

Examples 2 in Ratios and Rates

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What is a percentage?

It's a ratio. What ratio then is it?

Multiplying a ratio by 100, we get the percentage, so it's a ratio again, and is just another form of a ratio.

Suppose for instance, $X = 4$ and $Y = 8$.

Then first, taking the ratio of X to Y , we get $\frac{X}{Y} = \frac{4}{8} = \frac{1}{2}$.

So next, multiplying the ratio by 100, we get $\frac{1}{2} \times 100 = 50$.

Thus, 50 is the percentage of X with respect to Y .

In short, 50 is the percentage of X to Y .

And we use 'percent' to indicate the percentage, and use a symbol % after the percentage. % is read as percent.

So we say "X is 50 percent of Y.", and write "X is 50% of Y."

We know the ratio of X to Y is $\frac{1}{2}$. So 50% means a half.

Thus, X is a half of Y. What if the ratio is $\frac{1}{4}$?

Multiplying a ratio by 100, we get the percentage.

So we get $\frac{1}{4} \times 100 = 25$. Thus, $\frac{1}{4}$ is 25%.

So for instance, if $A = 3$ and $B = 12$, the ratio of A to B is

$\frac{A}{B} = \frac{3}{12} = \frac{1}{4}$. Thus, taking the percentage of A to B, we get

$\frac{1}{4} \times 100 = 25$. So we can say that A is 25% of B.

What then about B?

Taking the percentage of B to A, we take the ratio of B to A, and then, multiply the ratio by 100.

So taking the ratio of B to A, we get $\frac{B}{A} = \frac{12}{3} = 4$.

Thus, taking the percentage of B to A, we get $4 \times 100 = 400$.

In short, we get $\frac{B}{A} \times 100 = \frac{12}{3} \times 100 = 400$

Thus, B is 400% of A, which means B is four times A.

So what is percentage?

It is still a ratio, since it is the product of a ratio and 100.

That's because we take the reference as 100.

The reference is the amount we take the percentage from.

If for instance, $A:B:C = 2:2:1$, and the reference is $A + B + C$, what is the percentage of A?

We can assume that $A = 2$, $B = 2$, and $C = 1$, since the ratio is $A:B:C = 2:2:1$.

Then, since the reference we take the percentage from is $A + B + C$, which is 5, the ratio of A to $A + B + C$ is

$$\frac{A}{A+B+C} = \frac{2}{5}.$$

Thus, taking the percentage of A to the reference $A + B + C$,

we get $\frac{2}{5} \times 100 = 2 \times 20 = 40$.

In short, $\frac{A}{A + B + C} \times 100 = \frac{2}{5} \times 100 = 40$.

So A is 40% of $A + B + C$.

Note that for instance, the percentage of A from B is the same as the percentage of A to B, and also, is the same as the percentage of A in B. So in all the three cases, B is the reference.

Thus, we can ask a question as follows.

What percent of B is A?

Then, the answer can be for instance, A is 25% of B, which means the ratio of A to B is a quarter, which means A is a quarter of B, which means B is four times A.

It's because $\frac{A}{B} = \frac{1}{4}$, so $A = \frac{1}{4}B$, and thus, $B = 4A$.

Now, for another instance, taking the percentage of A from R , how can we get it?

Taking the product of 100 and the ratio of A to R , we get the percentage.

So the percentage of A from R is $\frac{A}{R} \times 100 \%$.

In other words, A is $\frac{A}{R} \times 100 \%$ of R .

In short, A is $\frac{100A}{R} \%$ of R .

Also, we can say that the percentage of A with respect to R is $\frac{A}{R} \times 100 \%$.

In short, the percentage of A to R is $\frac{100A}{R} \%$.

Thus, for instance, the percentage of 3 from 4 is as follows.

$$\frac{3}{4} \times 100 = 3 \times 25 = 75, \text{ so it's } 75\%.$$

Then, we usually say that 3 is 75% of 4.

For more instances, the percentage of 3 to 6 is $\frac{300}{6} = 50\%$,

so 3 is 50% of 6.

The percentage of 1 to 10 is $\frac{100}{10} = 10\%$, so 1 is 10% of 10.

The percentage of 3 to 100 is $\frac{300}{100} = 3\%$, so 3 is 3% of 100.

The percentage of 15 to 60 is $\frac{1500}{60} = \frac{150}{6} = \frac{75}{3} = 25\%$, so

15 is 25% of 60.

And the percentage of 15 to 20 is $\frac{1500}{20} = \frac{150}{2} = 75\%$,

so 15 is 75% of 20.

Let's now do more examples.

Examples 2 in Ratios and Rates

0. $A:B = 10:15$. What % of $A + B$ is A ?
1. $A:B = 25:10$. What % of $A + B$ is B ?
2. $A:B:C = 12:15:27$. What % of $A + B + C$ is B ?
3. Assuming $A:B:C:D = 5:17:39:24$,
 - 3.1 find the percentage of $B + D$ with respect to $A + C$,
 - 3.2 find the percentage of A with respect to $A + D$
4. Assuming $A:B:C:D:E = 16:32:48:64:80$,
 - 4.1 find the percentage of A to $A + C + E$,
 - 4.2 find the percentage of $B + D$ to $D + E$.

5. If $A:B:C = 3:4:5$,

5.1 what % of $C - A$ is A ?

5.2 what % of $A + B - C$ is $B + C$?

5.3 what % of $\frac{B}{A}$ is $\frac{C}{A}$?

5.4 What % of $A + \frac{C}{5}$ is $\frac{B}{4}$?

6. If $A:B:C:D = 2:3:5:1$ and $B:E = 2:5$, what % of E is D ?

7. If $A:B:C:D = 14:8:27:135$, $A:E = 2:3$, and $E:F = 4:3$,
what % of D is F ?

8. Four people get together and want to swap books among themselves in the situation as follows.

Each of the four people brought some books.

B brought 64 books.

A gave 12 books to B.

B received 4 books from C.

C got 7 books from D.

D has as many books as A has now.

C has 25% of the books B has now.

B has 80% of the books D has now.

What then is the percentage of the books that A has now with respect to all the books that the four people have now?

In other words, what % of all the books the four people have now is the books A has now?

Suggestions or Solutions To The Problems In Examples 2

0. $A:B = 10:15$. What % of $A + B$ is A ?

Taking the percentage, we don't need to know the actual amount of each of A and B . We only need the ratio between the two. So since the ratio is $A:B = 10:15$, we can set $A = 10$, and $B = 15$.

We know a percentage is the product of a ratio and 100.

So we may want to begin with the ratio of A to $A + B$.

Taking thus, the ratio, we get $\frac{A}{A+B} = \frac{10}{10+15} = \frac{10}{25} = \frac{2}{5}$.

So next, taking the percentage, we get $\frac{2}{5} \times 100 = 40$.

In short, we get

$$\frac{A}{A+B} \times 100 = \frac{10}{10+15} \times 100 = \frac{1000}{25} = \frac{200}{5} = 40$$

Thus, A is 40% of $A + B$.

1. $A:B = 25:10$. What % of $A + B$ is B ?

Again, taking the percentage, we don't need to know the actual amount of each of A and B . We only need the ratio between the two. So since the ratio is $A:B = 25:10$, we can set $A = 25$, and $B = 10$.

We know a percentage is the product of a ratio and 100.

So beginning with the ratio of B to $A + B$, we get

$$\frac{B}{A+B} = \frac{10}{25+10} = \frac{10}{35} = \frac{2}{7}.$$

So next, taking the percentage, we get $\frac{2}{7} \times 100 = \frac{200}{7}$.

In short, we get

$$\frac{B}{A+B} \times 100 = \frac{10}{25+10} \times 100 = \frac{1000}{35} = \frac{200}{7} = 28 + \frac{4}{7}$$

Thus, B is $(28 + \frac{4}{7})\%$ of $A + B$.

2. $A:B:C = 12:15:27$. What % of $A + B + C$ is B ?

Again, taking the percentage, we don't need to know the actual amount of each object. We only need the ratio.

Thus, since the ratio is $A:B:C = 12:15:27$, we can set $A = 12$, $B = 15$, and $C = 27$.

We know a percentage is the product of a ratio and 100.

So beginning with the ratio of B to $A + B + C$, we get

$$\frac{B}{A+B+C} = \frac{15}{12+15+27} = \frac{15}{54}.$$

So next, taking the percentage, we get

$$\frac{15}{54} \times 100 = \frac{1500}{54} = 27 + \frac{42}{54} = 27 + \frac{7}{9}.$$

In short, we get

$$\frac{B}{A+B+C} \times 100 = \frac{1500}{12+15+27} = \frac{1500}{54} = 27 + \frac{7}{9}$$

Thus, B is $(27 + \frac{7}{9})\%$ of $A + B + C$.

3.1 Assuming $A:B:C:D = 5:17:39:24$, find the percentage of $B + D$ with respect to $A + C$.

Again, taking the percentage, we don't need to know the actual amount of each object. We only need the ratio.

Thus, since the ratio is $A:B:C:D = 5:17:39:24$, we can set $A = 5$, $B = 17$, $C = 39$, and $D = 24$.

We know a percentage is the product of a ratio and 100.

So taking first, the ratio of $B + D$ to $A + C$, we get

$$\frac{B + D}{A + C} = \frac{17 + 24}{5 + 39} = \frac{41}{44}.$$

And next, taking the percentage, we get

$$\frac{41}{44} \times 100 = \frac{4100}{44} = 93 + \frac{8}{44} = 93 + \frac{2}{11}.$$

$$\text{In short, we get } \frac{B + D}{A + C} \times 100 = \frac{4100}{44} = 93 + \frac{2}{11}$$

Thus, $B + D$ is $(93 + \frac{2}{11})\%$ of $A + C$.

3.2 Assuming $A:B:C:D = 5:17:39:24$, find the percentage of A with respect to $A + D$

Again, taking the percentage, we don't need to know the actual amount of each object. We only need the ratio.

Thus, since the ratio is $A:B:C:D = 5:17:39:24$, we can set $A = 5$, $B = 17$, $C = 39$, and $D = 24$.

We know a percentage is the product of a ratio and 100.

So taking first, the ratio of A to $A + D$, we get

$$\frac{A}{A + D} = \frac{5}{5 + 24} = \frac{5}{29}.$$

And next, taking the percentage, we get

$$\frac{5}{29} \times 100 = \frac{500}{29} = 17 + \frac{7}{29}.$$

$$\text{In short, we get } \frac{A}{A + D} \times 100 = \frac{500}{29} = 17 + \frac{7}{29}$$

Thus, A is $(17 + \frac{7}{29})\%$ of $A + C$.

4.1 Assuming $A:B:C:D:E = 16:32:48:64:80$, find the percentage of A to $A + C + E$,

Getting a percentage, we only need the ratio.

Looking at the ratio $A:B:C:D:E = 16:32:48:64:80$, we can notice that it's not simplified. So simplifying it first, we get $A:B:C:D:E = 16:32:48:64:80 = 4:8:12:16:20 = 1:2:3:4:5$.

Thus, we can set $A = 1$, $B = 2$, $C = 3$, $D = 4$, and $E = 5$.

So now, since a percentage is the product of a ratio and 100, taking first, the ratio of A to $A + C + E$, we get

$$\frac{A}{A+C+E} = \frac{1}{1+3+5} = \frac{1}{9}.$$

Thus, taking the percentage, we get

$$\frac{1}{9} \times 100 = \frac{100}{9} = 11 + \frac{1}{9}.$$

$$\text{In short, we get } \frac{A}{A+C+E} \times 100 = \frac{100}{9} = 11 + \frac{1}{9}$$

Thus, A is $(11 + \frac{1}{9})\%$ of $A + C + E$.

4.2 Assuming $A:B:C:D:E = 16:32:48:64:80$, find the percentage of $B + D$ to $D + E$.

Getting a percentage, we only need the ratio.

Simplifying first, the ratio $A:B:C:D:E = 16:32:48:64:80$, we get $A:B:C:D:E = 16:32:48:64:80 = 4:8:12:16:20 = 1:2:3:4:5$.

Thus, we can set $A = 1$, $B = 2$, $C = 3$, $D = 4$, and $E = 5$.

So now, since a percentage is the product of a ratio and 100, taking first, the ratio of $B + D$ to $D + E$, we get

$$\frac{B + D}{D + E} = \frac{2 + 4}{4 + 5} = \frac{6}{9} = \frac{2}{3}.$$

Thus, taking the percentage, we get

$$\frac{2}{3} \times 100 = \frac{200}{3} = 66 + \frac{2}{3}.$$

$$\text{In short, we get } \frac{B + D}{D + E} \times 100 = \frac{600}{9} = \frac{200}{3} = 66 + \frac{2}{3}$$

Thus, $B + D$ is $(66 + \frac{2}{3})\%$ of $D + E$.

5.1 If $A:B:C = 3:4:5$, what % of $C - A$ is A ?

Getting a percentage, we only need the ratio.

Since the ratio is $A:B:C = 3:4:5$, we can set $A = 3$, $B = 4$, and $C = 5$.

A percentage is the product of a ratio and 100.

We want to get the percentage of A to $C - A$.

So taking first, the ratio of A to $C - A$, we get

$$\frac{A}{C - A} = \frac{3}{5 - 3} = \frac{3}{2}.$$

Thus, taking the percentage, we get $\frac{3}{2} \times 100 = \frac{300}{2} = 150$.

In short, we get $\frac{A}{C - A} \times 100 = \frac{300}{5 - 3} = \frac{300}{2} = 150$

Thus, A is 150% of $C - A$.

5.2 If $A:B:C = 3:4:5$, what % of $A + B - C$ is $B + C$?

Getting a percentage, we only need the ratio.

Since the ratio is $A:B:C = 3:4:5$, we can set $A = 3$, $B = 4$, and $C = 5$.

A percentage is the product of a ratio and 100.

We want to get the percentage of $B + C$ to $A + B - C$.

So taking first, the ratio of $B + C$ to $A + B - C$, we get

$$\frac{B + C}{A + B - C} = \frac{4 + 5}{3 + 4 - 5} = \frac{9}{2}.$$

Thus, taking the percentage, we get $\frac{9}{2} \times 100 = \frac{900}{2} = 450$.

In short, we get $\frac{B + C}{A + B - C} \times 100 = \frac{900}{2} = 450$

Thus, $B + C$ is 450% of $A + B - C$.

5.3 If $A:B:C = 3:4:5$, what % of $\frac{B}{A}$ is $\frac{C}{A}$?

A percentage is the product of a ratio and 100.

We want to get the percentage of $\frac{C}{A}$ to $\frac{B}{A}$.

So we want to get first, the ratio of $\frac{C}{A}$ to $\frac{B}{A}$.

Taking the ratio of $\frac{C}{A}$ to $\frac{B}{A}$, we just take the ratio of C to B.

It's because $\frac{C}{A} : \frac{B}{A} = C : B$.

Taking thus, the ratio of C to B, we get $\frac{C}{B} = \frac{5}{4}$.

Thus, taking the percentage, we get $\frac{5}{4} \times 100 = \frac{500}{4} = 125$.

In short, we get $\frac{C}{B} \times 100 = \frac{500}{4} = 125$

Thus, $\frac{C}{A}$ is 125% of $\frac{B}{A}$.

5.4 If $A:B:C = 3:4:5$, What % of $A + \frac{C}{5}$ is $\frac{B}{4}$?

A percentage is the product of a ratio and 100.

We want to get the percentage of $\frac{B}{4}$ to $A + \frac{C}{5}$.

So we want to take first, the ratio of $\frac{B}{4}$ to $A + \frac{C}{5}$.

We can however, have this:

$$\frac{B}{4} = \frac{5B}{20} \text{ and } A + \frac{C}{5} = \frac{5A + C}{5} = \frac{4(5A + C)}{20} = \frac{20A + 4C}{20}.$$

So we can get this:

$$\frac{B}{4} : A + \frac{C}{5} = \frac{5B}{20} : \frac{20A + 4C}{20} = 5B : 20A + 4C$$

Taking thus, the ratio of $\frac{B}{4}$ to $A + \frac{C}{5}$, we can take the ratio

of $5B$ to $20A + 4C$. So now, taking the ratio, we get

$$\frac{5B}{20A + 4C} = \frac{20}{60 + 20} = \frac{20}{80} = \frac{1}{4}.$$

Thus, taking the percentage, we get $\frac{1}{4} \times 100 = 25$.

In short, we get

$$\frac{5\mathbf{B}}{20\mathbf{A} + 4\mathbf{C}} \times 100 = \frac{2000}{60 + 20} = \frac{2000}{80} = \frac{100}{4} = 25$$

Thus, $\frac{\mathbf{B}}{4}$ is 25% of $\mathbf{A} + \frac{\mathbf{C}}{5}$.

6. If $A:B:C:D = 2:3:5:1$ and $B:E = 2:5$, what % of E is D?

A percentage is the product of a ratio and 100.

So we want to take first, the ratio of D to E.

We can get the ratio D to E using the ratio $B:E = 2:5$ and using the ratio $B:D = 3:1$ from $A:B:C:D = 2:3:5:1$.

Taking the ratio of B to D, we get $\frac{B}{D} = \frac{3}{1} = 3 \Rightarrow B = 3D$.

And we have $B:E = 2:5$. So we get $B:E = 3D:E = 2:5$.

Thus, taking the ratio of 3D to E, we get $\frac{3D}{E} = \frac{2}{5}$.

Then, we can get $\frac{3D}{E} = \frac{2}{5} \Rightarrow \frac{3D}{E} \cdot \frac{1}{3} = \frac{2}{5} \cdot \frac{1}{3} \Rightarrow \frac{D}{E} = \frac{2}{15}$

So the ratio of D to E is $\frac{2}{15}$.

Thus, next, taking the percentage of D to E, we get

$$\frac{2}{15} \times 100 = \frac{40}{3} = 13 + \frac{1}{3}$$

Thus, D is $(13 + \frac{1}{3})\%$ of E.

7. If $A:B:C:D = 14:8:27:135$, $A:E = 2:3$, and $E:F = 4:3$, what % of D is F?

A percentage is the product of a ratio and 100.

So we want to take first, the ratio of F to D.

We can get the ratio F to D using the ratio $A:E = 2:3$ and the ratio $E:F = 4:3$, and also, using the ratio $A:D = 14:135$ from $A:B:C:D = 14:8:27:135$.

Taking the ratio of A to E, we get $\frac{A}{E} = \frac{2}{3} \Rightarrow A = \frac{2}{3}E \dots (1)$

And we have $E:F = 4:3$. So we get $\frac{E}{F} = \frac{4}{3} \Rightarrow E = \frac{4}{3}F$

Thus, from (1), we get $A = \frac{2}{3}E = \frac{2}{3} \cdot \frac{4}{3}F = \frac{8}{9}F$.

So we get $A = \frac{8}{9}F \dots (2)$ Also, we have $A:D = 14:135$.

So we get $\frac{A}{D} = \frac{14}{135} \Rightarrow A = \frac{14}{135}D \dots (3)$

Now, from (2) and (3), we have $A = \frac{8}{9}F$ and $A = \frac{14}{135}D$.

So we get $\frac{8}{9}F = \frac{14}{135}D$. Thus, we get

$$\frac{8}{9}F = \frac{14}{135}D \Rightarrow \frac{8}{9}F \cdot \frac{9}{8} \cdot \frac{1}{D} = \frac{14}{135}D \cdot \frac{9}{8} \cdot \frac{1}{D}$$

$$\Rightarrow \frac{F}{D} = \frac{14}{135} \cdot \frac{9}{8} = \frac{7}{15} \cdot \frac{1}{4} = \frac{7}{60}$$

Thus, the ratio of F to D is $\frac{7}{60}$.

So next, taking the percentage of F to D, we get

$$\frac{7}{60} \times 100 = \frac{35}{3} = 11 + \frac{2}{3}$$

Thus, F is $(11 + \frac{2}{3})\%$ of D.

8. Four people get together and want to swap books among themselves in the situation as follows.

Each of the four people brought some books.

B brought 64 books.

A gave 12 books to B.

B received 4 books from C.

C got 7 books from D.

D has as many books as A has now.

C has 25% of the books B has now.

B has 80% of the books D has now.

What then is the percentage of the books that A has now with respect to all the books that the four people have now?

In other words, what % of all the books the four people have now is the books A has now?

A percentage is the product of a ratio and 100. In short, we can put it the way as follows.

A percentage is 100 times a ratio.

So do we need to find first, the amount of the books each person has now?

Not really. In fact, we can't. We cannot get the enough number of equations. We need four equations, because there are four people. We can only get three equations. What then can we do?

We can get ***the ratio between the amounts*** of the books the people have now. So we may want to consider rather ***the relationship between the amounts*** than the amounts themselves of the books that the four people have now.

We can use the names of the people for the numbers of the books that belong to the people with the names.

So suppose now

A has A books. B has B books. C has C books.

And D has D books.

We don't need to however, consider the statements as follows.

B brought 64 books. A gave 12 books to B.

B received 4 books from C. And C got 7 books from D.

That's because ***the relationship in quantity*** in the books that the individuals have now is in our consideration.

We know B brought 64 books, but not how many books each of A, C, and D brought.

However, the actual amounts of the books they brought are not in our consideration. They are just noise in the problem.

They are distractions. So we need to filter them out.

Now then, the statements in consideration are as follows.

D has as many books as A has now.

C has 25% of the amount of the books that B has now.

B has 80% of the amount of the books that D has now.

To begin with, D has as many books as A has now.

So we get $D = A$.

Next. C has 25% of the amount of books that B has now.

So we get $\frac{100C}{B} = 25$.

And next, B has 80% of the amount of books that D has

now. So we get $\frac{100B}{D} = 80$.

Then, we can get these: $D = A \Rightarrow D:A = 1:1$

$$\frac{100C}{B} = 25 \Rightarrow \frac{C}{B} = \frac{25}{100} = \frac{1}{4} \Rightarrow C:B = 1:4.$$

$$\frac{100B}{D} = 80 \Rightarrow \frac{B}{D} = \frac{80}{100} = \frac{4}{5} \Rightarrow B:D = 4:5.$$

So we have $D:A = 1:1$, that is, $A = D$, and have $B:D = 4:5$.

Thus, we get $A:B:D = 5:4:5$, since $A = D$.

And we have $C:B = 1:4$, and also, have $A:B:D = 5:4:5$.

In both ratios, we have $B = 4$.

So the ratios for A , C , and D with B get intact.

That is to say that since $A:B = 5:4$, $B:D = 4:5$, and $C:B = 1:4$, we can get $A:C = 5:1$ and $D:C = 5:1$.

Thus, we get $A:B:C:D = 5:4:1:5$.

Then, we can set $A = 5k$, $B = 4k$, $C = k$, and $D = 5k$ where k is a nonzero constant.

Then, the percentage of A with respect to $(A + B + C + D)$ is

$$\frac{100A}{A + B + C + D} = \frac{500k}{5k + 4k + k + 5k} = \frac{500}{15} = \frac{100}{3} = 33 + \frac{1}{3}$$

Thus, A has $(33 + \frac{1}{3})\%$ of all the books that all the four people have.