b \Rightarrow x + y \log_a b - \log_a b = 0 \Rightarrow x + (\log_a b)(y - 1) = 0$.

Thus, we get: $\log_a b = \dfrac{x}{1-y}$.

So we get $\log_a a^y b^x = y + x \log_a b = y + x \cdot \dfrac{x}{1-y} = y + \dfrac{x^2}{1-y} = \dfrac{y - y^2 + x^2}{1-y}$.

5. Assuming $\boldsymbol{a^3b^5 = b}$ where $\boldsymbol{a}$ and $\boldsymbol{b > 0}$ but $\boldsymbol{a \neq 1}$, find the value of $\boldsymbol{\log_a a^5b^3}$.

To begin with, we can get $\boldsymbol{\log_a a^5b^3 = \log_a a^5 + \log_a b^3 = 5 \log_a a + 3 \log_a b = 5 + 3 \log_a b}$.

So we get $\boldsymbol{\log_a a^5b^3 = 5 + 3 \log_a b}$. And thus, we want to get the value of $\boldsymbol{\log_a b}$.

First, we have $\boldsymbol{a^3b^5 = b}$ where $\boldsymbol{a}$ and $\boldsymbol{b > 0}$ but $\boldsymbol{a \neq 1}$.

So taking next, the log sub $\boldsymbol{a}$ of both sides in $\boldsymbol{a^3b^5 = b}$, we get $\boldsymbol{\log_a a^3b^5 = \log_a b}$.

Next, we can get $\boldsymbol{\log_a a^3b^5 = \log_a a^3 + \log_a b^5 = 3 \log_a a + 5 \log_a b = 3 + 5 \log_a b}$.

So we get $\boldsymbol{3 + 5 \log_a b = \log_a b \Rightarrow 3 + 5 \log_a b - \log_a b = 0 \Rightarrow 3 + 4 \log_a b = 0}$.

Thus, we get $\boldsymbol{\log_a b = -3/4}$. So we get $\boldsymbol{\log_a a^5b^3 = 5 + 3 \log_a b = 5 - 3\cdot(3/4) = 11/4}$.

6. Assuming $\boldsymbol{\frac{a^x}{b^y} = a}$ where $\boldsymbol{a}$ and $\boldsymbol{b > 0}$ but $\boldsymbol{b \neq 1}$, express $\boldsymbol{\log_b \frac{a^y}{b^x}}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.

First, we can get $\boldsymbol{\log_b \frac{a^y}{b^x} = \log_b a^y - \log_b b^x = y \log_b a - x \log_b b = y \log_b a - x}$.

What then do we want to get?

We want to put $\boldsymbol{\log_b a}$ in terms of $\boldsymbol{x}$ and $\boldsymbol{y}$.   How?

We have $\boldsymbol{\frac{a^x}{b^y} = a}$ where $\boldsymbol{a}$ and $\boldsymbol{b > 0}$ but $\boldsymbol{b \neq 1}$,

So taking first, the log sub $\boldsymbol{b}$ of both sides in $\boldsymbol{\frac{a^x}{b^y} = a}$, we get $\boldsymbol{\log_b \frac{a^x}{b^y} = \log_b a}$.

Next, we can get $\log_b \frac{a^x}{b^y} = \log_b a^x - \log_b b^y = x \log_b a - y \log_b b = x \log_b a - y$.

(Note that $\log_b (a - y) \neq \log_b a - y$.)

So we get $x \log_b a - y = \log_b a \Rightarrow x \log_b a - y - \log_b a = 0 \Rightarrow (\log_b a)(x - 1) - y = 0$.

Thus, we get $\log_b a = \frac{y}{x-1}$.

So we get $\log_b \frac{a^y}{b^x} = y \log_b a - x = y \cdot \frac{y}{x-1} - x = \frac{y^2}{x-1} - x = \frac{y^2 - x^2 + x}{x-1}$.

7. Assuming $\frac{a^3}{b^5} = a$ where a and $b > 0$ but $b \neq 1$, find the value of $\log_b \frac{a^5}{b^3}$.

First, we can get $\log_b \frac{a^5}{b^3} = \log_b a^5 - \log_b b^3 = 5 \log_b a - 3 \log_b b = 5 \log_b a - 3$.

So next, we want to get the value of $\log_b a$.

We have $\frac{a^3}{b^5} = a$ where a and $b > 0$ but $b \neq 1$,

So taking first, the log sub b of both sides in $\frac{a^3}{b^5} = a$, we get $\log_b \frac{a^3}{b^5} = \log_b a$.

Next, we can get $\log_b \frac{a^3}{b^5} = \log_b a^3 - \log_b b^5 = 3 \log_b a - 5 \log_b b = 3 \log_b a - 5$.

So we get $3 \log_b a - 5 = \log_b a \Rightarrow 3 \log_b a - 5 - \log_b a = 0 \Rightarrow 2 \log_b a - 5 = 0$.

Thus, we get $\log_b a = 5/2$.

So we get $\log_b \frac{a^5}{b^3} = 5 \log_b a - 3 = 5 \cdot \frac{5}{2} - 3 = \frac{25}{2} - 3 = \frac{19}{2}$.

8. Find the value of $\log_2 2\sqrt{6} - \frac{1}{2}\log_2 \frac{1}{7} - \frac{5}{2}\log_2 \sqrt[5]{21}$.

$$\log_2 2\sqrt{6} = \log_2 2 + \log_2 \sqrt{6} = 1 + \log_2 \sqrt{6}$$

$$\tfrac{1}{2}\log_2 \tfrac{1}{7} = \log_2 (\tfrac{1}{7})^{\frac{1}{2}} = \log_2 \sqrt{\tfrac{1}{7}} = \log_2 \tfrac{1}{\sqrt{7}}$$

$$\tfrac{5}{2}\log_2 \sqrt[5]{21} = \tfrac{5}{2}\log_2 21^{\frac{1}{5}} = \tfrac{5}{2}\cdot\tfrac{1}{5}\log_2 21 = \tfrac{1}{2}\log_2 21 = \log_2 \sqrt{21}$$

So we get

$$\log_2 2\sqrt{6} - \tfrac{1}{2}\log_2 \tfrac{1}{7} - \tfrac{5}{2}\log_2 \sqrt[5]{21} = 1 + \log_2 \sqrt{6} - \log_2 \tfrac{1}{\sqrt{7}} - \log_2 \sqrt{21}$$

$$= 1 + \log_2 \sqrt{6} + \log_2 \sqrt{7} - \log_2 \sqrt{21} = 1 + \log_2 \tfrac{\sqrt{6}\sqrt{7}}{\sqrt{21}} = 1 + \log_2 \tfrac{\sqrt{2}\sqrt{3}\sqrt{7}}{\sqrt{21}} = 1 + \log_2 \sqrt{2} = \tfrac{3}{2}.$$