Examples 2 in Approximations

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Examples 2 in Approximations

Getting an approximation of a log-value, we need to have two neighboring log-values sufficiently close to each other. What if we don't have such two?

There must be such two log-values if the problem is well defined, that is, solvable. So if not shown, they are probably hidden somewhere in the problem, in other words, implied. Somehow thus, we should be able to manage to find them. And then, we use ratios that apply to similar triangles. And the same is true for an approximation of an antilog, too.

Doing the examples below, use no calculator.

0. Assuming $\log 8 = p$, and $\log 81 = q$:

0.0 Find an approximation of $\log 8.4$.

0.1. Find an approximation of 0.84^5.

1. Using the values below

$\log 5.67 = 0.7536$, $\log 5.68 = 0.7543$, $\log 4.82 = 0.6830$, $\log 4.83 = 0.6839$,
$\log 13.8 = 1.1399$, and $\log 13.9 = 1.1430$,

find an approximate value of $\sqrt[3]{\dfrac{0.5673^2(48.26)}{0.1382}}$.

Suggestions or Solutions
To the Problem 0 in the Example 0

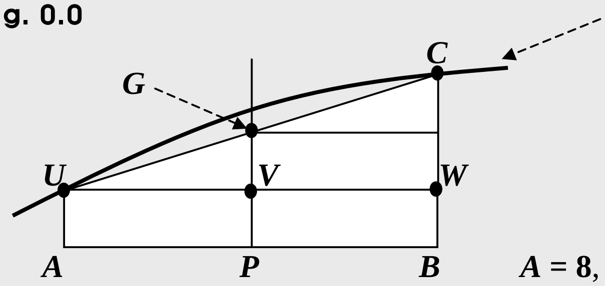
Assuming log 8 = *p*, and log 81 = *q*, find an approximation of log 8.4

We can have $\log 8 < \log 8.4 < \log 9$. And we have $\log 81 = q$.

So we get $\log 81 = \log 9^2 = 2\log 9 = q \Rightarrow \log 9 = 0.5q$.

And below is a diagram that has a part of the curve of $y = \log x$.

Fig. 0.0 a part of the curve of $y = \log x$.



$A = 8$, $P = 8.4$, and $B = 9$ are on the x-axis.

Assuming $UABW$ is a rectangle, and $(\log P)_a$ is the approximation, we can set

$G = (P, (\log P)_a)$, and $V = (P, \log A)$.

Then, setting $VG = d$, we get $\log P \cong (\log P)_a = \log A + d$.

In the graph above, we can see two right triangles UVG and UWC, which are similar to each other, since VG is parallel to WC. So we get

$$UV : VG = UV : d = UW : WC \Rightarrow \tfrac{UV}{d} = \tfrac{UW}{WC} \Rightarrow d = UV \cdot \tfrac{WC}{UW}.$$

Since $UABW$ is a rectangle, and U and C are two points in the curve, we can set

$WC = \log B - \log A = \log 9 - \log 8 = 0.5q - p$,

$UW = B - A = 9 - 8 = 1$, and $UV = P - A = 8.4 - 8 = 0.4$.

So we get $\boldsymbol{d = UV \cdot \frac{WC}{UW} = 0.4 \cdot \frac{0.5q - p}{1} = 0.2q - 0.4p}$.

And thus, $\boldsymbol{\log 8.4 \cong (\log P)_a = \log A + d = \log 8 + d = p + 0.2q - 0.4p = 0.6p + 0.2q}$.

If not quite sure of the idea behind the processes above, follow the steps below.

We need two neighboring log-values to **log 8.4**, and they can be **log 8** and **log 9**.

We have the value of **log 8**, but the value of **log 9** is not given, which is not quite the case though. Why not?

We can get the value of **log 9**, because we are given the value of **log 81**.
How then can we get the value of **log 9**?

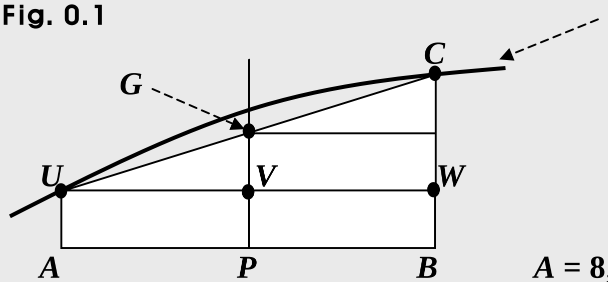
We know $\boldsymbol{\log 81 = \log 9^2 = 2 \log 9}$, and we have $\boldsymbol{\log 81 = q}$.
So we get $\boldsymbol{2 \log 9 = q \Rightarrow \log 9 = 0.5q}$.

Now that we've got the value of **log 9**, we can get an approximate value of **log 8.4**.

Setting first, $\boldsymbol{A = 8}$, $\boldsymbol{P = 8.4}$, and $\boldsymbol{B = 9}$, we can make a diagram as below.

Fig. 0.1

a part of the curve of $\boldsymbol{y = \log x}$.



$\boldsymbol{A = 8}$, $\boldsymbol{P = 8.4}$, and $\boldsymbol{B = 9}$ are on the $\boldsymbol{x}$-axis.

So next, assuming ***UABW*** is a rectangle, and $\boldsymbol{(\log P)_a}$ is the approximation, we can set

$\boldsymbol{G = (P, (\log P)_a)}$, and $\boldsymbol{V = (P, \log A)}$.

Then, setting $\boldsymbol{VG = d}$, we get $\boldsymbol{\log P \cong (\log P)_a = \log A + d}$. So we want to get $\boldsymbol{d}$ now.

In the diagram above, we can see two right triangles ***UVG*** and ***UWC***, which are similar to each other, since ***VG*** is parallel to ***WC***. So we get

$$UV : VG = UV : d = UW : WC \Rightarrow \frac{UV}{d} = \frac{UW}{WC} \Rightarrow UV = \frac{UW}{WC} \cdot d \Rightarrow d = UV \cdot \frac{WC}{UW}.$$

Then, first, since ***U*** and ***C*** are points in the curve, we can set

$U = (A, \log A)$, and $C = (B, \log B)$.

Next, since ***UABW*** is a rectangle, we get

$UV = P - A$, $UW = B - A$, and $WC = \log B - \log A$.

And in this example, we have

$A = 8$, $\log A = p$, $B = 9$, $\log B = 0.5q$, and $P = 8.4$.

Then, we get $UW = B - A = 9 - 8 = 1$, $UV = P - A = 8.4 - 8 = 0.4$,

and $WC = \log B - \log A = 0.5q - p$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.4 \cdot \frac{0.5q - p}{1} \Rightarrow d = 0.4(0.5q - p) = 0.2q - 0.4p$.

And thus, $\log 8.4 \cong (\log P)_a = \log A + d = p + 0.2q - 0.4p = 0.6p + 0.2q$.

In short:

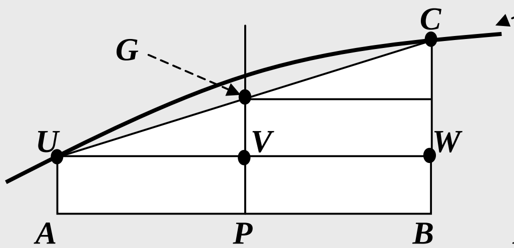
We can have $\log 8 < \log 8.4 < \log 9$. And we have $\log 81 = q$.

So we get $\log 81 = \log 9^2 = 2 \log 9 = q \Rightarrow \log 9 = 0.5q$.

And below is a diagram that has a part of the curve of $y = \log x$.

Fig. 0.2

a part of the curve of $y = \log x$.



$A = 8$, $P = 8.4$, and $B = 9$ are on the x-axis.

Assuming $UABW$ is a rectangle, and $(\log P)_a$ is the approximation, we can set

$G = (P, (\log P)_a)$, and $V = (P, \log A)$.

Then, setting $VG = d$, we get $\log P \cong (\log P)_a = \log A + d$.

In the graph above, we can see two right triangles UVG and UWC, which are similar to each other, since VG is parallel to WC. So we get

$$UV : VG = UV : d = UW : WC \Rightarrow \frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}.$$

Since $UABW$ is a rectangle, and U and C are two points in the curve, we can set

$WC = \log B - \log A = \log 9 - \log 8 = 0.5q - p$,

$UW = B - A = 9 - 8 = 1$, and $UV = P - A = 8.4 - 8 = 0.4$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.4 \cdot \frac{0.5q - p}{1} = 0.2q - 0.4p$.

And thus, $\log 8.4 \cong (\log P)_a = \log A + d = \log 8 + d = p + 0.2q - 0.4p = 0.6p + 0.2q$.

Suggestions or Solutions
To the Problem 1 in the Example 0

Assuming $\log 8 = p$, and $\log 81 = q$, find an approximation of 0.84^5.

$\log 0.84^5 = 5 \log 0.84 = 5\log\frac{8.4}{10} = 5(\log 8.4 - \log 10) = 5(\log 8.4 - 1)$

$\cong 5(0.6p + 0.2q - 1) = 3p + q - 5$. And we have $\log 8 = p$, and $\log 81 = q$.

So we get $\log 0.84^5 \cong 3p + q - 5 = 3 \log 8 + \log 81 - 5$

$= \log 512 + \log 81 - 5 \log 10 = \log (512 \cdot 81) - 5 \log 10 = \log 41472 + \log 10^{-5}$

$= \log (41472 \cdot 10^{-5}) = \log 0.41472$. Therefore, $0.84^5 \cong 0.41472$.

If not quite sure of the idea behind the processes above, follow the steps below.

For this example, we can use the approximation of **log 8.4** found above (Problem 0).

Let's first, express $\log 0.84^5$ using **log 8.4**. Then, we get

$\log 0.84^5 = 5 \log 0.84 = 5\log\frac{8.4}{10} = 5(\log 8.4 - \log 10) = 5(\log 8.4 - 1)$.

And we know $\log 8.4 \cong 0.6p + 0.2q$.

So we get $\log 0.84^5 \cong 5(0.6p + 0.2q - 1) = 3p + q - 5$.

And we have $\log 8 = p$, and $\log 81 = q$.

So we get $\log 0.84^5 \cong 3p + q - 5 = 3 \log 8 + \log 81 - 5$.

And we have $\mathbf{3 \log 8 = \log 8^3 = \log 512}$, and $\mathbf{5 = 5 \log 10}$.

So we get $\mathbf{3 \log 8 + \log 81 - 5 = \log 512 + \log 81 - 5 \log 10 = \log (512 \cdot 81) - 5 \log 10}$

$\mathbf{= \log 41472 + (-5) \log 10 = \log 41472 + \log 10^{-5} = \log (41472 \cdot 10^{-5}) = \log 0.41472}$.

Therefore, $\mathbf{0.84^5 \cong 0.41472}$. Using a calculator, we get $\mathbf{0.84^5 = 0.4182119424}$.

In short:

$\mathbf{\log 0.84^5 = 5 \log 0.84 = 5 \log \frac{8.4}{10} = 5(\log 8.4 - \log 10) = 5(\log 8.4 - 1)}$

$\mathbf{\cong 5(0.6\boldsymbol{p} + 0.2\boldsymbol{q} - 1) = 3\boldsymbol{p} + \boldsymbol{q} - 5}$. And we have $\mathbf{\log 8 = \boldsymbol{p}}$, and $\mathbf{\log 81 = \boldsymbol{q}}$.

So we get $\mathbf{\log 0.84^5 \cong 3\boldsymbol{p} + \boldsymbol{q} - 5 = 3 \log 8 + \log 81 - 5}$

$\mathbf{= \log 512 + \log 81 - 5 \log 10 = \log (512 \cdot 81) - 5 \log 10 = \log 41472 + \log 10^{-5}}$

$\mathbf{= \log (41472 \cdot 10^{-5}) = \log 0.41472}$. Therefore, $\mathbf{0.84^5 \cong 0.41472}$.

Suggestions or Solutions
To the Problem in the Example 1

We have these:

log 5.72 = 0.7574, log 5.73 = 0.7582, log 3.58 = 0.5539, log 3.59 = 0.5551,

log 29.5 = 1.4698, log 29.6 = 1.4713, log 3.30 = 0.5185, and log 3.31 = 0.5198.

Now, find an approximate value of $\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$.

All the values given are log-values.
And solving this problem, we are going to make use of them, of course.

However, the value we want to get the approximation of is not a log-value. So?

We may want to put it in a common log. That is, we take the log of $\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$.

Then, we get $\mathbf{log}\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$, the value of which is a log-value.   So what?

We can use some of the values given so that we can get an approximate value of the log above. And after taking the approximation, we can get the approximation of the antilog, which is this: $\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$. How?

We find first, the approximate value of $\mathbf{log}\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$ using the log values given, and then, using the approximate value found, we get the approximate value of $\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$.

So let’s now, get the approximation of the log value first.

Then, setting first $r = \sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$, and taking the common logs of both sides, we get

$$\log r = \log\{\tfrac{0.5724^2 \cdot 3583}{2.953}\}^{\frac{1}{5}} = \tfrac{1}{5}\log\{\tfrac{0.5724^2 \cdot 3583}{2.953}\} = \tfrac{1}{5}\{\log(0.5724^2 \cdot 3583) - \log 2.953\}$$

$$= \tfrac{1}{5}\{(\log 0.5724^2 + \log 3583) - \log 2.953\} = \tfrac{1}{5}(2\log 0.5724 + \log 3583 - \log 2.953).$$

Now, in the expression above, we have **log 0.5727**, **log 3583**, and **log 2.953**, the values of which are however, not given. Then, what are we going to do with those?

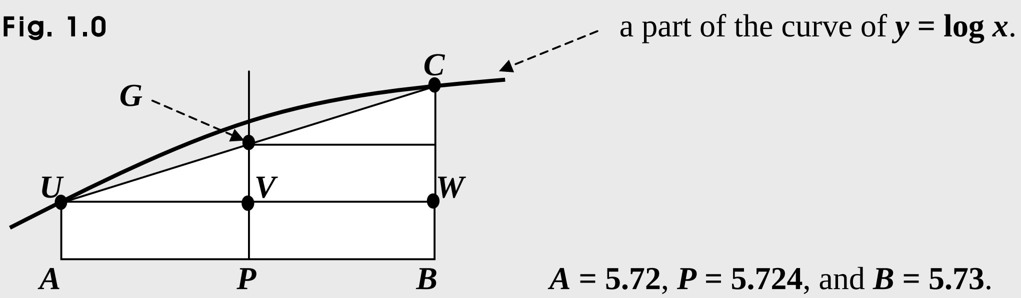
We want to take approximations of those. However, we are not given values close to them, that is, we are not given values neighboring each of those.

We are given values though, close to each of **log 5.724**, **log 3.583**, and **log 29.53**.

For instance, for **log 5.724**, we are given **log 5.72 = 0.7574**, and **log 5.73 = 0.7582**.

So let's now, begin with getting an approximation of **log 5.724**.
Then first, we can set up a diagram as follows.

Fig. 1.0



Assuming ***UABW*** is a rectangle, **(log *P*)*a*** is the approximation, and ***VG* = *d***, we can set **log *P* ≅ (log *P*)*a* = log *A* + *d***. So we want to get ***d*** now.

In the graph above, we can see two right triangles ***UVG*** and ***UWC***, which are similar to each other, since ***VG*** is parallel to ***WC***. So we get

$$UV : VG = UV : d = UW : WC \Rightarrow \tfrac{UV}{d} = \tfrac{UW}{WC} \Rightarrow d = UV \cdot \tfrac{WC}{UW}.$$

And since ***UABW*** is a rectangle, we get

$\boldsymbol{UV = P - A}$, $\boldsymbol{UW = B - A}$, and $\boldsymbol{WC = \log B - \log A}$.

And in this case, we have

$\boldsymbol{A = 5.72}$, $\boldsymbol{\log A = 0.7574}$, $\boldsymbol{B = 5.73}$, $\boldsymbol{\log B = 0.7582}$, and $\boldsymbol{P = 5.724}$.

Then, we get $\boldsymbol{UW = B - A = 5.73 - 5.72 = 0.01}$, $\boldsymbol{UV = P - A = 5.724 - 5.72 = 0.004}$,

and $\boldsymbol{WC = \log B - \log A = 0.7582 - 0.7574 = 0.0008}$.

So we get $\boldsymbol{d = UV \cdot \frac{WC}{UW} = 0.004 \cdot \frac{0.0008}{0.01} = 0.004 \cdot 0.08 = 0.00032}$.

Thus, we get $\boldsymbol{\log 5.724 \cong (\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577}$.

Therefore, we get $\boldsymbol{\log 0.5724 \cong -1 + 0.7577 = \bar{1}.7577}$. Why -1, though?

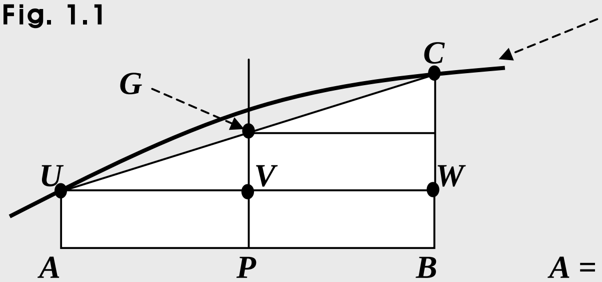
The highest place value in 0.5724 is $\boldsymbol{10^{-1}}$, and -1 is the characteristic.

Now, by the same token, we can get an approximation of **log 3.583**, and then, **log 3583**.

We have $\boldsymbol{\log 3.58 = 0.5539}$, and $\boldsymbol{\log 3.59 = 0.5551}$.

Then again, we can set up a diagram as follows.

Fig. 1.1



a part of the curve of $\boldsymbol{y = \log x}$.

$\boldsymbol{A = 3.58}$, $\boldsymbol{P = 3.583}$, and $\boldsymbol{B = 3.59}$.

Assuming first, $\boldsymbol{(\log P)_a}$ is the approximation, and $\boldsymbol{VG = d}$, we can set

$\boldsymbol{\log P \cong (\log P)_a = \log A + d}$. So we want to get ***d*** now.

Since two right triangles UVG and UWC are similar to each other, we get

$$UV : VG = UV : d = UW : WC \Rightarrow \frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}.$$

And since $UABW$ is a rectangle, we get

$UV = P - A$, $UW = B - A$, and $WC = \log B - \log A$.

And in this case, we have

$A = 3.58$, $\log A = 0.5539$, $B = 3.59$, $\log B = 0.5551$, and $P = 3.583$.

Then, we get $UW = B - A = 3.59 - 3.58 = 0.01$, $UV = P - A = 3.583 - 3.58 = 0.003$, and $WC = \log B - \log A = 0.5551 - 0.5539 = 0.0012$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.003 \cdot \frac{0.0012}{0.01} = 0.003 \cdot 0.12 = 0.00036$.

Thus, we get $\log 3.583 \cong (\log P)_a = \log A + d = 0.5539 + 0.00036 = 0.55426 \cong 0.5543$.

Therefore, we get $\log 3538 \cong 3 + 0.5543 = 3.5543$. Why 3 though, above the point?

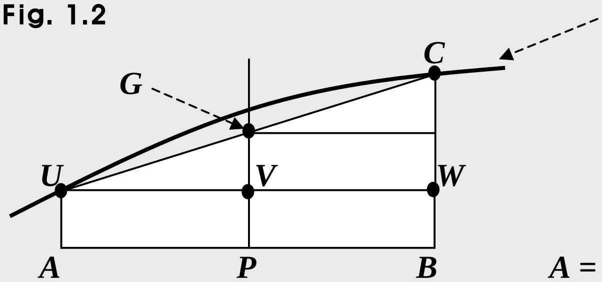
The highest place value in 3538 is 10^3, and 3 is the characteristic.

Likewise, we can get an approximation of $\log 29.53$, and then, $\log 2.953$.

We have $\log 29.5 = 1.4698$, and $\log 29.6 = 1.4713$.

Then again, we can set up a diagram as follows.

Fig. 1.2



$A = 29.5$, $P = 29.53$, and $B = 29.6$.

Assuming first, $(\log P)_a$ is the approximation, and $VG = d$, we can set

$\log P \cong (\log P)_a = \log A + d$. So we want to get d now.

Next, UVG and UWC are similar triangles, so we get $\frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}$.

And since $UABW$ is a rectangle, we get

$UV = P - A$, $UW = B - A$, and $WC = \log B - \log A$.

And in this case, we have

$A = 29.5$, $\log A = 1.4698$, $B = 29.6$, $\log B = 1.4713$, and $P = 29.53$.

Then, we get $UW = B - A = 29.6 - 29.5 = 0.01$, $UV = P - A = 29.53 - 29.5 = 0.03$,

and $WC = \log B - \log A = 1.4713 - 1.4698 = 0.0015$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.03 \cdot \frac{0.0015}{0.01} = 0.03 \cdot 0.15 = 0.0045$.

Thus, we get $\log 29.53 \cong (\log P)_a = \log A + d = 1.4698 + 0.0045 = 1.4743$.

Therefore, we get $\log 2.953 \cong 0.4743$.   Why 0 though, above the point?

The highest place value in 2.953 is 10^0, and 0 is the characteristic.

Now, we have $\log r = \frac{1}{5}(2\log 0.5724 + \log 3583 - \log 2.953)$.

And we have $\log 0.5724 \cong \bar{1}.7577$, $\log 3538 \cong 3.5543$, and $\log 2.953 \cong 0.4743$.

So we get $\log r \cong \frac{1}{5}\{2(-1 + 0.7577) + (3 + 0.5543) - 0.4743\}$

$= \frac{1}{5}(-2 + 1.5154 + 3 + 0.5543 - 0.4743) = \frac{2.5954}{5} = 0.51908 \cong 0.5191$.

Thus, we get $\log r \cong 0.5191$.   What then is the next?

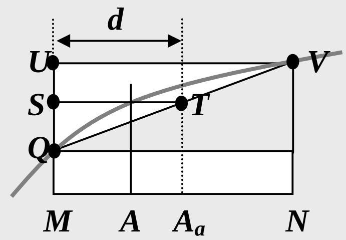
Finally, we want to take an approximation of $\boldsymbol{r}$, which is $\sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$.

We have **log 3.30 = 0.5185**, **log 3.31 = 0.5198**, and $\mathbf{log}\ \boldsymbol{r} \cong \mathbf{0.5191}$.

So we are now taking an approximation of an antilog.

To begin with, setting $\boldsymbol{r = A}$, we can set up a diagram as below, where the curve in gray is a part of the curve of $\boldsymbol{y} = \mathbf{log}\ \boldsymbol{x}$.

Fig. 1.3



$\boldsymbol{M}$, $\boldsymbol{A}$, $\boldsymbol{A_a}$, and $\boldsymbol{N}$ are on the $\boldsymbol{x}$-axis.

Then first, assuming $\boldsymbol{QMNB}$ is a rectangle, $\boldsymbol{A_a}$ is an approximation of $\boldsymbol{A}$, $\boldsymbol{T} = (\boldsymbol{A_a}, \mathbf{log}\ \boldsymbol{A})$, and $\boldsymbol{d = ST}$, we can get $\boldsymbol{A \cong A_a = M + d}$.

Next, assuming $\boldsymbol{BQUV}$ is a rectangle, we get

$\boldsymbol{QU} = \mathbf{log}\ \boldsymbol{N} - \mathbf{log}\ \boldsymbol{M}$, $\boldsymbol{UV = N - M}$, and $\boldsymbol{QS} = \mathbf{log}\ \boldsymbol{A} - \mathbf{log}\ \boldsymbol{M}$.

And next, we can see that two right triangles $\boldsymbol{QST}$ and $\boldsymbol{QUV}$ are similar to each other.

So we get $\dfrac{\boldsymbol{UV}}{\boldsymbol{ST}} = \dfrac{\boldsymbol{QU}}{\boldsymbol{QS}}$. And we can say these:

$\dfrac{\boldsymbol{UV}}{\boldsymbol{ST}}$ is the ratio between differences in antilogs.

$\dfrac{\boldsymbol{QU}}{\boldsymbol{QS}}$ is the ratio between differences in logs.

And we know $\boldsymbol{d = ST}$. So we get

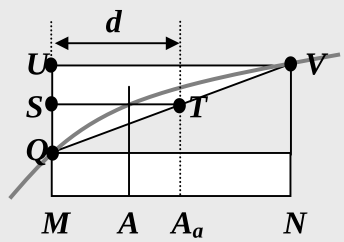
$$\frac{UV}{ST} = \frac{QU}{QS} \Rightarrow \frac{UV}{d} = \frac{QU}{QS} \Rightarrow UV = d \cdot \frac{QU}{QS} \Rightarrow d = UV \cdot \frac{QS}{QU}.$$

Now, in this example, we have

$\boldsymbol{M = 3.30}$, $\boldsymbol{\log M = 0.5185}$, $\boldsymbol{N = 3.31}$, $\boldsymbol{\log N = 0.5198}$, and $\boldsymbol{\log A = 0.5191}$.

So putting the information above in the diagram, we get this:

Fig. 1.4



$\boldsymbol{M = 3.30}$, $\boldsymbol{\log A = 0.5191}$, and $\boldsymbol{N = 3.31}$.

Then, taking the ratio between differences in antilogs, we get first,

$\boldsymbol{UV = N - M = 3.31 - 3.30 = 0.01}$, and $\boldsymbol{ST = d}$, so the ratio is $\dfrac{UV}{d} = \dfrac{0.01}{d}$.

Next, taking the ratio between differences in logs, we get first,

$\boldsymbol{QU = \log N - \log M = 0.0013}$, and $\boldsymbol{QS = \log A - \log M = 0.0006}$.

So the ratio is $\dfrac{QU}{QS} = \dfrac{0.0013}{0.0006} = \dfrac{13}{6}$. And we have $\dfrac{UV}{d} = \dfrac{QU}{QS}$.

So we get $\dfrac{0.01}{d} = \dfrac{13}{6} \Rightarrow d = 0.01 \cdot \dfrac{6}{13} = \dfrac{6}{1300} = 0.00462$.

Thus, we get $\boldsymbol{A \cong A_a = M + d \cong 3.30 + 0.00462 = 3.30462}$

$\Rightarrow \boldsymbol{A = r = \sqrt[5]{\tfrac{0.5724^2 \cdot 3583}{2.953}} \cong 3.3046}$. Using a calculator, we get $\boldsymbol{\sqrt[5]{\tfrac{0.5724^2 \cdot 3583}{2.953}} \cong 3.3104}$.

In short:

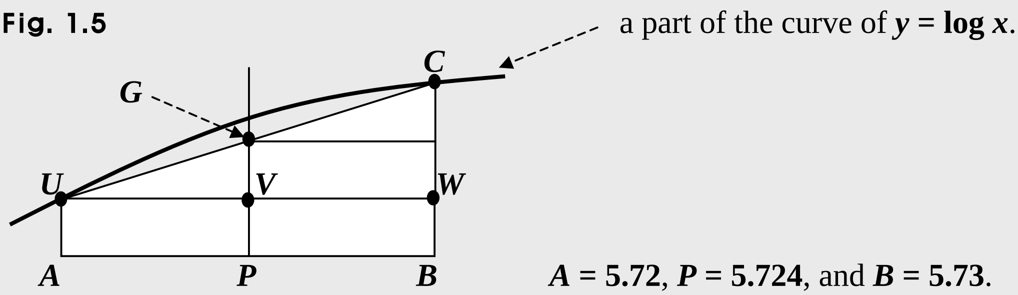
Setting first, $\boldsymbol{r} = \sqrt[5]{\frac{0.5724^2 \cdot 3583}{2.953}}$, and taking the common logs of both sides, we get

$$\log r = \log\{\tfrac{0.5724^2 \cdot 3583}{2.953}\}^{\frac{1}{5}} = \tfrac{1}{5}\log\{\tfrac{0.5724^2 \cdot 3583}{2.953}\} = \tfrac{1}{5}\{\log(0.5724^2 \cdot 3583) - \log 2.953\}$$

$$= \tfrac{1}{5}\{(\log 0.5724^2 + \log 3583) - \log 2.953\} = \tfrac{1}{5}(2\log 0.5724 + \log 3583 - \log 2.953).$$

Let's next, get an approximation of **log 5.724**, and then, **log 0.5724**.
Then first, we can set up a diagram as follows.

Fig. 1.5



A **= 5.72**, ***P*** **= 5.724**, and ***B*** **= 5.73**.

Assuming ***UABW*** is a rectangle, **(log *P*)*a*** is the approximation, and ***VG*** **=** ***d***, we can set
$\log P \cong (\log P)_a = \log A + d$.

Since ***UVG*** and ***UWC*** are similar triangles, we get $\frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}$.

And since ***UABW*** is a rectangle, we get

$UV = P - A = 5.724 - 5.72 = 0.004$,
$UW = B - A = 5.73 - 5.72 = 0.01$, and $WC = \log B - \log A = 0.7582 - 0.7574 = 0.0008$.

So we get: $d = UV \cdot \frac{WC}{UW} = 0.004 \cdot \frac{0.0008}{0.01} = 0.004 \cdot 0.08 = 0.00032$.

Thus, we get $\log 5.724 \cong (\log P)_a = \log A + d = 0.7574 + 0.00032 = 0.75772 \cong 0.7577$.

Therefore, we get $\log 0.5724 \cong -1 + 0.7577 = \bar{1}.7577$.

Next, by the same token, we can get an approximation of **log 3.583**, and then, **log 3583**.

We have **log 3.58 = 0.5539**, and **log 3.59 = 0.5551**.

Then again, uisng the diagram above, taking $(\log P)_a$ as the approximation, and assuming $VG = d$, we can set $\log P \cong (\log P)_a = \log A + d$.

And we get $\frac{UV}{d} = \frac{UW}{WC} \Rightarrow d = UV \cdot \frac{WC}{UW}$.

And in this case, we have
$A = 3.58$, $\log A = 0.5539$, $B = 3.59$, $\log B = 0.5551$, and $P = 3.583$.

Then, $UW = B - A = 0.01$, $UV = P - A = 0.003$, and $WC = \log B - \log A = 0.0012$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.003 \cdot \frac{0.0012}{0.01} = 0.003 \cdot 0.12 = 0.00036$.

Thus, we get $\log 3.583 \cong (\log P)_a = \log A + d = 0.5539 + 0.00036 = 0.55426 \cong 0.5543$.

Therefore, we get $\log 3538 \cong 3 + 0.5543 = 3.5543$.

Likewise, we can get an approximation of **log 29.53**, and then, **log 2.953**.

And in this case, we have
$A = 29.5$, $\log A = 1.4698$, $B = 29.6$, $\log B = 1.4713$, and $P = 29.53$.

Then, $UW = B - A = 0.01$, $UV = P - A = 0.03$, and $WC = \log B - \log A = 0.0015$.

So we get $d = UV \cdot \frac{WC}{UW} = 0.03 \cdot \frac{0.0015}{0.01} = 0.03 \cdot 0.15 = 0.0045$.

Thus, we get $\log 29.53 \cong (\log P)_a = \log A + d = 1.4698 + 0.0045 = 1.4743$.

Therefore, we get $\log 2.953 \cong 0.4743$.

Now, we have $\log r = \frac{1}{5}(2\log 0.5724 + \log 3583 - \log 2.953)$.

And we have $\log 0.5724 \cong \bar{1}.7577$, $\log 3538 \cong 3.5543$, and $\log 2.953 \cong 0.4743$.

So we get $\mathbf{\log} \boldsymbol{r} \cong \frac{1}{5}\mathbf{\{2(-1 + 0.7577) + (3 + 0.5543) - 0.4743\}} = \frac{\mathbf{2.5954}}{\mathbf{5}} = \mathbf{0.51908} \cong \mathbf{0.5191}$.

Thus, we get $\mathbf{\log}\ \boldsymbol{r} \cong \mathbf{0.5191}$.

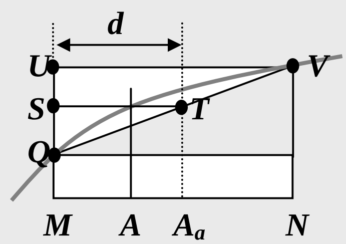
So we can now take an approximation of $\boldsymbol{r}$, which is $\sqrt[5]{\frac{\mathbf{0.5724^2 \cdot 3583}}{\mathbf{2.953}}}$.

We have $\mathbf{\log 3.30 = 0.5185}$, $\mathbf{\log 3.31 = 0.5198}$, and $\mathbf{\log}\ \boldsymbol{r} \cong \mathbf{0.5191}$.

So we are now taking an approximation of an antilog.

To begin with, setting $\boldsymbol{r = A}$, we can set up a diagram as below, where the curve in gray is a part of the curve of $\boldsymbol{y} = \mathbf{\log}\ \boldsymbol{x}$.

Fig. 1.6



$\boldsymbol{M}$, $\boldsymbol{A}$, $\boldsymbol{A_a}$, and $\boldsymbol{N}$ are on the $\boldsymbol{x}$-axis.

Then first, assuming $\boldsymbol{QMNB}$ is a rectangle, $\boldsymbol{A_a}$ is an approximation of $\boldsymbol{A}$, $\boldsymbol{T} = (\boldsymbol{A_a}, \mathbf{\log}\ \boldsymbol{A})$, and $\boldsymbol{d = ST}$, we can get $\boldsymbol{A \cong A_a = M + d}$.

And next, since $\boldsymbol{QST}$ and $\boldsymbol{QUV}$ are similar triangles, we get $\boldsymbol{d = UV} \cdot \frac{\boldsymbol{QS}}{\boldsymbol{QU}}$.

Now, in this example, we have

$\boldsymbol{M} = \mathbf{3.30}$, $\mathbf{\log}\ \boldsymbol{M} = \mathbf{0.5185}$, $\boldsymbol{N} = \mathbf{3.31}$, $\mathbf{\log}\ \boldsymbol{N} = \mathbf{0.5198}$, and $\mathbf{\log}\ \boldsymbol{A} = \mathbf{0.5191}$.

Then, taking the ratio between differences in antilogs, we get first,

$\boldsymbol{UV = N - M} = \mathbf{3.31 - 3.30 = 0.01}$, and $\boldsymbol{ST = d}$, so the ratio is $\frac{\boldsymbol{UV}}{\boldsymbol{d}} = \frac{\mathbf{0.01}}{\boldsymbol{d}}$.

Next, taking the ratio between differences in logs, we get first,

$\boldsymbol{QU} = \mathbf{\log}\ \boldsymbol{N} - \mathbf{\log}\ \boldsymbol{M} = \mathbf{0.0013}$, and $\boldsymbol{QS} = \mathbf{\log}\ \boldsymbol{A} - \mathbf{\log}\ \boldsymbol{M} = \mathbf{0.0006}$.

So the ratio is $\frac{\boldsymbol{QU}}{\boldsymbol{QS}} = \frac{\mathbf{0.0013}}{\mathbf{0.0006}} = \frac{\mathbf{13}}{\mathbf{6}}$. And we have $\frac{\boldsymbol{UV}}{\boldsymbol{d}} = \frac{\boldsymbol{QU}}{\boldsymbol{QS}}$.

So we get $\frac{\mathbf{0.01}}{\boldsymbol{d}} = \frac{\mathbf{13}}{\mathbf{6}} \Rightarrow \boldsymbol{d} = \mathbf{0.01} \cdot \frac{\mathbf{6}}{\mathbf{13}} = \frac{\mathbf{6}}{\mathbf{1300}} = \mathbf{0.00462}$.

Thus, we get $\boldsymbol{A} \cong \boldsymbol{A_a} = \boldsymbol{M} + \boldsymbol{d} \cong \mathbf{3.30 + 0.00462 = 3.30462}$

$\Rightarrow \boldsymbol{A} = \boldsymbol{r} = \sqrt[5]{\frac{\mathbf{0.5724^2 \cdot 3583}}{\mathbf{2.953}}} \cong \mathbf{3.3046}$.

Using a calculator, we get $\sqrt[5]{\frac{\mathbf{0.5724^2 \cdot 3583}}{\mathbf{2.953}}} \cong \mathbf{3.3104}$.