Examples 1 in Ratios and Rates

Click this to start.

What is a ratio?

A dictionary says it's a relationship in quantity, amount, or size between two or more things. It is.
So it is ***a math expression*** where we can see ***a relationship in quantity***, amount, or size between two or more objects.

Usually or oftentimes though, a ratio is an amount.
What amount then is it?

> ***A ratio*** indicates how much an amount is of the other amount or other amounts. So it's ***a relative amount***.

Thus, a ratio says how big or small in proportion an amount is compared to other amount. So it's not a difference but ***a proportion as being twice or a half***.

Using thus, a ratio, we can compare two or more amounts in proportion.

Oftentimes though, we use a ratio between two amounts.

So comparing two amounts in proportion, we use a ratio.

Suppose for instance, X = 6 and Y = 2.

Then, we can put ***the ratio of X to Y*** the way as follows.

X:Y = 3:1, which is a math expression read as

“X to Y equals 3 to 1.”, or just simply “X to Y is 3 to 1.”

Then, taking the amount indicating ***the ratio of X to Y***, we do this: X ÷ Y, and we get $\frac{X}{Y} = \frac{6}{2} = \frac{3}{1} = 3$,

which is ***the relative amount of X to Y***,

and thus, is the amount of X relative to the amount of Y.

In short, 3 is X relative to Y. So 3 is ***the ratio of X to Y***.

Thus, the ratio of X to Y is $\frac{X}{Y}$.

So “X:Y = 3:1” is saying that ***the ratio of X to Y*** is 3,
which means X is three times Y,
that is, X = 3Y. What then about the ratio of Y to X?

We know X = 6 and Y = 2. So the ratio of Y to X is $\frac{1}{3}$,
because taking the ratio of Y to X, we get $\frac{\boldsymbol{Y}}{\boldsymbol{X}} = \frac{2}{6} = \frac{1}{3}$.

Then, we can put it the way as follows. Y:X = 1:3, which is read as “Y to X equals 1 to 3.”, or simply “Y to X is 1 to 3.”.
Thus, “Y:X = 1:3” is saying that ***the ratio of Y to X*** is $\frac{1}{3}$,
which means Y is a third of X, that is, $\boldsymbol{Y} = \frac{1}{3}\boldsymbol{X} = \frac{\boldsymbol{X}}{3}$.

So what is a ratio?

A ratio is a relative amount.
And of course, it’s a math expression that shows a relationship in quantity between two or more things.

For instance, we can have A:B = 2:1 and A:B:C = 1:2:3, which is saying that C is three times A, and also, is three halves of B, that is, $\frac{3}{2}$B. Why?

Since B:C = 2:3, we get C:B = 3:2, so the ratio of C to B is $\frac{\boldsymbol{C}}{\boldsymbol{B}} = \frac{3}{2}$, and thus, we get $\boldsymbol{C} = \frac{3}{2}\boldsymbol{B}$.

So what is a ratio?

> ***A ratio*** indicates how much an amount is of the other amount or other amounts. So it's ***a relative amount***.

And of course, a ratio is a math expression indicating a relationship in quantity between two or more objects.

Let's now do more examples.

Examples 1 in Ratios and Rates

Find and show how much one is of the other using the ratio stated in each of the problems 0 - 8 below.

0. A:B = 20:10

1. C:D = 25:5

2. A:C = 35:15

3. A:B = 17:7

4. B:D = 19:5

5. A:B:C = 10:20:30

6. A:C:D = 15:35:45

7. C:D:E = 25:50:183

8. A:B:C:D = 2:4:2:8

9. Assuming A:B = 1:2, and B:C = 2:1, find A:B:C.

10. Assuming A:B = 1:2, and B:C = 3:1, find A:C.

11. Assuming A:B = 2:3, and A:C = 5:1, find B:C.

12. Assuming A:B:C = 3:4:5, and B:D = 2:1, find A:B:C:D.

13. Assuming A:B:C = 3:5:17, and B:D = 3:2,
find A:B:C:D.

14. Assuming A:B:C = 2:8:9, B:E = 2:3, and B:D = 4:3,
find A:B:C:D:E.

Suggestions or Solutions
To the Problems in the Examples 1

0. A:B = 20:10

We can put the same ratio many different ways.

For instance, A:B = 20:10 = 2:1 = 4:2 = 12:6 = 3:1.5

And taking the ratio of A to B, we get

A:B = 20:10 $\Rightarrow$ $\frac{A}{B}=\frac{20}{10}=\frac{2}{1}=2$ $\Rightarrow$ A = 2B.

So the ratio of A to B is 2.

Thus, the amount of A is twice the amount of B.

In short, A is twice B.

Usually, expressing a ratio, we simplify the ratio.

And simplifying a ratio, we keep dividing by a common divisor all the numbers used in the ratio until all the numbers are prime to each other.

For instance, 3 is a divisor common to 24 and 30, so we can call 3 a common divisor.
And if numbers are prime to each other, the numbers have no common divisor other than 1. For instance, 2 and 1 are prime to each other, 8 and 21 are prime to each other, and 15 and 14 are prime to each other.

Now, for instance, simplifying a ratio A:B = 168:210, we reduce the numbers 168 and 210 so that the numbers used in the ratio get prime to each other.

And reducing numbers, we keep dividing the numbers by a divisor common to the numbers until the numbers are prime to each other.

So for instance, simplifying A:B = 168:210, we can reduce them the way as follows. 168:210, 84:105, 28:35, 4:5, which is now fully simplified, since 4 and 5 are prime to each other.

Thus, simplifying A:B = 168:210, we get A:B = 4:5.

1. C:D = 25:5

We can put the same ratio many different ways.

So we can have this:

C:D = 25:5 = 5:1 = 0.5:0.1 = 5000:1000

5 and 1 are prime to each other, which means 5 and 1 share no common divisor other than 1.

Normally, we use the simplest form, so we usually put the ratio the way as follows. C:D = 5:1.

C:D = 25:5 $\Rightarrow \frac{C}{D} = \frac{25}{5} = \frac{5}{1} = 5 \Rightarrow$ C = 5D.

So the ratio of C to D is 5.

Thus, the amount of C is five times the amount of D, and in short, C is five times D.

2. A:C = 35:15

We can put the same ratio many different ways.

So we can have A:C = 35:15 = 7:3 = 70:30 = 140:60

Now, 7 and 3 are prime to each other.

So we normally put the ratio the way as follows. A:C = 7:3.

And taking the ratio of A to C, we get

A:C = 35:15 $\Rightarrow \frac{A}{C} = \frac{35}{15} = \frac{7}{3} \Rightarrow$ A = $\frac{7}{3}$C.

So the ratio of A to C is $\frac{7}{3}$.

Thus, the amount of A is seven thirds of the amount of C.

In short, A is seven thirds of C.

3. A:B = 17:7

17 and 7 are prime to each other.

We can put the same ratio many different ways.

So we can have A:B = 17:7 = 34:14 = 17x3:7x3 = 51:21

And taking the ratio of A to B, we get

A:B = 17:7 $\Rightarrow \frac{A}{B} = \frac{17}{7} \Rightarrow$ A = $\frac{17}{7}$B

So the ratio of A to B is $\frac{17}{7}$.

Thus, the amount of A is seventeen sevenths of the amount of B, and in short, A is $\frac{17}{7}$ of B.

4. B:D = 19:5

19 and 5 are prime to each other.

We can put the same ratio many different ways.

B:D = 19:5 = 9.5:2.5 = 19 x 2:5 x 2 = 38:10, but is not equal to (19+1):(5+1) = 20:6 = 10:3

Now, taking the ratio of B to D, we get $\frac{\boldsymbol{B}}{\boldsymbol{D}} = \frac{19}{5}$.

So we get $\boldsymbol{B} = \frac{19}{5}\boldsymbol{D}$

Thus, B is nineteen fifths of D.

What then about the ratio of D to B?

Now, taking the ratio of D to B, we get $\frac{\boldsymbol{D}}{\boldsymbol{B}} = \frac{5}{19}$.

So we get $\boldsymbol{D} = \frac{5}{19}\boldsymbol{B}$

Thus, D is five nineteenths of B.

5. A:B:C = 10:20:30

A:B:C = 10:20:30 = 1 x 10:2 x 10:3 x 10 = 1:2:3

1, 2, and 3 are prime to each other.

So we get A:B:C = 1:2:3.

Now, taking the ratio of A to B, we get A:B:C = 1:2:3

$\Rightarrow$ A:B = 1:2 $\Rightarrow$ $\frac{A}{B} = \frac{1}{2}$ $\Rightarrow$ A = $\frac{1}{2}$B $\Rightarrow$ B = 2A.

So the ratio of A to B is $\frac{1}{2}$, and also, we can say that the ratio of B to A is 2. Thus, A is a half of B, and B is twice A.

Next, taking the ratio of B to C, we get A:B:C = 1:2:3

$\Rightarrow$ B:C = 2:3 $\Rightarrow$ $\frac{B}{C} = \frac{2}{3}$ $\Rightarrow$ B = $\frac{2}{3}$C $\Rightarrow$ C = $\frac{3}{2}$B. So the ratio of B to C is $\frac{2}{3}$, and also, the ratio of C to B is $\frac{3}{2}$.

Thus, B is $\frac{2}{3}$ of C, or C is $\frac{3}{2}$ of B.

And next, taking the ratio of A to C, we get A:B:C = 1:2:3

$\Rightarrow$ A:C = 1:3 $\Rightarrow$ $\frac{A}{C} = \frac{1}{3}$ $\Rightarrow$ A = $\frac{1}{3}$C $\Rightarrow$ C = 3A

So the ratio of A to C is $\frac{1}{3}$, and also, the ratio of C to A is 3.

Thus, A is a third of C, and C is three times A.

And we can put the ratio several different ways as follows.

A:B:C = 1:2:3 $\Rightarrow$ A:C:B = 1:3:2 $\Rightarrow$ B:A:C = 2:1:3

6. A:C:D = 15:35:45

Simplifying the ratio, we get A:C:D = 15:35:45 = 3:7:9, because 15:35:45 = 5x3:5x7:5x9, where the common divisor is 5, and 3, 7, and 9 are prime to each other, since there is no common divisor to 3, 7, and 9.

Next, taking the ratio of A to C, we get

A:C = 3:7 $\Rightarrow$ $\frac{A}{C}=\frac{3}{7}$ $\Rightarrow$ A = $\frac{3}{7}$C $\Rightarrow$ C = $\frac{7}{3}$A. So the ratio of

A to C is $\frac{3}{7}$, and also, the ratio of C to A is $\frac{7}{3}$.

Thus, A is a three sevenths of C, and C is seven thirds of A.

Next, taking the ratio of A to D, we get

A:D = 3:9 $\Rightarrow$ $\frac{A}{D}=\frac{3}{9}=\frac{1}{3}$ $\Rightarrow$ A = $\frac{1}{3}$D $\Rightarrow$ D = 3A.

So the ratio of A to D is $\frac{1}{3}$, and also, the ratio of D to A is 3.

Thus, A is a third of D, and D is three times A.

Next, taking the ratio of C to D, we get

C:D = 7:9 $\Rightarrow$ $\frac{C}{D}=\frac{7}{9}$ $\Rightarrow$ C = $\frac{7}{9}$D $\Rightarrow$ D = $\frac{9}{7}$C. So the ratio of

C to D is $\frac{7}{9}$, and also, the ratio of D to C is $\frac{9}{7}$.

Thus, C is seven ninths of D, and D is nine sevenths of C.

7. C:D:E = 25:50:183

25, 50, and 183 are prime to each other, although 25 and 50 are not. So no further simplification is needed.

First, taking the ratio of C to D, we get

C:D = 25:50 = 1:2 $\Rightarrow \frac{C}{D} = \frac{1}{2} \Rightarrow$ C = $\frac{1}{2}$D $\Rightarrow$ D = 2C.

So the ratio of C to D is $\frac{1}{2}$, and also, the ratio of D to C is 2.

Thus, C is a half of D, and D is twice C.

Next, taking the ratio of C to E, we get

C:E = 25:183 $\Rightarrow \frac{C}{E} = \frac{25}{183} \Rightarrow$ C = $\frac{25}{183}$E $\Rightarrow$ E = $\frac{183}{25}$C.

So the ratio of C to E is $\frac{25}{183}$, and the ratio of E to C is $\frac{183}{25}$.

Thus, C is twenty five of a hundred and eighty thirds of E, and E is a hundred and eighty three of twenty fifths of C.

And next, taking the ratio of D to E, we get

D:E = 50:183 $\Rightarrow$ $\frac{D}{E} = \frac{50}{183}$ $\Rightarrow$ D = $\frac{50}{183}$E $\Rightarrow$ E = $\frac{183}{50}$D. So

the ratio of D to E is $\frac{50}{183}$, and the ratio of E to D is $\frac{183}{50}$.

Thus, D is fifty of a hundred and eighty thirds of D, and D is a hundred and eighty three of fiftieths of C.

8. A:B:C:D = 2:4:2:8

Simplifying the ratio, we get A:B:C:D = 2:4:2:8 = 1:2:1:4.

So now, 1, 2, and 4 are prime to each other, because other than 1, no divisor is common to 1, 2, and 4.

First, taking the ratio of A to B, we get

A:B = 1:2 $\Rightarrow$ $\frac{A}{B} = \frac{1}{2}$ $\Rightarrow$ A = $\frac{1}{2}$B $\Rightarrow$ B = 2A.

So the ratio of A to B is $\frac{1}{2}$, and also, the ratio of B to A is 2.

Thus, A is a half of B, and B is twice A.

Next, taking the ratio of B to D, we get A:B:C:D = 1:2:1:4

$\Rightarrow$ B:D = 2:4 $\Rightarrow$ $\frac{\boldsymbol{B}}{\boldsymbol{D}} = \frac{2}{4} = \frac{1}{2}$ $\Rightarrow$ B = $\frac{1}{2}$D $\Rightarrow$ D = 2B.

So the ratio of B to D is $\frac{1}{2}$, and the ratio of D to B is 2.

Thus, B is a half of D, and D is twice B.

Next, taking the ratio of A to C, we get

A:C = 1:1 $\Rightarrow$ $\frac{\boldsymbol{A}}{\boldsymbol{C}} = \frac{1}{1} = 1$ $\Rightarrow$ A = C. So the ratio of A to C is 1,

and of course, the ratio of C to A is 1, too.

Thus, A and C are equal in amount.

And next, since A:B:C:D = 1:2:1:4, the ratio of A to D is

$\frac{\boldsymbol{A}}{\boldsymbol{D}} = \frac{1}{4}$, and the ratio of D to A is $\frac{\boldsymbol{D}}{\boldsymbol{A}} = \frac{4}{1} = 4$.

Thus, A is a quarter of D, and D is four times A.

And the same is true for the ratio between C and D, too.

9. Assuming A:B = 1:2, and B:C = 2:1, find A:B:C.

We have A:B = 1:2, so we get B = 2A.

And we have B:C = 2:1, and B = 2A, so we get

B:C = 2:1 $\Rightarrow$ 2A:C = 2:1 $\Rightarrow$ $\frac{2\boldsymbol{A}}{\boldsymbol{C}} = \frac{2}{1} = 2$.

Thus, we get 2A = 2C $\Rightarrow$ A = C.

So we get A:C = 1:1.

And we have A:B = 1:2.

Thus, we get A:B:C = 1:2:1.

We can also see that A:C = 1:1 from the fact that A:B = 1:2.

That's because A:B = 1:2 $\Rightarrow$ B:A = 2:1, which is the same ratio of B to C, since B:C = 2:1,

10. Assuming A:B = 1:2, and B:C = 3:1, find A:C.

We may want to express the amounts of all the objects in terms of one of the objects.

Then, we can express B and C in terms of A.

So to begin with, we have A:B = 1:2, so we get

$\frac{A}{B} = \frac{1}{2} \Rightarrow$ B = 2A.

And we have B:C = 3:1, and also, B = 2A, so we get

B:C = 3:1 $\Rightarrow$ 2A:C = 3:1 $\Rightarrow \frac{2A}{C} = \frac{3}{1} = 3 \Rightarrow$ 2A = 3C

Thus, we get C = $\frac{2}{3}$A $\Rightarrow$ A:B:C = A:2A:$\frac{2}{3}$A = 1:2:$\frac{2}{3}$

= 1x3:2x3:$\frac{2}{3}$x3 = 3:6:2. So we get A:B:C = 3:6:2.

Thus, we get A:C = 3:2.

Note that we normally express a ratio in terms of integers. However, we do not have to use integers only unless specified otherwise in the problem.

11. Assuming A:B = 2:3, and A:C = 5:1, find B:C.

This example is no other than the example 10 above. So again, expressing each of A, B, and C in terms of one of the three, we can get a ratio between A, B, and C.

Thus again, we can express B and C in terms of A.

So to begin with, we have A:B = 2:3, so we get

$$B:A = 3:2 \Rightarrow \frac{B}{A} = \frac{3}{2} \Rightarrow B = \frac{3}{2}A.$$

Next, we have A:C = 5:1, so we get

$$A:C = 5:1 \Rightarrow C:A = 1:5 \Rightarrow \frac{C}{A} = \frac{1}{5} \Rightarrow C = \frac{1}{5}A.$$

Thus, we get

$$B:C = \frac{3}{2}A:\frac{1}{5}A = \frac{3}{2}:\frac{1}{5} = \frac{3}{2}x10:\frac{1}{5}x10 = \frac{30}{2}:\frac{10}{5} = 15:2$$

What then about A:B:C?

$$A:B:C = A:\frac{3}{2}A:\frac{1}{5}A = 1:\frac{3}{2}:\frac{1}{5} = 10:\frac{3}{2}x10:\frac{1}{5}x10 = 10:15:2$$

12. Assuming A:B:C = 3:4:5, and B:D = 2:1, find A:B:C:D.

We have B:D = 2:1, so we can get B:D = 4:2.

And we have A:B:C = 3:4:5, so we get A:B:C:D = 3:4:5:2.

How?

Since A:B:C = 3:4:5, we get B:A = 4:3 and B:C = 4:5.

And we have B:D = 2:1 = 4:2.

That is to say that if B = 4, we get A = 3, C = 5, and D = 2.

Thus, we get A:B:C:D = 3:4:5:2.

What if B:D = 7:4?

We may want to express the ratio B:D converting B to 4.

Then, we get B:D = 7:4 $\Rightarrow$ B:D = 1:$\frac{4}{7}$ = 1x4:$\frac{4}{7}$x4 = 4:$\frac{16}{7}$.

So we get

A:B:C:D = 3:4:5:$\frac{16}{7}$ = 3x7:4x7:5x7:$\frac{16}{7}$x7= 21:28:35:16.

13. Assuming A:B:C = 3:5:17, and B:D = 3:2, find A:B:C:D.

Since A:B:C = 3:5:17, we may want to express the ratio B:D converting B to 5.

So converting B to 5 in the ratio B:D, we can get

B:D = 3:2 = 1:$\frac{2}{3}$ = 5:$\frac{10}{3}$.

Then, we get

A:B:C:D = 3:5:17:$\frac{10}{3}$ = 3x3:5x3:17x3:$\frac{10}{3}$x3 = 9:15:51:10.

14. Assuming A:B:C = 2:8:9, B:E = 2:3, and B:D = 4:3, find A:B:C:D:E.

We have 5 items, which are A, B, C, D, and E.

We need to select one item to be the reference.

We may want to choose B to be the reference.

That’s because B appears in each of all the ratios given.

In fact, we can choose any item as long as the item can be related to all of the other items.

Choosing B, we need to convert B to 8, since B is 8 in A:B:C.

So converting B to 8, in all the other ratios, we get

B:E = 2:3 = 8:12, and B:D = 4:3 = 8:6

Thus, we get A:B:C:D:E = 2:8:9:6:12.