Examples 4 in Common Logs

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Note that a small dot $\cdot$ is a multiplication operator unless specified otherwise, e.g. $3 \times 5 = 3\cdot5 = 15$.

Note also, that no use of calculator is allowed in this set of examples.

Now, doing the examples below, use **log 2 = 0.3010**, and **log 3 = 0.4771**.

0. Find the number of digits in $\mathbf{6^{100}}$.

1. Find the highest place value in $\left(\frac{1}{\sqrt{2}}\right)^{25}$.

2. Find the highest place value and the number in the highest digit in $\frac{27^{100}}{5^{200}}$.

Suggestions or Solutions
To the Problem in the Example 0

Taking the common log of a positive integer, we get a log value, which is composed of two parts. One of the two has to do with the sequence of digits in the integer. Then, what does the other have to do with?

The other is an integer called the characteristic, which can tell us the highest place value in the integer, so what else can the characteristic tell us?

The lowest place value in an integer is 1, and the second lowest place value in it is 10 if it has more than one digit, of course, so the characteristic can tell us the number of all the digits in the integer, too.

Now, assuming $\mathbf{\log 2 = 0.3010}$ and $\mathbf{\log 3 = 0.4771}$, find the number of digits in $\mathbf{6^{100}}$.

$\mathbf{\log 6^{100} = 100 \log (2)(3) = 100(\log 2 + \log 3) = 100(0.3010+0.4771)}$
$\mathbf{= 100(0.7781) = 77.8100}$.

So the characteristic is 77, and therefore, 6^{100} has 78 digits.

If not quite sure of the idea behind the processes above, follow the steps below.

We know 6^{100} is a positive integer.   And we have $\boldsymbol{k \log_b N = \log_b N^k}$.

Thus, taking the common log of it, we get $\mathbf{\log 6^{100}}$, which is $\mathbf{100 \log 6}$.

So what do we need, now?

We need the value of **log 6**. How then can we get it?

We have log 2 = 0.3010, log 3 = 0.4771, and $\mathbf{log}_{\boldsymbol{b}}\ \boldsymbol{MN} = \mathbf{log}_{\boldsymbol{b}}\ \boldsymbol{N} + \mathbf{log}_{\boldsymbol{b}}\ \boldsymbol{M}$, too.

So we get **log 6 = log 2 + log 3 = 0.3010 + 0.4771 = 0.7781.**

Thus, we get **100 log 6 = 100(0.7781) = 77.81 = 77 + 0.8100**.

So the highest place value in 6^{100} is 10^{77}.

The lowest place value in it is 10^0, which is 1.

Therefore, the number of all the digits in it is 78.

In short:

$\mathbf{log\ 6^{100} = 100\ log\ (2)(3) = 100(log\ 2 + log\ 3) = 100(0.3010+0.4771)}$
$\mathbf{= 100(0.7781) = 77.8100}$.

So the characteristic is 77, and therefore, 6^{100} has 78 digits.

Suggestions or Solutions
To the Problem in the Example 1

Assuming **log 2 = 0.3010** and **log 3 = 0.4771**, find the highest place value in $(\frac{1}{\sqrt{2}})^{25}$.

$\log(\frac{1}{\sqrt{2}})^{25} = 25\log 2^{-\frac{1}{2}} = -\frac{25}{2}\log 2 = -\frac{25}{2}\cdot 0.3010 = -3.7625 = -4 + 0.2375.$

$\Rightarrow$ The highest place value is 10^{-4}.

If not quite sure of the idea behind the processes above, follow the steps below.

Taking the common log of a positive number, we get a log value, which has two parts. Then, of the two parts, what does the highest place value in the number have to do with?

It is the part called the characteristic, which is an integer.
So we want to find the characteristic of the number in question.

Now, the number in question is $(\frac{1}{\sqrt{2}})^{25}$.

So taking the common log of $(\frac{1}{\sqrt{2}})^{25}$, we get $\log(\frac{1}{\sqrt{2}})^{25} = 25\log 2^{-\frac{1}{2}} = -\frac{25}{2}\log 2$.

We know **log 2 = 0.3010**. So we get this:

$$-\frac{25}{2}\log 2 = -\frac{25}{2}\cdot 0.3010 = -3.7625 = -3 - 0.7625 = -3 - 1 + 1 - 0.7625$$

$$= -4 + 0.2375 = \bar{4}.2375.$$

Therefore, the highest place value is 10^{-4}, which is one ten thousandth.

In short:

$\log(\frac{1}{\sqrt{2}})^{25} = 25\log 2^{-\frac{1}{2}} = -\frac{25}{2}\log 2 = -\frac{25}{2}\cdot 0.3010 = -3.7625 = -4 + 0.2375.$

$\Rightarrow$ The highest place value is 10^{-4}.

Suggestions or Solutions
To the Problem in the Example 2

Assuming **log 2 = 0.3010** and **log 3 = 0.4771**, find the highest place value and the number in the highest digit in $\frac{27^{100}}{5^{200}}$.

We have **log 2 = 0.3010**, and **log 3 = 0.4771**.

Let $A = \frac{27^{100}}{5^{200}}$. Then, taking **log** of A, we get this:

$$\log A = \log \frac{27^{100}}{5^{200}} = \log \frac{3^{300}}{5^{200}} = \log 3^{300} - \log 5^{200} = 300 \log 3 - 200 \log 5.$$

Meanwhile, $\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - \log 2$.

So we get this:

$$\log A = 300 \log 3 - 200(1 - \log 2) = 300 \cdot 0.4771 - 200(1 - 0.3010)$$
$$= 143.13 - 139.8 = 3.3300.$$

Therefore, the highest place value is 10^3, which is 1000.

We have this: **log 2 = 0.3010**, and **log 3 = 0.4771**.

So **log 2000 = 3.3010**, and **log 3000 = 3.4771**.

Thus, we get $\log 2000 < \log A < \log 3000 \Rightarrow 2000 < A < 3000$.

Therefore, the number in the highest digit is 2.

If not quite sure of the idea behind the processes above, follow the steps below.

Taking the common log of a positive number, we get a log value, which has two parts. Then, of the two parts, one is an integer called the characteristic, which can tell us the highest place value in the number.

So to begin with, setting $\boldsymbol{A} = \frac{27^{100}}{5^{200}}$, and taking the common log of $\boldsymbol{A}$, we get this:

$\log \frac{27^{100}}{5^{200}}$, which equals $\log \frac{3^{300}}{5^{200}}$ since $27 = 3^3$.

Next, we have this: $\log_b \frac{M}{N} = \log_b M - \log_b N$, and $k \log_b N = \log_b N^k$, too.

So we get this: $\log \frac{3^{300}}{5^{200}} = \log 3^{300} - \log 5^{200} = 300 \log 3 - 200 \log 5$.

Meanwhile, $\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - \log 2$.

Thus, we get $\log A = 300 \log 3 - 200\,(1 - \log 2)$.

We have $\log 2 = 0.3010$ and $\log 3 = 0.4771$, too. So we get this:

$\log A = 300(0.4771) - 200(1 - 0.3010) = 143.13 - 200(0.699)$

$= 143.13 - 139.8 = 3.3300$.

Therefore, the highest place value in $\boldsymbol{A}$ is 10^3, which means 1000's place.

Next, we want to find the number in the highest digit in $\boldsymbol{A}$.
That is, what's in the 1000's place?

We have $\log A = 3.3300$, $\log 2 = 0.3010$, and $\log 3 = 0.4771$.

So we may want to begin with whereabouts the commmon log of $\boldsymbol{A}$ is in a number line.

To begin with, we know the highest place value in both 2000 and 3000 is 3.

So we can see that $\log 2000 = 3.3010$ and that $\log 3000 = 3.4771$.

Thus, since $\mathbf{log}\ \boldsymbol{A} = \mathbf{3.3300}$, we get this:

$$\mathbf{3.3010 < 3.3300 < 3.4771 \Rightarrow \log 2000 < \log \boldsymbol{A} < \log 3000 \Rightarrow 2000 < \boldsymbol{A} < 3000.}$$

Therefore, the number in the highest digit is 2. The actual results from a calculator are as follows. $\mathbf{\frac{27^{100}}{5^{200}} = (\frac{27}{25})^{100} \cong 2199.7612563}$, and $\mathbf{\log 2199.7612563 \cong 3.3424}$.

Besides, using a calculator, we can get $\mathbf{\log \boldsymbol{A} = 3.3300 \Rightarrow \boldsymbol{A} = 10^{3.33} \cong 2137.9620895}$.

In short:

We have $\mathbf{\log 2 = 0.3010}$, and $\mathbf{\log 3 = 0.4771}$.

Let $\boldsymbol{A} = \mathbf{\frac{27^{100}}{5^{200}}}$. Then, taking **log** of $\boldsymbol{A}$, we get this:

$$\mathbf{\log \boldsymbol{A} = \log \frac{27^{100}}{5^{200}} = \log \frac{3^{300}}{5^{200}} = \log 3^{300} - \log 5^{200} = 300 \log 3 - 200 \log 5}.$$

Meanwhile, $\mathbf{\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - \log 2}$.

So we get this:

$$\mathbf{\log \boldsymbol{A} = 300 \log 3 - 200(1 - \log 2) = 300 \cdot 0.4771 - 200(1 - 0.3010)}$$
$$\mathbf{= 143.13 - 139.8 = 3.3300}.$$

Therefore, the highest place value is 10^3, which is 1000.

We have $\mathbf{\log 2 = 0.3010}$, and $\mathbf{\log 3 = 0.4771}$.

So $\mathbf{\log 2000 = 3.3010}$, and $\mathbf{\log 3000 = 3.4771}$.

Thus, we get $\mathbf{\log 2000 < \log \boldsymbol{A} < \log 3000 \Rightarrow 2000 < \boldsymbol{A} < 3000}$.

Therefore, the number in the highest digit is 2.