Examples 3 in Common Logs

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Note that a small dot $\cdot$ is a multiplication operator unless specified otherwise, e.g. $3 \times 5 = 3 \cdot 5 = 15$.

Note also, that no use of calculator is allowed in this set of examples.

3. Using **log 412 = 2.6149**, find ***x*** in each case below.

3.0. $\log x = 3.6149$

3.1. $\log x = \bar{1}.6149$

3.2. $\log x = \bar{2}.6149$

3.3. $\log 41.2 = x$

3.4. $\log 0.412^2 = x$

3.5. $\log \frac{1}{\sqrt{0.0412}} = x$

3.6. $\log 412^x = 1.3542$

3.7. $\log 412^{2x} = \bar{2}.3462$

Suggestions or Solutions
To the Problems in the Examples 3

Let's begin with the basics of common logs.

Suppose first, K is a number positive.
Then, the common log of K is $\log K = c + m$ where c is an integer called the characteristic indicating the highest place value of K, m is the mantissa, and $0 \le m < 1$.
And the highest place value of K is 10^c.

Suppose next, two different positive numbers have the same sequence of digits.
For instance, one is 1.4809, and the other is 148.09.

Then, the common logs of the two numbers have different characteristics, but their mantissas are the same. That is to say that, the log values share the same mantissa, but their characteristics are different.

Now in this example, we use **log 412 = 2.6149**, which is the value of a common log.

So in the log value 2.6149, 2 is the characteristic, and thus, indicates that the highest place value of the antilog is 10^2. The antilog is 412, and the common log of it is **log 412**, of course. So if the common log of x is 0.6149, x is 4.12. That's for two reasons below.

One is that the characteristic is 0, so the highest place value is 10^0, which is 1.

And the other is that the mantissa is the same as that of **log 412**, so the sequence of digits has 4, 1, and 2 in order. What then if the common log of x is 1.6149?

Then, x is 41.2 because the characteristic is 1, so the highest place value is 10^1, and since the mantissa is the same as that of **log 412**, the sequence of digits has 4, 1, and 2 in order.

Now, the first three logs given in this set of examples have the same mantissa, which is 0.6149.

In each of the first three examples, therefore, $\boldsymbol{x}$ needs to have the same sequence of digits, which has 4, 1, and 2 in order, since the mantissa is the same as that of **log 412**.

So using the characteristic in each of the three logs, we can get the value of $\boldsymbol{x}$.

Let's next, move on to the forth example: $\mathbf{log\ 41.2} = \boldsymbol{x}$.

The antilog 41.2 has the same sequence of digits as the one in 412, of which the common log is 2.6149, that is, log 412 = 2.6149.

So the mantissa of **log 41.2** is the same as that of the log of 412.

Also, from the highest place value in 41.2, we can get the characteristic of **log 41.2**.

Next, in the last four examples, all the antilogs in the logs are powers.
(Note that a radical can be taken as a power, too. For instance, $\sqrt{2} = 2^{\frac{1}{2}}$, and $\sqrt[3]{5} = 5^{\frac{1}{3}}$.)

And in all the antilogs, the bases share the same sequence of digits, which has 4, 1, and 2 in order. We have **log 412 = 2.6149**. So the sequence the bases share is the same as the one in 412, of which the log is 2 + 0.6149, where 2 is the characteristic and 0.6149 is the mantissa. So using such information, we can find $\boldsymbol{x}$ in each of the examples.

Besides, in this example, we get to use two log identities as follows.

$\mathbf{log}_{\boldsymbol{b}}\, \boldsymbol{b} = \mathbf{1}$.

$\boldsymbol{x}\, \mathbf{log}_{\boldsymbol{b}}\, \boldsymbol{A} = \mathbf{log}_{\boldsymbol{b}}\, \boldsymbol{A}^{\boldsymbol{x}}$.

In the identities above, $\boldsymbol{A}$ and $\boldsymbol{b} > \mathbf{0}$, but $\boldsymbol{b} \neq \mathbf{1}$, of course.

3.0. log x = 3.6149

We know **log 4.12 = 0.6149**.
The log value, 3.6149 has the same mantissa as that of **log 4.12**, and indicates that the antilog has the same sequence of digits as the one in 4.12, but the highest place value in the antilog is 10^3, which is 1000. So the sequence of digits in ***x*** is 4, 1, and 2 in order, and the highest place value in ***x*** is 1000. Therefore, ***x* = 4120**.

3.1. log x = $\bar{1}$.6149

We know **log 4.12 = 0.6149**.
Thus, the log value **$\bar{1}$.6149** has the same mantissa as that of **log 4.12**, and thus, indicates that ***x*** has the same sequence of digits as the one in 4.12, but the characteristic of the log value is -1, so the highest place value in ***x*** is 10^{-1}. Therefore, ***x* = 0.412**.

3.2. log x = $\bar{2}$.6149

We know **log 4.12 = 0.6149**.
Thus, the log value **$\bar{2}$.6149** indicates that ***x*** has the same sequence of digits as the one in 4.12, and the highest place value in ***x*** is 10^{-2}.

Therefore, ***x* = 0.0412**.

3.3. log 41.2 = x

We know **log 4.12 = 0.6149**.
To begin with, 41.2 has the same sequence of digits as the one in 4.12.

Thus, **log 41.2** has to have the same mantissa as that of **log 4.12**, which is 0.6149.

Next, the highest place value in 41.2 is 10^1.

So the characteristic of **log 41.2** is 1.
Therefore, ***x* = 1.6149**.

3.4. $\log 0.412^2 = x$

We know **log 4.12 = 0.6149**.

To begin with, 0.412 has the same sequence of digits as the one in 4.12, and the highest place value in it is 10^{-1}.

So the mantissa of the log of 0.412 is 0.6149, and the characteristic of the log is -1.

Thus, we get **log 0.412 = -1 + 0.6149**.

Next, we have $P \log_b N = \log_b N^P$.

So we get $\log 0.412^2 = 2\log 0.412 = 2(-1 + 0.6149) = -2 + 1.2298 = -1 + 0.2298$

$= \bar{1}.2298$, which is x.

3.5. $\log \frac{1}{\sqrt{0.0412}} = x$

We know **log 4.12 = 0.6149**.

To begin with, 0.0412 has the same sequence of digits as the one in 4.12, and the highest place value in it is 10^{-2}.

So the mantissa of the log of 0.0412 is 0.6149, and the characteristic of the log is -2.

Thus, we get **log 0.0412 = -2 + 0.6149**.

Next, we have $P \log_b N = \log_b N^P$. So we get this:

$\log \frac{1}{\sqrt{0.0412}} = -\frac{1}{2}\log 0.0412 = -\frac{1}{2}(-2 + 0.6149) = 1 - 0.30745 = 0.69255$, which is x.

3.6. **$\log 412^x = 1.3542$**

We know **$\log 4.12 = 0.6149$**.

To begin with, 412 has the same sequence of digits as the one in 4.12, and the highest place value in it is 10^2.

So the mantissa of the log of 412 is 0.6149, and the characteristic of the log is 2.

Thus, we get **$\log 412 = 2 + 0.6149$**.

Next, we have **$P \log_b N = \log_b N^P$**.

So we get **$\log 412^x = x \log 412 = x(2.6149) = 1.3542$**.

Therefore, $x = \frac{1.3542}{2.6149} = \frac{13542}{26149} \cong \mathbf{0.5179}$.

3.7. **$\log 412^{2x} = \bar{2}.3462$**

We know **$\log 4.12 = 0.6149$**.

The antilog, 412 has the same sequence of digits as in 4.12, but has the highest place value of 10^2.

So we get **$\log 412 = 2.6149$**.

Next, we have **$P \log_b N = \log_b N^P$**.

Thus, **$\log 412^{2x} = 2x \log 412 = 2x(2.6149) = 5.2258x = \bar{2}.3462$.**

So we get **$5.2258x = -2 + 0.3462 = -1 - 1 + 0.3462 = -1 - 0.6538 = -1.6538$**.

Therefore, $x = \frac{-1.6538}{5.2258} = -\frac{16538}{52258} \cong \mathbf{-0.3165}$.