# Examples 3 in Ratios and Rates



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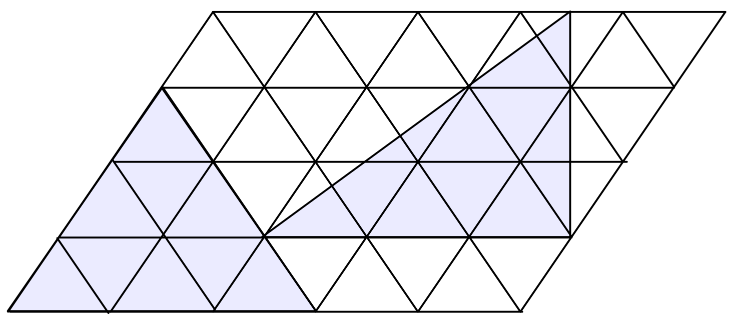
0. All the smallest triangles in Fig. 0 below are regular triangles.

Fig. 0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.

0.0. What is the sum of the prices of the areas in white?

0.1. What is the percentage of the price of the areas in white to the price of the whole panel?

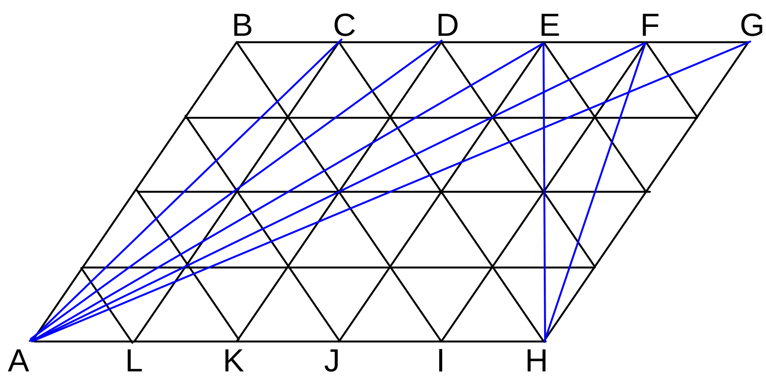
0.2. What is the percentage of the sum of the areas in white to the amount of the whole panel?

0.3. What is the percentage of the price of one of the smallest triangles with respect to the sum of the prices of the areas in white?

0.4. If we need to purchase at least 78% of the whole panel, how many of the smallest triangles do we have to buy? The store does not sell any part of one of the smallest triangle.

0.5. If we have 18 dollars, then what is the percentage of the maximum amount of the panel we can buy to the whole panel? The store does not sell any part of one of the smallest triangle.

1. All the line segments in black are parallel to each other, and they are equally spaced both horizontally and vertically. Therefore, all the triangles made by the line segments in black are regular triangles.




A triangle can be specified by the labels at all the vertices of the triangle. So ABI means the triangle ABI.

Then, find the ratio between the areas of the triangles specified as follows.

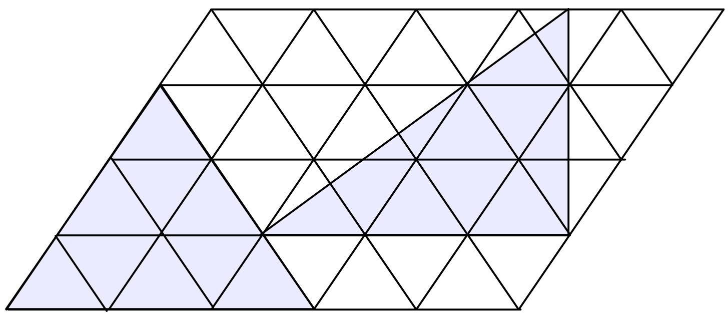
1.0. ABI : ACH

1.1. ABI : ACH : AEH

## Suggestions or Solutions
## To The Problems In Examples 3

0 All the smallest triangles in Fig. 0 below are regular triangles.

Fig. 0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.
What then is the sum of the prices of the areas in white?

We need to find the ratio of the sum of the areas in white to the area of the whole panel.
We can see that the values of the areas can be expressed in terms of the numbers of the smallest triangles.

So we may want to begin with finding the number of the smallest triangles taken up by the areas in light blue.

Next, we subtract the number from the total number of all the smallest triangles in the whole panel.
Then, the difference is the number of all the triangles taken up by the areas in white.

And then, we proceed with finding the total price of the areas in white.

The total number of all the smallest triangles in the whole panel = 10 x 4 = 40.

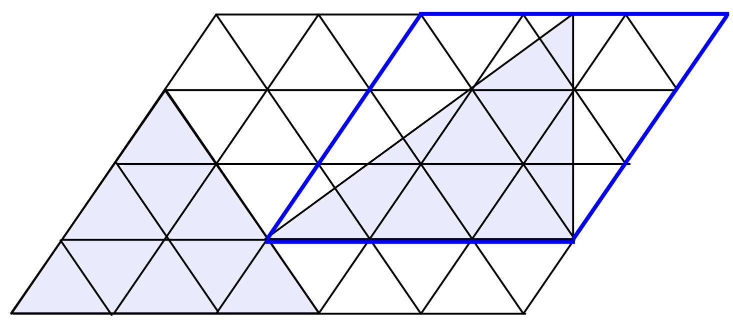
We can simply count the number of the triangles in the large regular triangle, and the number is 9.
What about the ones in the right triangle?　A right triangle is a triangle with an angle of $90^{o}$.

We can calculate the area of a triangle if we are given the required information on the triangle.

The usual information required is as follows.

1. The length of a line segment in a triangle.
2. The distance from the line segment to the vertex facing the line segment. So the distance is the height of the triangle from the line segment.

Fig. 1



We can see the right triangle is part of the parallelogram in blue. In fact, the area of the right triangle is half the area of the parallelogram. That’s because the distance between the two horizontal line segments in the parallelogram is the height of the right triangle.

The number of the smallest triangles in the parallelogram is 18. So the right triangle has 9 of the smallest triangles.

Thus, we can see that the total number of the smallest triangles taken up by the areas in light blue = 9 + 9 = 18.
We know that the total number of all the smallest triangles in the whole panel = 40.
So the sum of the areas in white takes up 22 (= 40 – 18) of the smallest triangles.

Thus, assuming

P = the price of the sum of the areas in white,

P1 = the price of the sum of the areas in white,

P2 = the number of the smallest triangles taken up by the areas in white, and

P3 = the number of the smallest triangles taken up by the whole panel,

We get $\mathrm{P} = \mathrm{P1}\ \mathrm{X}\ \dfrac{\mathrm{P2}}{\mathrm{P3}}$

$$= 26.75\ \mathrm{X}\ \frac{22}{40} = \frac{2675}{100}\ \mathrm{X}\ \frac{22}{40} = \frac{535}{20}\ \mathrm{X}\ \frac{11}{20}$$

$$= \frac{5350+535}{400} = \frac{5885}{400} = \frac{5600+285}{400} = 14 + \frac{285}{400} = 14.7125$$

Thus, the price is 14 dollars and 71.25 cents.

Fig. 2

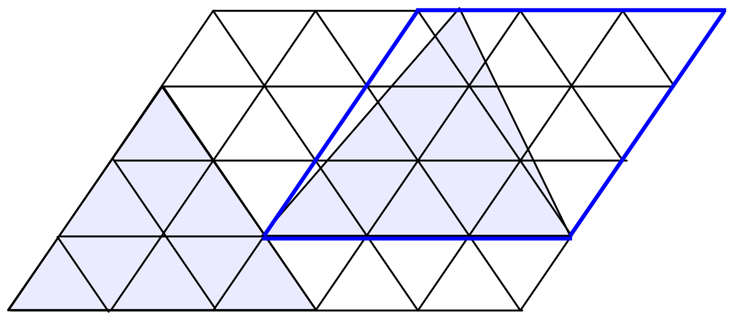
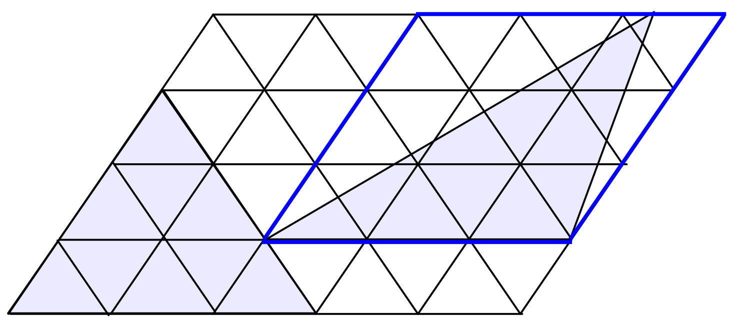


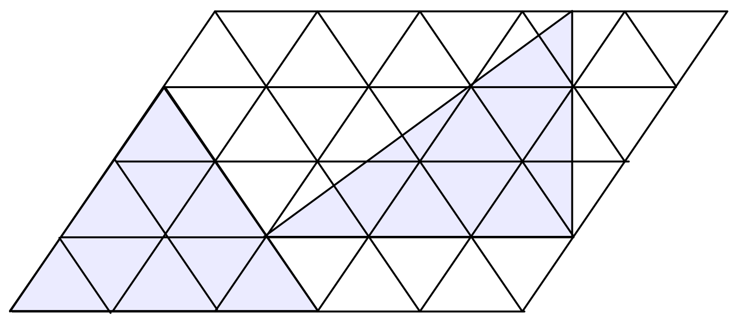
Fig. 3



We can see that the area of the triangle in light blue in the blue parallelogram in Fig. 2 is the same as the area of the triangle in light blue in the blue parallelogram in Fig 3.
The area of each triangle is the same as half the area of the parallelogram.
Also, the area is the same as the area taken up by the regular triangle on the left in light blue.

0.1 All the smallest triangles in Fig. 0.0 below are regular triangles.

Fig. 0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.
What then is the percentage of the price of the areas in white to the price of the whole panel?

Suppose
The price of the areas in white = W
The price of the whole panel = H
The percentage of W to H = P

Then, W = $\frac{5885}{400}$ dollars = 100x$\frac{5885}{400}$ cents = $\frac{5885}{4}$ cents
And H = 26.75 dollars = 2675 cents

So we get $P = \frac{100W}{H} = 100W \times \frac{1}{H}$

$$= 100 \times \frac{5885}{4} \times \frac{1}{2675} = \frac{25 \times 5885}{2675} = \frac{5 \times 5885}{535}$$

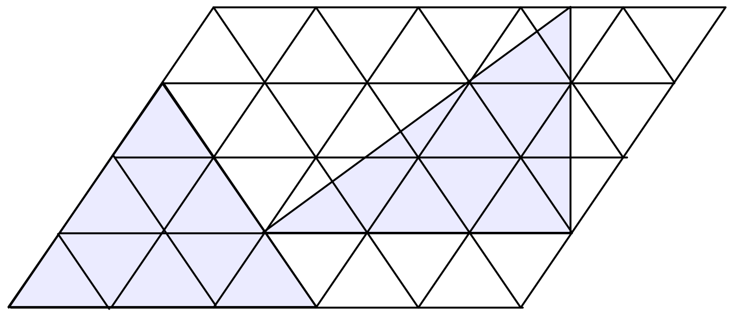
$$= 50 \times \frac{5885}{5350} = = 50 \times \frac{5350 + 535}{5350} = 50 \times (1 + \frac{535}{5350})$$

$$= 50(1 + 0.1) = 55$$

Thus, the price of the areas in white is 55% of the total price of the whole panel.

0.2 All the smallest triangles in Fig. 0.0 below are regular triangles.

Fig. 0.0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.
What then is the percentage of the sum of the areas in white to the area of the whole panel?

We know a percentage is 100 times a ratio, so it’s 100 times a ratio of a part to the whole. Thus, it’s still a ratio.

The ratio of the price of a part to the price of the whole is no other than the ratio of the same part to the same whole.

The price is proportional to the area of the part with respect to the area of the whole in this case.

We are talking about the same proportion.
So the percentage remains the same.
Therefore, the percentage = 55%.

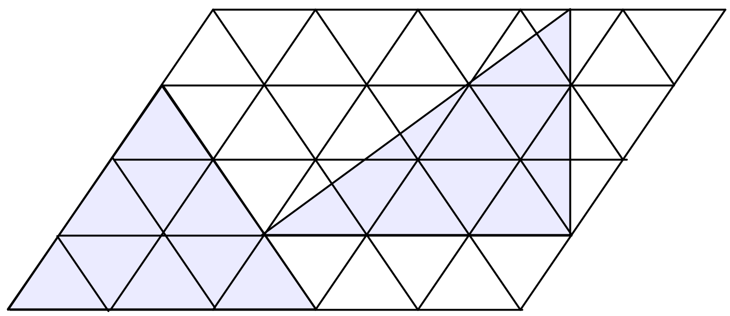
Suppose for instance, a store sells wood panels for 10 dollars per $m^2$.

Then, the purchase price is proportional to the area of the panel, and the percentage of the price of 20 $m^2$ to the price of 100 $m^2$ is the same as the percentage of 20 $m^2$ to 100 $m^2$.

It's because the percentage of 20 dollars to 100 dollars is the same as 20 $m^2$ to 100 $m^2$.

0.3 All the smallest triangles in Fig. 0.0 below are regular triangles.

Fig. 0.0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.
What then is the percentage of the price of one of the smallest triangles with respect to the sum of the prices of the areas in white?

We know the price is proportional to the area of the part with respect to the area of the whole in this case.
So the ratio between the prices is the same as the ratio between the areas.

The number of the smallest triangles taken up by the areas in white = 18.

Suppose that P = the percentage of the price of one of the smallest triangles with respect to the sum of the prices of the areas in white.

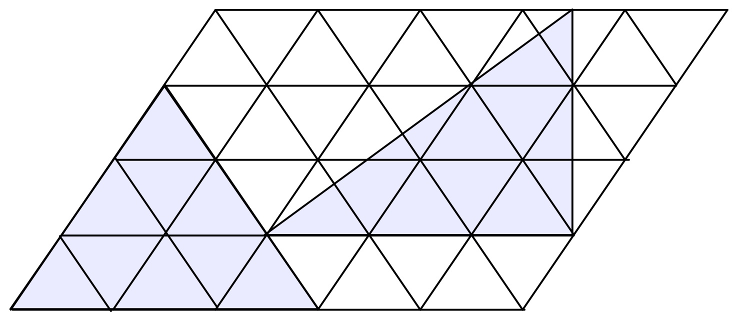
Then, we get

$$P = 100 \times \frac{1}{18} = \frac{100}{18} = \frac{50}{9} = \frac{45+5}{9} = 5 + \frac{5}{9}$$

So it is $5+\frac{5}{9}$ %.

0.4. All the smallest triangles in Fig. 0.0 below are regular triangles.

Fig. 0.0

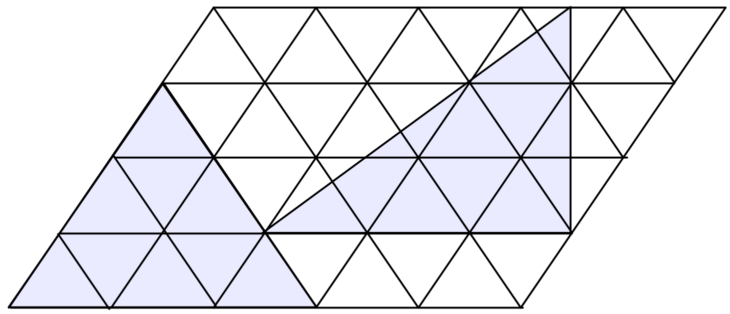


Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.
Then, if we need to purchase at least 78% of the whole panel, how many of the smallest triangles do we have to buy? The store does not sell any part of one of the smallest triangles. It only sells one or multiple of the smallest triangles.

We may want to find first, 78% of the total number of the smallest triangles.
The total number of the smallest triangles = 40.
78% of 40 = 0.78 x 40= 7.8 x 4 = 28 + 3.2 = 31.2
Thus, we need to buy at least 32 of the triangles.

0.5 All the smallest triangles in Fig. 0 below are regular triangles.

Fig. 0



Suppose that the price of the whole panel in parallelogram above is 26 dollars and 75 cents. Note that all the smallest pieces in triangles are the same.

Then, if we have 18 dollars, then what is the percentage of the maximum amount of the panel we can buy to the whole panel? The store does not sell any part of one of the smallest triangles. It only sells one or multiple of the smallest triangles.

The ratio for the prices is the same as the ratio for the amounts of the panel.

That's because the price is proportional to the area of the part with respect to the area of the whole in this case.

So we may want to begin with finding the percentage of the fund we have with respect to the price of the whole panel.

Assuming thus, P is the percentage of the fund to the price of the whole, we get

$$P = 100 \text{ x } \frac{18}{26.75} = \frac{1800}{26.75} = \frac{180000}{2675} = \frac{36000}{535} = \frac{7200}{107}$$

So we can get $\frac{7200}{107}$% of the total number of the smallest triangles in the whole panel.

We know the total number of the smallest triangles is 40.

And note that $\frac{7200}{107}\% = \frac{7200}{107} \text{x} \frac{1}{100} = \frac{72}{107}$. Why?

We know, for instance,

2% of 50 is 2 x$\frac{1}{100}$x 50 = 0.02 x 50 = 1.

So we get $\frac{7200}{107}$% of 40 =$\frac{72}{107}$x 40 instead of $\frac{7200}{107}$x 40.

Thus, assuming N is the amount worth $\frac{7200}{107}$% of the smallest triangles in the whole panel, we get

$$N = \frac{72}{107} \text{ x } 40 = \frac{2800+80}{107} = \frac{2880}{107} = \frac{2140+740}{107}$$

$$= 20 + \frac{740}{107} = 20 + \frac{749-9}{107} = 20 + 7 - \frac{9}{107} = 26 + 1 - \frac{9}{107}.$$

So the maximum number of the triangles we can buy = 26.

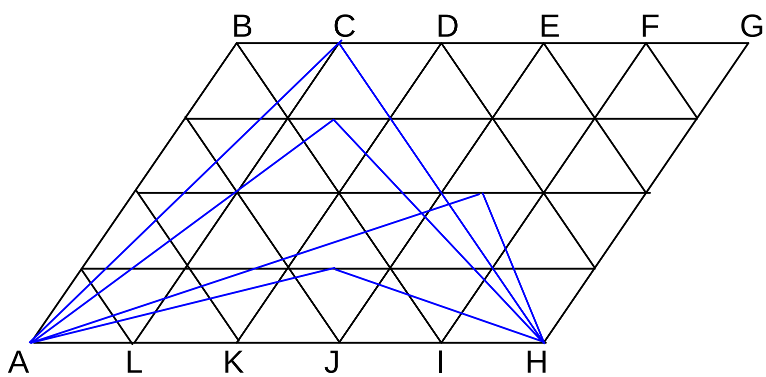
Thus, we need to find the percentage of 26 to 40.

So we get 100 x $\frac{26}{40} = \frac{2600}{40} = \frac{260}{4} = \frac{130}{2}$ = 65.

Therefore, it is 65%.

1.0. In Fig. 1, all the line segments in black are parallel to each other, and they are equally spaced both horizontally and vertically. Therefore, all the triangles made by the line segments in black are regular triangles.

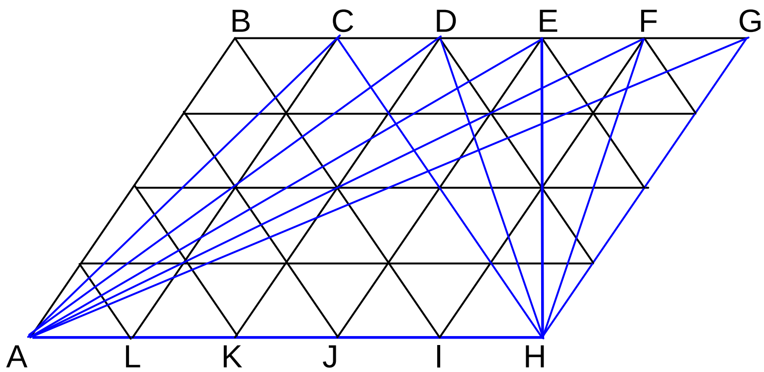
Fig. 1




A triangle can be specified by the labels at all the vertices of the triangle. So ABI means the triangle ABI.
Then, find the ratio between the areas of the triangles specified as follows. ABI : ACH

Suppose as the bases, triangles share a line segment. Then, the areas of the triangles are proportional to the heights of the triangles. So the higher the height, the bigger the area if the base is constant. It's because such an area is half the product of the base and the height, and the triangles share the same base as shown in Fig. 1 above.

Fig. 2

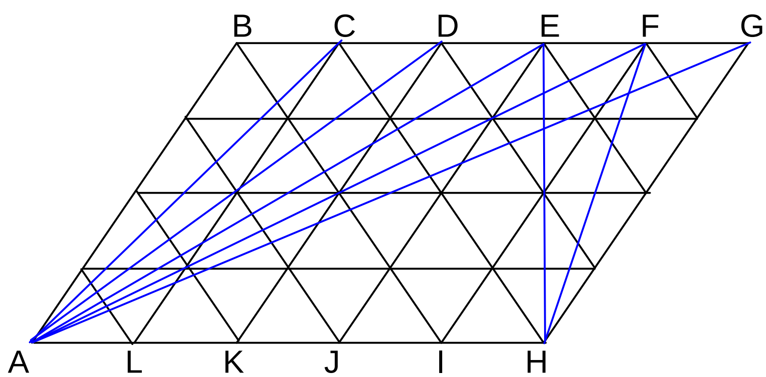



On the other hand, suppose triangles share the same height. Then, the areas of the triangles are proportional to the lengths of the bases. So the longer the base, the larger the area if the height is constant.

In Fig. 2, the triangles in blue and some triangles share the same height, which is the line segment EH, since EH is perpendicular (in $90^o$) to the line segment AH, which is the base the triangles in blue share.

ABI and ACH have the same height.
So the ratio is proportional to the lengths of the bases, the line segments perpendicular to the height.
And AI:AH = 4:5, where AI is the base of triangle ABI, and AH is the base of triangle ACH.
Thus, ABI : ACH = 4:5, since both share the same height.

1.1. All the line segments in black are parallel to each other, and they are equally spaced both horizontally and vertically. Therefore, all the triangles made by the line segments in black are regular triangles.




A triangle can be specified by the labels at all the vertices of the triangle. So ABI means the triangle ABI.
Then, find the ratio between the areas of the triangles specified as follows. ABI : ACH : AEH

We can see that the areas of ACH and AEH are the same, since they share the same height and the same base.

We know ABI : ACH = 4:5.
So ABI : ACH : AEH = 4:5:5, since ACH : AEH = 1:1.