# Examples 2 on Log Identities



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# Examples 2 on Log Identities

To begin with, we have some log identities as follows.

$\boldsymbol{y \log_b A = \log_b A^y}$. So for instance, $2 \log_3 9 = \log_3 9^2$.

$\boldsymbol{\log_b M + \log_b N = \log_b MN}$. So for instance, $\log_3 9 + \log_3 27 = \log_3 9 \cdot 27 = \log_3 243$.

$\boldsymbol{\log_b M - \log_b N = \log_b \frac{M}{N}}$. So for instance, $\log_3 27 - \log_3 9 = \log_3 27/9 = \log_3 3$.

$\boldsymbol{\log_a C = \log_b D = \log_{ab} CD}$. So for instance, $\log_3 9 = \log_2 4 = \log_6 36$.

$\boldsymbol{\log_a C = \log_b D = \log_{\frac{a}{b}} \frac{C}{D} = \log_{\frac{b}{a}} \frac{D}{C}}$. So for instance, $\log_4 16 = \log_2 4 = \log_2 4 = \log_{\frac{1}{2}} \frac{1}{4}$.

$\boldsymbol{\log_b b = 1}$, and $\boldsymbol{\log_b 1 = 0}$. So for instance, $\log_{10} 10 = \log_2 2 = 1$, and $\log_7 1 = \log_3 1 = 0$.

$\boldsymbol{\log_b A = \frac{\log_c A}{\log_c b}}$. So for instnace, $\log_4 16 = \frac{\log_2 16}{\log_2 4} = \frac{\log_3 16}{\log_3 4} = \frac{\log_4 16}{\log_4 4} = \frac{\log_5 16}{\log_5 4} = \ldots$

$\boldsymbol{\log_b A = \frac{1}{\log_A b}}$. So for instance, $\log_4 16 = \frac{1}{\log_{16} 4}$.

Now, do the examples as follows.

0. Assuming $\boldsymbol{a > b > 0}$, simplify the expression below.

$$\sqrt{\log_c \sqrt{ab} + \sqrt{\log_c a \cdot \log_c b}} + \sqrt{\log_c \sqrt{ab} - \sqrt{\log_c a \cdot \log_c b}}$$

1. Assuming $\boldsymbol{x > 0}$, simplify $\sqrt{\log_2 8x - \sqrt{\log_2 x^{12}}}$.

2. Assuming $x = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}})$, find the value of $y \log_9 3(\sqrt{x + \frac{1}{18}} + \sqrt{x - \frac{1}{18}})$.

3. Assuming $\boldsymbol{x > 1}$, put $\sqrt{\log 10\sqrt{x} - \sqrt{\log x^2}}$ in terms of $\sqrt{\log x}$.

4. Assuming $\boldsymbol{x}$ and $\boldsymbol{y}$ are rational numbers, but $\log_3 2$ is not, and $x \log_3 2 + y \log_3 6 = 1$, find $\boldsymbol{x}$ and $\boldsymbol{y}$.

## Suggestions or Solutions
## To the Problem in the Example 0

**Assuming $a > b > 0$, simplify the expression below.**

$$\sqrt{\log_c \sqrt{ab} + \sqrt{\log_c a \cdot \log_c b}} + \sqrt{\log_c \sqrt{ab} - \sqrt{\log_c a \cdot \log_c b}}$$

To begin with, we can get $\log_c \sqrt{ab} = \log_c(\sqrt{a}\sqrt{b}) = \log_c \sqrt{a} + \log_c \sqrt{b}$

Next, $\sqrt{\log_c a \cdot \log_c b} = 2 \cdot \frac{1}{2}\sqrt{\log_c a \cdot \log_c b} = 2\sqrt{\frac{1}{4}(\log_c a \cdot \log_c b)} = 2\sqrt{\frac{1}{2}\log_c a \cdot \frac{1}{2}\log_c b}$

$= 2\sqrt{\log_c \sqrt{a} \cdot \log_c \sqrt{b}}$

So we get $\log_c \sqrt{ab} - \sqrt{\log_c a \cdot \log_c b} = \log_c \sqrt{a} + \log_c \sqrt{b} - 2\sqrt{\log_c \sqrt{a} \cdot \log_c \sqrt{b}}$

$= (\sqrt{\log_c \sqrt{a}} - \sqrt{\log_c \sqrt{b}})^2$. And we know $a > b > 0$, so we get $\sqrt{\log_c \sqrt{a}} > \sqrt{\log_c \sqrt{b}}$.

Thus, we get $\sqrt{\log_c \sqrt{ab} - \sqrt{\log_c a \cdot \log_c b}} = \sqrt{\log_c \sqrt{a}} - \sqrt{\log_c \sqrt{b}}$.

And by the same token, we can get $\sqrt{\log_c \sqrt{ab} + \sqrt{\log_c a \cdot \log_c b}} = \sqrt{\log_c \sqrt{a}} + \sqrt{\log_c \sqrt{b}}$.

So assuming the expression given is $P$, we get $P = 2\sqrt{\log_c \sqrt{a}} = \sqrt{4\log_c \sqrt{a}}$

$= \sqrt{4 \cdot \frac{1}{2}\log_c a} = \sqrt{2\log_c a} = \sqrt{\log_c a^2}$.

## Suggestions or Solutions
## To the Problem in the Example 1

**Assuming $x > 0$, simplify $\sqrt{\log_2 8x - \sqrt{\log_2 x^{12}}}$.**

First, we can get $\log_2 x^{12} = 12 \log_2 x$.

So we get $\sqrt{\log_2 x^{12}} = \sqrt{12 \log_2 x} = \sqrt{4 \cdot 3 \log_2 x} = \sqrt{4}\sqrt{3 \log_2 x} = 2\sqrt{3 \log_2 x}$.

And next, we can get
$\log_2 8x = \log_2 8 + \log_2 x = \log_2 2^3 + \log_2 x = 3 \log_2 2 + \log_2 x = 3 + \log_2 x$.

Thus, we get $\log_2 8x - \sqrt{\log_2 x^{12}} = 3 + \log_2 x - 2\sqrt{3 \log_2 x} = 3 + \log_2 x - 2\sqrt{3}\sqrt{\log_2 x}$

$= (\sqrt{3} - \sqrt{\log_2 x})^2$.

And we know what's inside a square root sign is positive or 0.

So we get $\sqrt{\log_2 8x - \sqrt{\log_2 x^{12}}} = \left|\sqrt{3} - \sqrt{\log_2 x}\right|$.

## Suggestions or Solutions
## To the Problem in the Example 2

**Assuming $x = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}})$, find the value of $y\log_9 3(\sqrt{x+\frac{1}{18}} + \sqrt{x-\frac{1}{18}})$.**

First, $x + \frac{1}{18} = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}}) + \frac{1}{18} = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}} + 2) = \frac{1}{36}(3^{\frac{2}{y}} + 3^{-\frac{2}{y}} + 2) = \frac{1}{36}(3^{\frac{1}{y}} + 3^{-\frac{1}{y}})^2$.

So we get $\sqrt{x+\frac{1}{18}} = \frac{1}{6}(3^{\frac{1}{y}} + 3^{-\frac{1}{y}})$.

Next, $x - \frac{1}{18} = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}}) - \frac{1}{18} = \frac{1}{36}(9^{\frac{1}{y}} + 9^{-\frac{1}{y}} - 2) = \frac{1}{36}(3^{\frac{2}{y}} + 3^{-\frac{2}{y}} - 2) = \frac{1}{36}(3^{\frac{1}{y}} - 3^{-\frac{1}{y}})^2$.

So we get $\sqrt{x-\frac{1}{18}} = \frac{1}{6}(3^{\frac{1}{y}} - 3^{-\frac{1}{y}})$, because $3^{\frac{1}{y}} - 3^{-\frac{1}{y}} > 0$, since $y > 0$.

Thus, we get $3(\sqrt{x+\frac{1}{18}} + \sqrt{x-\frac{1}{18}}) = 3 \cdot \frac{1}{3} \cdot 3^{\frac{1}{y}} = 3^{\frac{1}{y}}$.

So we get $y\log_9 3(\sqrt{x+\frac{1}{18}} + \sqrt{x-\frac{1}{18}}) = y\log_9 3^{\frac{1}{y}} = \log_9 (3^{\frac{1}{y}})^y = \log_9 3 = \log_9 9^{1/2} = 1/2$.

## Suggestions or Solutions
## To the Problem in the Example 3

**Assuming $x > 1$, put $\sqrt{\log 10\sqrt{x} - \sqrt{\log x^2}}$ in terms of $\sqrt{\log x}$ .**

To begin with, $\log 10\sqrt{x} = \log 10 + \log\sqrt{x} = 1 + \frac{1}{2}\log x$, and $\log x^2 = 2\log x$.

Thus next, we can get

$$\log 10\sqrt{x} - \sqrt{\log x^2} = 1 + \tfrac{1}{2}\log x - \sqrt{2\log x} = \tfrac{1}{2}(2 + \log x - 2\sqrt{2\log x}) = \tfrac{1}{2}(\sqrt{2} - \sqrt{\log x})^2.$$

Meanwhile, $\sqrt{2} - \sqrt{\log x} \geq 0 \Rightarrow \sqrt{\log x} \leq \sqrt{2} \Rightarrow \log x \leq 2 \Rightarrow x \leq 10^2$.

So we get $\sqrt{2} - \sqrt{\log x} < 0 \Rightarrow x > 10^2$. And we have $x > 1$, too.

Thus, we get

If $1 \leq x \leq 100$, $\sqrt{\log 10\sqrt{x} - \sqrt{\log x^2}} = \frac{1}{\sqrt{2}}(\sqrt{2} - \sqrt{\log x})$.

If $x > 10^2$, $\sqrt{\log 10\sqrt{x} - \sqrt{\log x^2}} = \frac{1}{\sqrt{2}}(\sqrt{\log x} - \sqrt{2})$.

*If not quite sure of the idea behind the processes above, follow the steps below.*

To begin with, we can get

$\log 10\sqrt{x} = \log 10 + \log\sqrt{x} = 1 + \frac{1}{2}\log x$, and $\log x^2 = 2\log x$.

And next, we have $x > 1$. So we can get $\sqrt{\log x}$, since $\log x > 0$.

Thus next, we can get

$$\log 10\sqrt{x} - \sqrt{\log x^2} = 1 + \tfrac{1}{2}\log x - \sqrt{2\log x} = \tfrac{1}{2}(2 + \log x - 2\sqrt{2\log x}).$$

Meanwhile, we can have $(\sqrt{2}-\sqrt{\log x})^2 = 2+\log x - 2\sqrt{\log x}$.

So we get $\sqrt{\log 10\sqrt{x}-\sqrt{\log x^2}} = \sqrt{\frac{1}{2}(\sqrt{2}-\sqrt{\log x})^2} = \frac{1}{\sqrt{2}}\sqrt{(\sqrt{2}-\sqrt{\log x})^2}$.

And we know $\sqrt{(\sqrt{2}-\sqrt{\log x})^2} \geq 0$. So we get two cases.

One is $\sqrt{(\sqrt{2}-\sqrt{\log x})^2} = \sqrt{2}-\sqrt{\log x}$ if $\sqrt{2}-\sqrt{\log x} \geq 0$.

And the other is $\sqrt{(\sqrt{2}-\sqrt{\log x})^2} = \sqrt{\log x}-\sqrt{2}$ if $\sqrt{2}-\sqrt{\log x} < 0$.

What then is the next?

We want to get the extent of $x$ in each case.

To begin with, we have $x > 1$. And next, we can have these:

$\sqrt{2}-\sqrt{\log x} \geq 0 \Rightarrow \sqrt{\log x} \leq \sqrt{2} \Rightarrow \log x \leq 2 \Rightarrow x \leq 10^2$. So we get $1 \leq x \leq 100$.
$\sqrt{2}-\sqrt{\log x} < 0 \Rightarrow \sqrt{\log x} > \sqrt{2} \Rightarrow \log x > 2 \Rightarrow x > 10^2$.

Thus, we get these:

If $1 \leq x \leq 100$, $\sqrt{\log 10\sqrt{x}-\sqrt{\log x^2}} = \frac{1}{\sqrt{2}}\sqrt{(\sqrt{2}-\sqrt{\log x})^2} = \frac{1}{\sqrt{2}}(\sqrt{2}-\sqrt{\log x})$.

If $x > 10^2$, $\sqrt{\log 10\sqrt{x}-\sqrt{\log x^2}} = \frac{1}{\sqrt{2}}(\sqrt{\log x}-\sqrt{2})$.

## Suggestions or Solutions
## To the Problem in the Example 4

**Assuming $x$ and $y$ are rational numbers, but $\log_3 2$ is not, and $x \log_3 2 + y \log_3 6 = 1$, find $x$ and $y$.**

To begin with, we have $\mathbf{\log_3 6 = \log_3 2{\cdot}3 = \log_3 2 + \log_3 3 = \log_3 2 + 1}$.

So next, we can get $x \log_3 2 + y \log_3 6 = x \log_3 2 + y (\log_3 2 + 1) = (x + y)\log_3 2 + y$.

And we have $x \log_3 2 + y \log_3 6 = 1$.

So we get $(x + y)\log_3 2 + y = 1 \Rightarrow (x + y)\log_3 2 + y - 1 = 0$.

And we know that $\log_3 2$ is not a rational number, but that $x$ and $y$ are.

So we get $x + y = 0$, and $y - 1 = 0$. Thus, we get $x = -y$, and $y = 1$.

So we get $x = -1$, and $y = 1$.