Examples 2 in Common Logs

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Note that a small dot · is a multiplication operator unless specified otherwise,

e.g. 3 x 5 = 3·5 = 15.

Note also, that no use of calculator is allowed in this set of examples.

Use **log 1.35 = 0.1303**, and find the characteristic and mantissa of the common log of each of the numbers as follows.

0. **13.5**

1. **135**

2. **0.135**

3. **0.0135**

4. $\mathbf{1.35^2}$

5. $\sqrt{\mathbf{13.5}}$

6. $\mathbf{0.135^{-2}}$

7. $\frac{1}{\sqrt{0.0135}}$

Suggestions or Solutions
To the Problems in the Examples 2

Suppose that two different positive numbers have the same sequence of digits.

Then, the common logs of the two numbers have different characteristics, but their mantissas are the same.

The first four numbers given in this set of examples have the same sequence of digits, and the sequence has 1, 3, and 5 in order. Therefore, their mantissas are the same.

What then about the next four numbers?

Those four numbers are powers and radicals, and their bases have the same sequence of digits, but if we expand those four numbers, the expansions will probably have different sequences of digits.
For instance, if we expand $\mathbf{2^3}$ and $\sqrt{\mathbf{2}}$, the expansions are 8 and 1.4142135…

And also, the last four numbers in this set of examples have exponents other than 1. Technically, we can take the first four numbers for antilogs in power notation, too. For instance, 13.5 can be taken for an antilog where the base is 13.5, and the exponent is 1.

Now in this example, we use **log 1.35 = 0.1303** doing each problem.

So we may want to first, take a common log of each number given in each problem and then, extract the log of 1.35. Thus, we get to use some identities on logarithms, and they are as follows.

Assuming $\boldsymbol{M}$, $\boldsymbol{N}$, and $\boldsymbol{b} > \mathbf{0}$, but $\boldsymbol{b} \neq \mathbf{1}$, we have these:

$\mathbf{log}_b\, \boldsymbol{b} = \mathbf{1}$.

$\mathbf{log}_b\, \boldsymbol{MN} = \mathbf{log}_b\, \boldsymbol{N} + \mathbf{log}_b\, \boldsymbol{M}$.

$\boldsymbol{A} = \mathbf{log}_b\, \boldsymbol{N} \Rightarrow \boldsymbol{PA} = \boldsymbol{P}\, \mathbf{log}_b\, \boldsymbol{N} = \mathbf{log}_b\, \boldsymbol{N}^{\boldsymbol{P}}$.

0. 13.5

We know 13.5 = 10·1.35. Taking the common log of it, we get $\mathbf{log_{10}\ 10{\cdot}1.35}$, which is normally though, put this way: **log 10·1.35**. So the base 10 is usually omitted.

And we have $\mathbf{\log_b b = 1}$, and $\mathbf{\log_b MN = \log_b N + \log_b M}$.

So we get **log 13.5 = log 10 + log 1.35 = 1 + 0.1303**.
Therefore, the characteristic is 1, and the mantissa is 0.1303.

1. 135

We know $135 = 10^2(1.35)$. Taking the common log of it, we get $\mathbf{\log\ 10^2(1.35)}$.
And we have $\mathbf{\log_b b = 1}$, $\mathbf{\log_b MN = \log_b N + \log_b M}$, and $\mathbf{P \log_b N = \log_b N^P}$.

So we get $\mathbf{\log\ 13.5 = \log\ 10^2 + \log\ 1.35 = 2\ \log\ 10 + \log\ 1.35 = 2 + 0.1303}$.
Therefore, the characteristic is 2, and the mantissa is 0.1303.

2. 0.135

We know $0.135 = 10^{-1}(1.35)$. Taking the common log of it, we get $\mathbf{\log\ 10^{-1}(1.35)}$.
And we have $\mathbf{\log_b b = 1}$, $\mathbf{\log_b MN = \log_b N + \log_b M}$, and $\mathbf{P \log_b N = \log_b N^P}$.

So we get $\mathbf{\log\ 0.135 = \log\ 10^{-1} + \log\ 1.35 = (-1)\log\ 10 + \log\ 1.35 = -1 + 0.1303}$.
Therefore, the characteristic is -1, and the mantissa is 0.1303.

3. 0.0135

We know $0.0135 = 10^{-2}(1.35)$. Taking the common log of it, we get $\mathbf{\log\ 10^{-2}(1.35)}$.
And we have $\mathbf{\log_b b = 1}$, $\mathbf{\log_b MN = \log_b N + \log_b M}$, and $\mathbf{P \log_b N = \log_b N^P}$.

So we get $\mathbf{\log\ 0.0135 = \log\ 10^{-2} + \log\ 1.35 = (-2)\log\ 10 + \log\ 1.35 = -2 + 0.1303}$.
Therefore, the characteristic is -2, and the mantissa is 0.1303.

4. 1.35^2

We have $1.35^2 = (1.35)^2$. Taking the common log of it, we get $\mathbf{\log (1.35)^2}$.
And we have $\log_b b = 1$, $\log_b MN = \log_b N + \log_b M$, and $P \log_b N = \log_b N^P$.

So we get $\mathbf{\log (1.35)^2 = 2 \log 1.35 = 2(0.1303) = 0.2606}$.
Therefore, the characteristic is 0, and the mantissa is 0.2606.

5. $\sqrt{13.5}$

We have $\sqrt{13.5} = (10 \cdot 1.35)^{\frac{1}{2}}$. Taking the common log of it, we get $\log(10 \cdot 1.35)^{\frac{1}{2}}$.
And we have $\log_b b = 1$, $\log_b MN = \log_b N + \log_b M$, and $P \log_b N = \log_b N^P$.

So we get $\log(10 \cdot 1.35)^{\frac{1}{2}} = \frac{1}{2}\log(10 \cdot 1.35) = \frac{1}{2}(1 + 0.1303) = 0.5 + 0.06515 = 0.56515$.
Therefore, the characteristic is 0, and the mantissa is 0.56515.

6. 0.135^{-2}

We have $(0.135)^{-2} = \{10^{-1}(1.35)\}^{-2}$.
Taking the common log of it, we get $\log \{10^{-1}(1.35)\}^{-2}$.
And we have $\log_b b = 1$, $\log_b MN = \log_b N + \log_b M$, and $P \log_b N = \log_b N^P$.

So we get $\log \{10^{-1}(1.35)\}^{-2} = (-2)\log 10^{-1}(1.35) = -2(-1 + 0.1303)$
$= 2 - 0.2606 = 2 - 1 + 1 - 0.2606 = 1 + 0.7394$.
Therefore, the characteristic is 1, and the mantissa is 0.7394.

7. $\frac{1}{\sqrt{0.0135}}$

We have $\frac{1}{\sqrt{0.0135}} = (10^{-2} \cdot 1.35)^{-\frac{1}{2}}$.

Taking the common log of it, we get $\log(10^{-2} \cdot 1.35)^{-\frac{1}{2}}$.
And we have $\log_b b = 1$, $\log_b MN = \log_b N + \log_b M$, and $P \log_b N = \log_b N^P$.

So we get $\log(10^{-2} \cdot 1.35)^{-\frac{1}{2}} = -\frac{1}{2}\log(10^{-2} \cdot 1.35) = -\frac{1}{2}(-2 + 0.1303)$
$= 1 - 0.06515 = 0.93485.$

Therefore, the characteristic is 0, and the mantissa is 0.93485.

In short:

0. $\log 13.5 = \log 10(1.35) = \log 10 + \log 1.35 = 1 + \log 1.35 = 1 + 0.1303 = 1.1303$.

Therefore, the characteristic = 1, and the mantissa = 0.1303.

1. $\log 135 = \log 10^2 1.35 = \log 10^2 + \log 1.35 = 2 + \log 1.35 = 2 + 0.1303 = 2.1303$.

Therefore, the characteristic = 2, and the mantissa = 0.1303.

2. $\log 0.135 = \log 10^{-1} 1.35 = \log 10^{-1} + \log 1.35 = -1 + \log 1.35 = -1 + 0.1303$.

Therefore, the characteristic = -1, and the mantissa = 0.1303.

3. $\log 0.0135 = \log 10^{-2} 1.35 = \log 10^{-2} + \log 1.35 = -2 + \log 1.35 = -2 + 0.1303$.

Therefore, the characteristic = -2, and the mantissa = 0.1303.

4. $\log 1.35^2 = 2 \log (1.35) = 2(0.1303) = 0.2606.$

Therefore, the characteristic = 0, and the mantissa = 0.2606.

5. $\log \sqrt{13.5} = \log(10 \cdot 1.35)^{\frac{1}{2}} = \frac{1}{2}\log(10 \cdot 1.35) = \frac{1}{2}(1 + 0.1303) = 0.5 + 0.06515$

$= 0.56515$.

Therefore, the characteristic is 0, and the mantissa is 0.5652.

6. $\log 0.135^{-2} = \log \{10^{-1}(1.35)\}^{-2} = (-2)\log 10^{-1}(1.35) = -2(-1 + 0.1303) = 2 - 0.2606$

$= 2 - 1 + 1 - 0.2606 = 1 + 0.7394$.

Therefore, the characteristic is 1, and the mantissa is 0.7394.

7. $\log \frac{1}{\sqrt{0.0135}} = \log(10^{-2} \cdot 1.35)^{-\frac{1}{2}} = -\frac{1}{2}\log(10^{-2} \cdot 1.35) = -\frac{1}{2}(-2 + 0.1303)$

$= 1 - 0.06515 = 0.93485$.

Therefore, the characteristic is 0, and the mantissa is 0.9348.