Examples 1 in Approximations

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0. Using **log 5.72 = 0.7574** and **log 5.73 = 0.7582**, find an approximation of **log 5.724** without calculator.

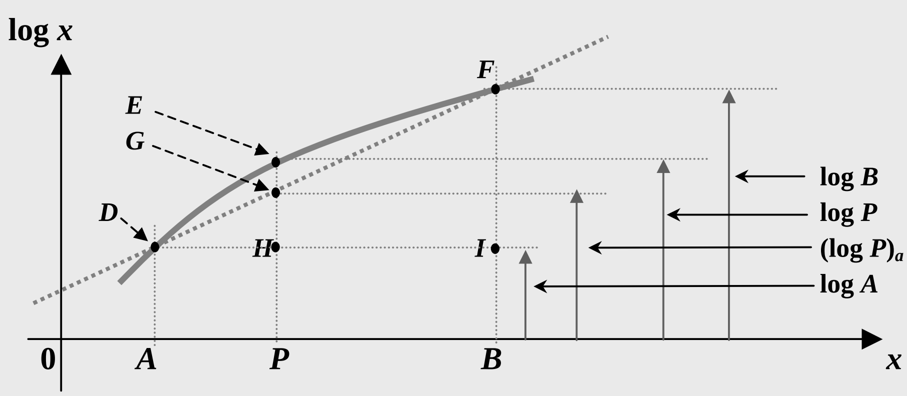
1. Using **log 5.72 = 0.7574** and **log 5.73 = 0.7582**, find an approximation of the antilog ***A*** in **log *A* = 0.7579** with no calculator.

Suggestions or Solutions
To the Problem in the Example 0

Using log 5.72 = 0.7574 and log 5.73 = 0.7582, find an approximation of log 5.724 without a calculator.

Setting $\boldsymbol{A}$ **= 5.72**, and $\boldsymbol{B}$ **= 5.73**, and putting all the values in a graph, we can get this:

Fig. 0.0 **log 5.72 = 0.7574**, **log 5.73 = 0.7582**, and $\boldsymbol{P}$ **= 5.724**



Since $\boldsymbol{FI}$ is parallel to $\boldsymbol{GH}$, two triangles $\boldsymbol{DIF}$ and $\boldsymbol{DHG}$ are similar to each other.

So setting $\boldsymbol{GH = d}$, and letting $\mathbf{(log}\ \boldsymbol{P})_a$ be the approximation, we can set

$\mathbf{(\log}\ \boldsymbol{P})_a = \log \boldsymbol{A} + \boldsymbol{d}$, where $\boldsymbol{d} = \frac{P-A}{B-A}\cdot(\log B - \log A)$.

We have $\boldsymbol{P} - \boldsymbol{A}$ **= 5.724 – 5.72 = 0.004**, $\boldsymbol{B} - \boldsymbol{A}$ **= 5.73 – 5.72 = 0.01**, and

$\log \boldsymbol{B} - \log \boldsymbol{A} = 0.7582 - 0.7574 = 0.0008$.

So we get $\boldsymbol{d} = \frac{P-A}{B-A}\cdot(\log B - \log A) = \frac{0.004}{0.01}\cdot 0.0008 = 0.4\cdot 0.0008 = 0.00032$.

Thus, $\log 5.724 \cong (\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577$.

If not quite sure of the idea behind the processes above, follow the steps below.

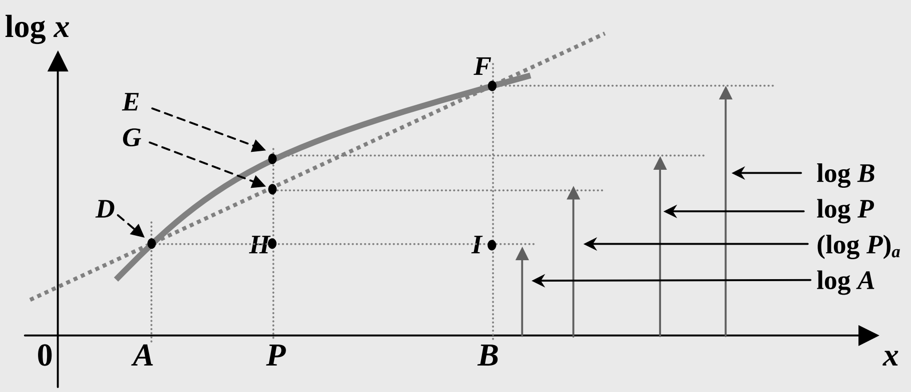
Let's first, set $\boldsymbol{A}$ **= 5.72**, $\boldsymbol{B}$ **= 5.73**, and $\boldsymbol{P}$ **= 5.724**.

Next, taking the log of both sides in each equality, we get this:

log $\boldsymbol{A}$ **= 0.7574**, and **log** $\boldsymbol{B}$ **= 0.7582**. That is, **log 5.72 = 0.7574**, and **log 5.73 = 0.7582**

And putting next, the values above in a graph, we can get a graph as shown below.

Fig. 0.1 **log 5.72 = 0.7574**, **log 5.73 = 0.7582**, and $\boldsymbol{P}$ **= 5.724**



Next, assuming that the distance from $\boldsymbol{E}$ to $\boldsymbol{G}$ is negligible, which means small enough, we can get an approximate value of **log** $\boldsymbol{P}$ using ratios that apply to similar triangles.

In the graph above, we have two triangles ***DIF*** and ***DHG*** similar to each other, because $\overline{FI}$ and $\overline{GH}$ are parallel to each other. Therefore, we can see this:

$$\overline{AB} : \overline{AP} = \overline{DI} : \overline{DH} = \overline{FI} : \overline{GH} \Rightarrow \overline{AB} : \overline{AP} = \overline{FI} : \overline{GH}.$$

That is to say that assuming ***AB*** is the distance from $\boldsymbol{A}$ to $\boldsymbol{B}$, we can get this:

$$\frac{AB}{AP} = \frac{DI}{DH} = \frac{FI}{GH} \Rightarrow \frac{AB}{AP} = \frac{FI}{GH}.$$

So next, assuming for simplicity, $\boldsymbol{d = GH}$, we can see that $\mathbf{log}\ \boldsymbol{A} + \boldsymbol{d} = (\mathbf{log}\ \boldsymbol{P})_{\boldsymbol{a}}$, which is an approximation of **log** $\boldsymbol{P}$. And thus, we want to find $\boldsymbol{d}$.

To begin with, we can get $\frac{AB}{AP} = \frac{FI}{d} \Rightarrow d \cdot \frac{AB}{AP} = FI \Rightarrow d = FI \cdot \frac{AP}{AB}$.

And next, we can get $FI = \log B - \log A$, $AP = P - A$, and $AB = B - A$.

So we get $d = (\log B - \log A) \cdot \frac{P - A}{B - A}$. And we have this:

$A = 5.72$, $\log A = 0.7574$, $B = 5.73$, $\log B = 0.7582$, and $P = 5.724$.

So first, we can get these:

$P - A = 5.724 - 5.72 = 0.004$, $\log B - \log A = 0.7582 - 0.7574 = 0.0008$, and

$B - A = 5.73 - 5.72 = 0.01$.

So next, we can get this:

$$d = \frac{P - A}{B - A} \cdot (\log B - \log A) = \frac{0.004}{0.01} \cdot 0.0008 = 0.4 \cdot 0.0008 = 0.00032.$$

Now, we have $\log A + d = (\log P)_a$, which is an approximation of $\log P = \log 5.724$.

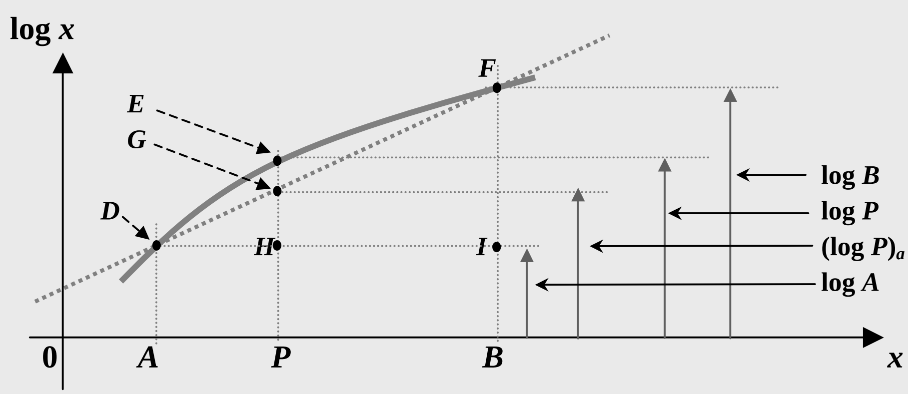
So we get $(\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577$.

Therefore, $\log 5.724 \cong 0.7577$, which is in fact, just about the same as the value we can get using a calculator, too.

In short:

Setting $A = 5.72$, and $B = 5.73$, and putting all the values in a graph, we can get this:

Fig. 0.2 **log 5.72 = 0.7574**, **log 5.73 = 0.7582**, and ***P* = 5.724**



Since ***FI*** is parallel to ***GH***, two triangles ***DIF*** and ***DHG*** are similar to each other.

So setting ***GH* = *d***, and letting **(log *P*)*a*** be the approximation, we can set

$(\log P)_a = \log A + d$, where $d = \frac{P-A}{B-A}\cdot(\log B - \log A)$.

We have these: $P - A = 5.724 - 5.72 = 0.004$, $B - A = 5.73 - 5.72 = 0.01$, and

$\log B - \log A = 0.7582 - 0.7574 = 0.0008$.

So we get $d = \frac{P-A}{B-A}\cdot(\log B - \log A) = \frac{0.004}{0.01}\cdot 0.0008 = 0.4\cdot 0.0008 = 0.00032$.

Thus, $\log 5.724 \cong (\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577$.

Note:

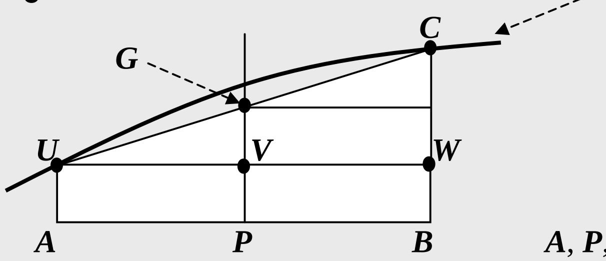
We don't have to take as a formula, $(\log P)_a = \log A + d$ where $d = \frac{P-A}{B-A}\cdot(\log B - \log A)$, nor yet to memorize it. Not using it often, we get to forget it soon anyway.

Like any others in math, it's an idea.
So understanding the idea, and doing some examples, we can get used to it. And then, we can readily derive it by simple geometry on similar triangles.
And getting used to the idea, we can calculate separately the two ratios using two right triangles similar to each other:

One is the ratio between differences in antilogs, and the other is the ratio in logs.

For simplicity,we can make a diagram as below.

Fig. 0.3



a part of the curve of $\boldsymbol{y = \log x}$.

$\boldsymbol{A}$, $\boldsymbol{P}$, and $\boldsymbol{B}$ are on the $\boldsymbol{x}$-axis.

Assuming $\boldsymbol{UABW}$ is a rectangle, and $\boldsymbol{(\log P)_a}$ is the approximation, we can set

$\boldsymbol{G = (P, (\log P)_a)}$, and $\boldsymbol{V = (P, \log A)}$.

Then, setting $\boldsymbol{VG = d}$, we get $\boldsymbol{\log P \cong (\log P)_a = \log A + d}$. So we want to get $\boldsymbol{d}$ now.

In the graph above, we can see two right triangles $\boldsymbol{UVG}$ and $\boldsymbol{UWC}$, which are similar to each other, since $\boldsymbol{VG}$ is parallel to $\boldsymbol{WC}$. So we get this:

$$\boldsymbol{UV : VG = UV : d = UW : WC \Rightarrow \frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}}.$$

Since $\boldsymbol{U}$ and $\boldsymbol{C}$ are points in the curve, we can set

$\boldsymbol{U = (A, \log A)}$, and $\boldsymbol{C = (B, \log B)}$.

And since $\boldsymbol{UABW}$ is a rectangle, we get these :

$\boldsymbol{UV = P - A}$, $\boldsymbol{UW = B - A}$, and $\boldsymbol{WC = \log B - \log A}$.

And in this example, we have these:
$\boldsymbol{A = 5.72}$, $\boldsymbol{\log A = 0.7574}$, $\boldsymbol{B = 5.73}$, $\boldsymbol{\log B = 0.7582}$, and $\boldsymbol{P = 5.724}$.

Then, we get $\boldsymbol{UW = B - A = 5.73 - 5.72 = 0.01}$, $\boldsymbol{UV = P - A = 5.724 - 5.72 = 0.004}$,

and $\boldsymbol{WC = \log B - \log A = 0.7582 - 0.7574 = 0.0008}$,

So we get $\boldsymbol{d = UV \cdot \frac{WC}{UW} = 0.004 \cdot \frac{0.0008}{0.01} = 0.004 \cdot 0.08 = 0.00032}$.

Thus, we get $\boldsymbol{\log 5.724 \cong (\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577}$.

Suggestions or Solutions
To the Problem in the Example 1

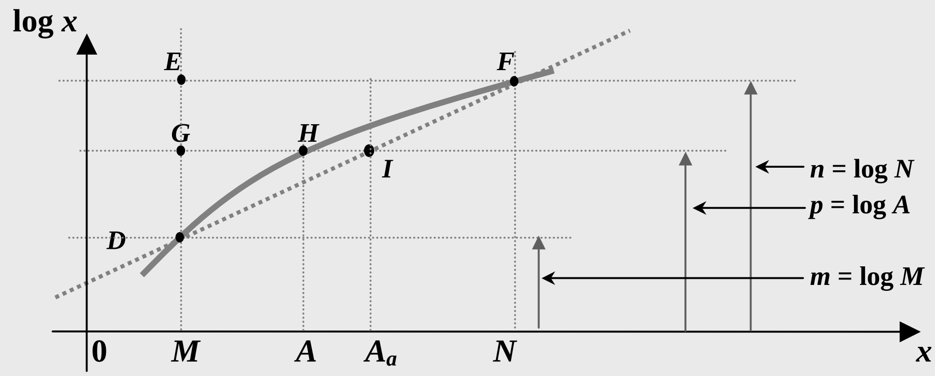
Using log 5.72 = 0.7574 and log 5.73 = 0.7582, find an approximation of the antilog *A* in log *A* = 0.7579 with no calculator.

We have **log 5.72 = 0.7574**, **log 5.73 = 0.7582**, and **log *A* = 0.7579**.

Setting ***M* = 5.72**, and ***N* = 5.73**, we get **log *M* = 0.7574**, and **log *N* = 0.7582**.

Assuming A_a is the approximation, and putting in a graph all the values above, we get this:

Fig. 1.0



Two triangles ***DGI*** and ***DEF*** are similar to each other, because ***GI*** is parallel to ***EF***.

Assuming $GI = d$, we get $A_a = M + d$, where $d = (N - M)\cdot\dfrac{\log A - \log M}{\log N - \log M}$.

We have $N - M = 5.73 - 5.72 = 0.01$, $\log A - \log M = 0.7579 - 0.7574 = 0.0005$, and $\log N - \log M = 0.7582 - 0.7574 = 0.0008$.

Thus, we get $d = (N - M)\cdot\dfrac{\log A - \log M}{\log N - \log M} = 0.01\cdot\dfrac{0.0005}{0.0008} = \dfrac{0.05}{8} = 0.00625$.

Therefore, $A \cong A_a = M + d \cong 5.72 + 0.00625 = 5.72625 \Rightarrow A \cong 5.7263$.

If not quite sure of the idea behind the processes above, follow the steps below.

To begin with, what do we mean by an approximation of an antilog?

By the definition for logs, we get $\boldsymbol{A = 10^{0.7579} \Leftrightarrow 0.7579 = \log A}$, where $\boldsymbol{A}$ is the antilog.

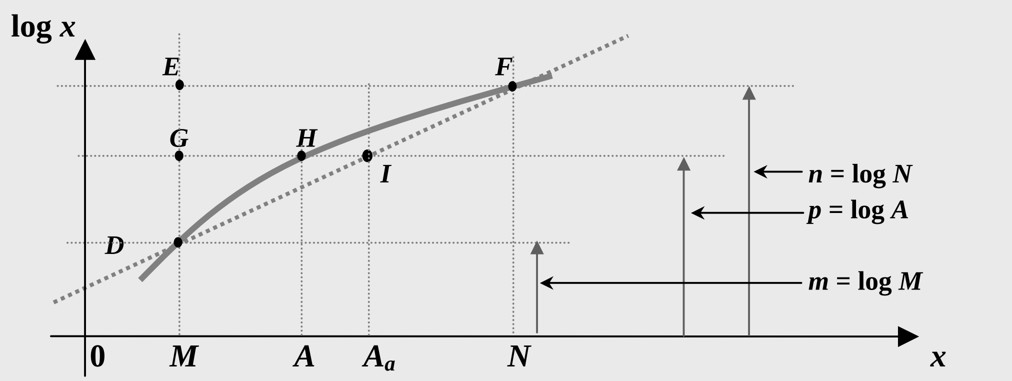
So this problem is asking us to get an approximate value of the power of 10, which is $10^{0.7579}$. How then can we get such an approximation, without a calculator, of course?

Using two ratios that apply to similar triangles, we can get it the way as follows.

Setting first, $\boldsymbol{M = 5.72}$, and $\boldsymbol{N = 5.73}$, we get $\boldsymbol{\log M = 0.7574}$, and $\boldsymbol{\log N = 0.7582}$.

Next, assuming $\boldsymbol{A_a}$ is an approximation of $\boldsymbol{A}$, and putting in a graph all the values above, we can get one as shown below.

Fig. 1.1



In the graph above, assuming that the distance between $\boldsymbol{H}$ and $\boldsymbol{I}$ is sufficiently small, we can get an approximation of $\boldsymbol{A}$ by means of ratios that apply to similar triangles.
And we can get it the way below.

First, two triangles $\boldsymbol{DGI}$ and $\boldsymbol{DEF}$ are similar to each other, since $\boldsymbol{GI}$ is parallel to $\boldsymbol{EF}$.

So we get $\boldsymbol{DG : DE = GI : EF}$. That is, we get $\dfrac{\boldsymbol{DG}}{\boldsymbol{DE}} = \dfrac{\boldsymbol{GI}}{\boldsymbol{EF}}$.

And assuming for simplicity, $\boldsymbol{d = GI}$, we can get this:

$\boldsymbol{M + d = A_a}$, which is an approximation of $\boldsymbol{A}$. So we need to find $\boldsymbol{d}$ now.

First, we can get $\dfrac{\boldsymbol{DG}}{\boldsymbol{DE}} = \dfrac{\boldsymbol{GI}}{\boldsymbol{EF}} \Rightarrow \boldsymbol{GI = EF} \cdot \dfrac{\boldsymbol{DG}}{\boldsymbol{DE}}$, which is $\boldsymbol{d}$.

And next, we can get $\boldsymbol{EF = N - M}$, $\boldsymbol{DG = \log A - \log M}$, and $\boldsymbol{DE = \log N - \log M}$.

So we get $\boldsymbol{d = (N - M) \cdot \dfrac{\log A - \log M}{\log N - \log M}}$.

Now, we put the following values into the equation above.

$\boldsymbol{M = 5.72}$, $\boldsymbol{\log M = 0.7574}$, $\boldsymbol{N = 5.73}$, $\boldsymbol{\log N = 0.7582}$, and $\boldsymbol{\log A = 0.7579}$.

Then, we get $\boldsymbol{N - M} = 5.73 - 5.72 = 0.01$, $\boldsymbol{\log A - \log M} = 0.7579 - 0.7574 = 0.0005$,

and $\boldsymbol{\log N - \log M} = 0.7582 - 0.7574 = 0.0008$.

So we get $\boldsymbol{d = (N - M) \cdot \dfrac{\log A - \log M}{\log N - \log M} = 0.01 \cdot \dfrac{0.0005}{0.0008} = \dfrac{0.05}{8} = 0.00625}$.

Thus, we get $\boldsymbol{A \cong A_a = M + d \cong 5.72 + 0.00625 = 5.72625}$.

Therefore, $\boldsymbol{A \cong 5.7263}$. And in fact, using a calculator, we get $\boldsymbol{\log 5.7263 \cong 0.7579}$.

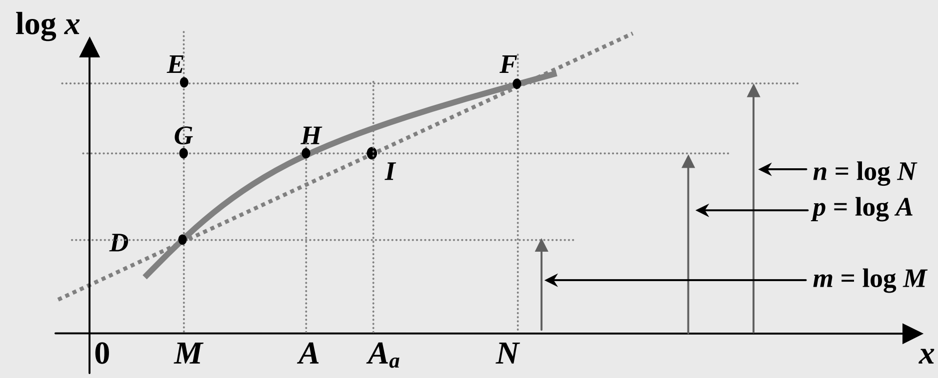
In short:

We have **log 5.72 = 0.7574**, **log 5.73 = 0.7582**, and **log *A* = 0.7579**.

Setting ***M* = 5.72**, and ***N* = 5.73**, we get **log *M* = 0.7574**, and **log *N* = 0.7582**.

Assuming $\boldsymbol{A_a}$ is the approximation, and putting in a graph all the values above, we get this:

Fig. 1.2



Two triangles ***DGI*** and ***DEF*** are similar to each other, because ***GI*** is parallel to ***EF***.

Assuming ***GI = d***, we get $\boldsymbol{A_a = M + d}$, where $\boldsymbol{d = (N - M) \cdot \dfrac{\log A - \log M}{\log N - \log M}}$.

We have ***N* – *M*** = 5.73 – 5.72 = 0.01, **log *A* – log *M*** = 0.7579 – 0.7574 = 0.0005, and

log *N* – log *M* = 0.7582 – 0.7574 = 0.0008.

Thus, we get $\boldsymbol{d = (N - M) \cdot \dfrac{\log A - \log M}{\log N - \log M} = 0.01 \cdot \dfrac{0.0005}{0.0008} = \dfrac{0.05}{8} = 0.00625}$.

Therefore, $\boldsymbol{A \cong A_a = M + d \cong 5.72 + 0.00625 = 5.72625 \Rightarrow A \cong 5.7263}$.

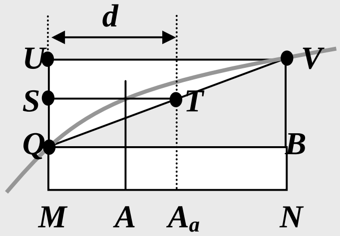
Note:

We don't have to take for a formula, $A_a = M + d$, where $d = (N - M)\cdot\dfrac{\log A - \log M}{\log N - \log M}$,

nor yet to memorize it. Not using it often, we forget it quickly anyway. Understanding the idea, we can readily derive it by simple geometry on similar triangles.
Getting used to the idea, we can calculate separately two ratios that apply to similar right triangles.
One is the ratio between differences in antilogs, and the other is the ratio in logs.

Solving problems in this kind, we may want to use a diagram as below, where the curve in gray is a part of the curve of $y = \log x$.

Fig. 1.3



M, A, A_a, and N are on the x-axis.

Then first, assuming $QMNB$ is a rectangle, A_a is an approximation of A, $T = (A_a, \log A)$, and $d = ST$, we can get $A \cong A_a = M + d$.

Next, assuming $BQUV$ is a rectangle, we get these:

$QU = \log N - \log M$, $UV = N - M$, and $QS = \log A - \log M$.

And next, we can see that two right triangles QST and QUV are similar to each other.

So we get $\dfrac{UV}{ST} = \dfrac{QU}{QS}$. And we can say these:

$\dfrac{UV}{ST}$ is the ratio between differences in antilogs.

$\dfrac{QU}{QS}$ is the ratio between differences in logs.

And we know $\boldsymbol{d = ST}$. So we get this:

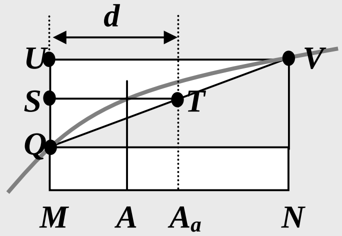
$$\frac{UV}{ST} = \frac{QU}{QS} \Rightarrow \frac{UV}{d} = \frac{QU}{QS} \Rightarrow UV = d \cdot \frac{QU}{QS} \Rightarrow d = UV \cdot \frac{QS}{QU}.$$

Now, in this example, we have these:

$\boldsymbol{M = 5.72}$, $\boldsymbol{\log M = 0.7574}$, $\boldsymbol{N = 5.73}$, $\boldsymbol{\log N = 0.7582}$, and $\boldsymbol{\log A = 0.7579}$.

So putting the information above in the diagram, we get this:

Fig. 1.4



$\boldsymbol{M = 5.72}$, $\boldsymbol{\log A = 0.7579}$, and $\boldsymbol{N = 5.73}$.

Then, taking the ratio between differences in antilogs, we get first,

$\boldsymbol{UV = N - M = 5.73 - 5.72 = 0.01}$, and $\boldsymbol{ST = d}$, so the ratio is $\frac{UV}{d} = \frac{0.01}{d}$.

Next, taking the ratio between differences in logs, we get first,

$\boldsymbol{QU = \log N - \log M = 0.0008}$, and $\boldsymbol{QS = \log A - \log M = 0.0005}$.

So the ratio is $\frac{QU}{QS} = \frac{0.0008}{0.0005} = \frac{8}{5}$.

And we have $\frac{UV}{d} = \frac{QU}{QS}$.

So we get $\frac{0.01}{d} = \frac{8}{5} \Rightarrow d = 0.01 \cdot \frac{5}{8} = \frac{5}{800} = 0.00625$.

Thus, we get $\boldsymbol{A \cong A_a = M + d \cong 5.72 + 0.00625 = 5.72625 \Rightarrow A \cong 5.7263}$.