Examples 4 in Ratios and Rates

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Examples 4 in Ratios and Rates

0. Suppose we are making two medals. One is in gold and weighs $\frac{8}{9}$ Kg. The other is in silver, and its weight is $\frac{5}{8}$ of the one in gold. What then is the weight of the silver medal?

1. When pure water frizzes, the volume of the ice is the sum of 1/11 of the water's volume and the original volume of the water. Then, what part (or portion) of the volume of the ice gets lost when it completely melts to become water?

2. A train left Chicago for Boston. Among the passengers, children took up 25% of the seats taken. For adults, the ratio between male and female is 3:4. The number of children is 35 less than the number of female adults. What is the number of children in the train?

3. There are two buses running on a straight road. Bus A left a station A and traveled 5 Km to the west. Bus B left a station B and ran 7 km to the east. The two buses have already passed each other, and the distance between the buses is 1/5 of the distance between the stations.
What then is the distance between the two stations?

4. Two children, a boy and a girl have apples. The girl has 3 more apples than the boy has. If the two children eat 5 apples each, half the girl's apples will be the same as 5/7 of those the boy will have. How many apples will each of the children have after their eating?

5. There are two classes at a school. Class A has 5 less students than 3/4 of the students that class B has. If class B sends 5 students to class A, then class A will have 21/23 of the students that class B will have. How many students are there now in each of the two classes?

6. It takes 20 days for A to complete a project, and it takes 25 days for B to complete the same project. For the first 5 days, A and B will work together on the project. After the five days, B will work alone to complete the rest of the project. How many days then will it take to complete the entire project?

7. It takes 30 days for A to complete a work, and B can complete the same work in 25 days. A and B worked together on the same work and completed it in 20 days. During the 20 days period, A took 5 days off. Then, how many days did B take off during the 20-day period?

8. Two trains run between two cities, A and B. It takes 4 hours for one train to get to one city from the other. The other train takes 1/10 more of the time to travel the same distance. The two trains left each city at the same time and ran in the opposite directions. They both stopped at the same time after three hours. Suppose now that the distance between the two trains is 195 Km. Then, how far is the city A from B?

Suggestions or Solutions

To The Problem 0 In Examples 4

0. Suppose we are making two medals. One is in gold and weighs $\frac{8}{9}$ Kg. The other is in silver, and its weight is $\frac{5}{8}$ of the one in gold. What then is the weight of the silver medal?

Assuming G is the weight of the gold medal, and S is the weight of the silver medal, we can set G = $\frac{8}{9}$, and get

S = $\frac{5}{8}$ x $\frac{8}{9}$ Kg = $\frac{5}{9}$ Kg.

Suggestions or Solutions

To The Problem 1 In Examples 4

1. When pure water frizzes, the volume of the ice is the sum of 1/11 of the water's volume and the original volume of the water. Then, what part (or portion) of the volume of the ice gets lost when it completely melts to become water?

To begin with, what are we after in this problem?
Is it the volume of the ice lost?

It is a ratio. What ratio?

We are after the ratio of the volume lost to the volume of the ice.
When water gets frozen, it becomes ice, and the volume of the ice is bigger than that of water’s.

Suppose now, W is the volume of the water, V is the volume of the ice. Then, V – W is the increase in volume when water becomes ice.

And assuming U = V – W, we get U = $\frac{1}{11}$W = $\frac{W}{11}$.

Thus, we get V = U + W = $\frac{W}{11}$+ W = $\frac{12W}{11} = \frac{12}{11}$W

And we know the decrease in volume when ice completely melted is the same as the increase in volume when water gets completely frozen.

So when ice melted completely to become water, the decrease in volume is $\frac{1}{11}$ of W.

Assuming thus, D is the decrease in volume, and P is the ratio of D to the volume of ice V, we can set D = U, and get the ratio P the way as follows.

We get D = $\frac{W}{11}$, since D = U. And we know V =$\frac{12W}{11}$.

So we get P = $\frac{D}{V} = \frac{W}{11}$ x $\frac{11}{12W}$ = $\frac{1}{12}$.

Thus, the ratio is $\frac{1}{12}$, so $\frac{1}{12}$ of the volume of the ice will be decreased when it gets completely melted to water.

Suggestions or Solutions

To The Problem 2 In Examples 4

2. A train left Chicago for Boston. Among the passengers, children took up 25% of the seats taken. For adults, the ratio of the male to female is 3:4. And the number of children is 35 less than the number of female adults.
What then is the number of children in the train?

We need to assume that everyone in the train has taken a seat, of course. So to begin with, let's set
t = the total number of passengers in the train
c = the number of children
m = the number of male adults
f = the number of female adults

Then, we can get c + m + f = t.
We know c is 25% of t.
So we get c = 0.25t = $\frac{25}{100} = \frac{1}{4}t \Rightarrow$ t = 4c

We know c is 35 less than f.

So we get $c = f - 35 \Rightarrow f = c + 35$

And the ratio of the male adults to the female adults = 3:4.

So $m:f = 3:4 \Rightarrow \frac{m}{f} = \frac{3}{4} \Rightarrow m = \frac{3}{4}f$.

Also, we know $f = c + 35$. So we get $m = \frac{3}{4}f = \frac{3}{4}(c + 35)$.

Thus, we get $c + m + f = t \Rightarrow c + \frac{3}{4}(c + 35) + c + 35 = 4c$

So solving the equation for c now, we get

$$c + \frac{3}{4}(c + 35) + c + 35 = 4c$$

$$\Rightarrow 4c + 3(c + 35) + 4(c + 35) = 16c \Rightarrow 7 \times 35 = 5c \Rightarrow c = 49$$

Therefore, the number of the children is 49.

Suggestions or Solutions

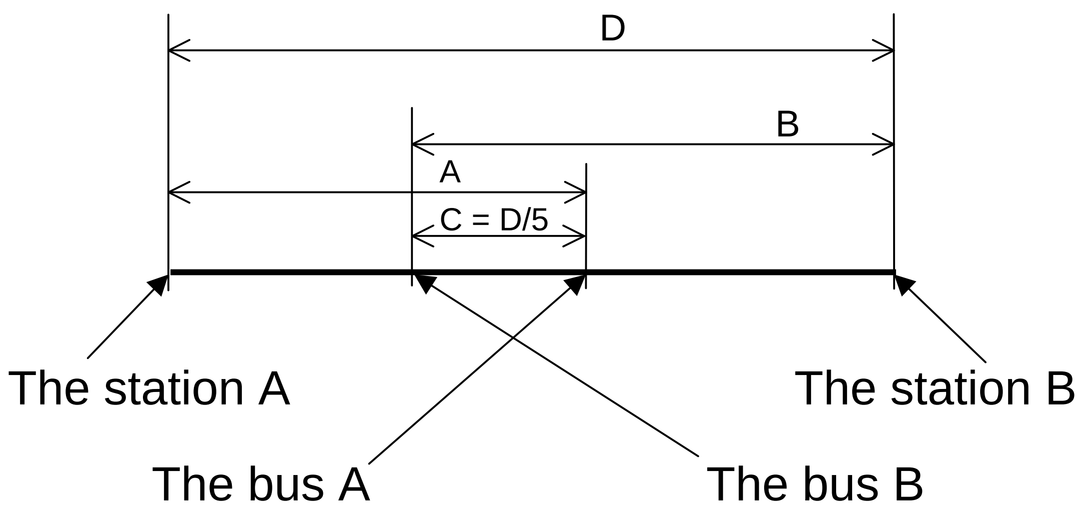
To The Problem 3 In Examples 4

3. There are two buses running on a straight road. Bus A left a station A and traveled 5 Km to the west. Bus B left a station B and ran 7 km to the east. The two buses have already passed each other, and the distance between the buses is 1/5 of the distance between the stations. What is the distance between the two stations?

To begin with, we may want to name first, what we are working with in the problem definition.
What is the problem definition though?

It is what the problem is made of or just what the problem is saying.
So suppose first, A is the distance covered by bus A,
B is the distance covered by bus B
D is the distance between the stations A and B.
C is the distance between the buses

And we may not want to just keep thinking about the problem, so it’s a good idea to the problem in a diagram the way as follows. Then, we can see better what the problem is talking about, and how we can get the solution.



The sum of the distances covered by both buses is greater than D, since they passed each other.

And the difference between the sum and D is C.

So we get C = A + B – D.

Then, $D/5 = 5 + 7 - D = 12 - D \Rightarrow (6/5)D = 12 \Rightarrow D = 10$.

Therefore, the station A is 10 Km away from the station B.

Suggestions or Solutions

To The Problem 4 In Examples 4

4. Two children, a boy and a girl have apples. The girl has 3 more apples than the boy has. If the two children eat 5 apples each, half the girl's apples will be the same as 5/7 of those the boy will have. How many apples will each of the children have after eating their apples?

To begin with, what is the problem is asking?

The leftover each child will have after eating 5 apples, but we may want to find first the number of apples each child has before eating the apples.
Finding it first, and then, subtracting 5 from each number of apples each child, we can get the leftover each child will have.

And we may want to begin with naming what we are working with doing the problem.

So assuming first, B is the number of apples the boy has, and G is the number of apples the girl has, we get $G = B + 3$, since the girl has 3 more apples than the boy has.

Next, after the two children eat 5 apples each, half the girl's apples will be the same as 5/7 of those the boy will have.
So we get $(G - 5)(1/2) = (B - 5)(5/7)$.

Thus, we have $G = B + 3$ and $(G - 5)(1/2) = (B - 5)(5/7)$.

First, we may want to remove the fractions, so multiplying both sides by the denominators in the fractions, we can get
$7(G - 5)2(1/2) = 2(B - 5)7(5/7) \Rightarrow 7(G - 5) = 10(B - 5)$
$\Rightarrow 7G - 10B = 35 - 50 = -15 \Rightarrow 7G - 10B = -15$
$\Rightarrow 7B + 21 - 10B = -15$, since $G = B + 3$.
$\Rightarrow 3B = 36 \Rightarrow B = 12 \Rightarrow G = B + 3 = 12 + 3 = 15$.

So the girl will have $15 - 5 = 10$ apples, and the boy will have $12 - 5 = 7$ apples after eating their apples.

Suggestions or Solutions

To The Problem 5 In Examples 4

5. There are two classes at a school. Class A has 5 less students than 3/4 of the students that class B has. If class B sends 5 students to class A, then class A will have 21/23 of the students that class B will have. How many students are there now in each of the two classes?

Suppose that:

The number of students in the class A = A
The number of students in the class B = B

Then:

The class A has 5 less students than 3/4 of the students that the class B has.
→ A = (3/4)B – 5

If the class B sends 5 students to the class A, then the class

B will have B – 5, and the class A will have A + 5.

After the sending and receiving, the class A will have 21/23 of the students that the class B will have. → A + 5 = (B – 5)(21/23).

Then, we have two equations in A and B as follows:
A = (3/4)B – 5 and A + 5 = (B – 5)(21/23).

Now then, we need to solve the simultaneous equations as above for A and B.

In the first equation, we have A = (3/4)B – 5 → A + 5 = (3/4)B
In the second equation, we have A + 5 = (B – 5)(21/23) → (3/4)B = (B – 5)(21/23), which is now in B alone.

(B – 5)(21/23) = (21/23)B – (21/23)5.

Then, (3/4)B = (21/23)B – (21/23)5

Then, (21/23)5 = (21/23)B – (3/4)B = (21 x 4 – 23 x 3)B / (23 x 4) = (84 – 69)B/(23 x 4) = 15B/(23 x 4).

Then, (21/23)5 = 15B/(23 x 4) → 21 x 5 = 15B/4 → 21 x 5 x 4 = 15B = 3 x 5B → 3 x 7 x 4 = 3B → B = 28.

We know that A = (3/4)B – 5.
Then, (3/4)28 – 5 = (3/4)(4 x 7) – 5 = 21 – 5 = 16.

Therefore, the class B will have B – 5 = 28 – 5 = 23 students, and the class A will have A + 5 = 16 + 5 = 21 students.

Suggestions or Solutions

To The Problem 6 In Examples 4

6. It takes 20 days for A to complete a project, and it takes 25 days for B to complete the same project. For the first 5 days, A and B will work together on the project. After the five days, B will work alone to complete the rest of the project. How many days will it take to complete the entire project?

Suppose that:

The amount of the project = P

Then:

It takes 20 days for A to complete a project. → We may assume that A can cover P/20 a day, and that the daily speed of the work done by A = A = P/20.

It takes 25 days for B to complete the same project. → We may assume that B can cover P/25 a day, and that the daily speed of the work done by B = B = P/25.

For the first 5 days, A and B will work together on the project.
→ The amount of the work done by A and B together for the five days = A x 5 + B x 5
= (A + B)5
= (P/20 + P/25)5
= {5P/(20 x 5) + 4P/(25 x 4)}5
= (9P/100)5
= 45P/100

Then, the rest of the work to be done = P – 45P/100 = 55P/100.

Suppose that the number of days it takes for B to complete the rest = D.

Then, D = (55P/100)/(P/25) because the number of days = the amount of work / the speed of the work.

Then:

D

= (55P/100)/(P/25)

= (55/100)/(1/25)

= {55/(4 x 25)}/(1/25)

= 55/4 = (44 + 11)/4 = 11 + 11/4 = 11 + (8 + 3)/4 = 11 + 2 + 3/4 = (13 + 3/4) days.

Then, B will have to work 14 more days alone to complete the rest of the work.

Therefore, it will take 5 + 14 = 19 days for the entire project to be completed.

Note:

{55/(4 x 25)}/(1/25) = {(55/4)(1/25)}/(1/25) =

{(55/4)(1/25)}/(1/25) = 55/4.

Suggestions or Solutions

To The Problem 7 In Examples 4

7. It takes 30 days for A to complete a work, and B can complete the same work in 25 days. A and B worked together on the same work and completed it in 20 days. During the 20 days period, A took 5 days off. Then, how many days did B take off during the 20-day period?

Suppose that:

The amount of work = W

Then:

It takes 30 days for A to complete a work. → We may assume that A can cover W/30 a day, and that the daily speed of the work done by A = the daily rate of A = A = W/30.

B can complete the same work in 25 days. → We may assume that B can cover W/25 a day, and that the daily speed of the work done by B = the daily rate of B = B = W/25.

During the 20 days period, A took 5 days off. → A worked 15 days.

Suppose that the total amount of work done by A = C.

Then:

C
= (W/20) x 15 since the amount of work done
= the daily amount of work done x the number of days.
Now then, C = 15W/20 = 3W/4.

Suppose that the rest of the work to be done by B alone = D.
Then, D = W – (3W/4) since B = the entire work – the total amount of work done by A alone for the 15 days.

Now then, D = W – (3W/4) = W/4.

Suppose now that the number of days it will take for B to do the amount of $W/4 = E$.

Then, $E = (W/4)/(W/25)$ since the number of days = the amount of work / the daily rate.

Then, $E = (W/4)/(W/25) = (1/4)/(1/25) = (1/4)25 = 25/4 = (24 + 1)/4 = (6 + 1/4)$ days.

Then, B worked 7 days, and took off $20 – 7 = 13$ days during the 20-day period.

Suggestions or Solutions

To The Problem 8 In Examples 4

8. Two trains run between two cities, A and B. It takes 4 hours for one train to get to one city from the other. The other train takes 1/10 more of the time to travel the same distance. The two trains left each city at the same time and ran in the opposite directions. They both stopped at the same time after three hours. Suppose now that the distance between the two trains is 195 Km. Then, how far is the city A from B?

Suppose that:

The distance between the cities = D

Then:

It takes 4 hours for one train to get to one city from the other. Suppose that the train is the train A and that the average speed of the train = S. Then, S = D/4 Km/hour.

The other train takes 1/10 more of the time to travel the same distance. Suppose that the other train is the train B and that the average speed of the train = T. Then, it takes 4 + 4 x 1/10 = 4 + 4/10 hours for the train B to travel the D. Then, T = D/(4 + 4/10) Km/hour.

4 + 4/10 = 4 + 2/5 = 20/5 + 2/5 = 22/5 → D/(4 + 4/10) = D/(22/5) = D x (5/22) = 5D/22
→ T = 5D/22 Km/hour.

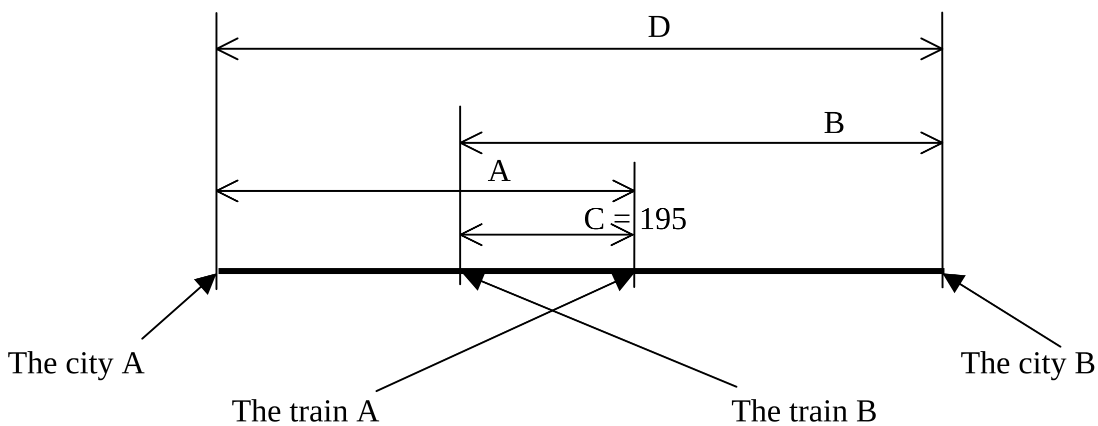
The two trains left each city at the same time and ran in the opposite directions. They both stopped at the same time after three hours.

→ The distance covered by the train A = A = S x 3 = 3S, and the distance covered by the train B = B = T x 3 = 3T.

We know that the distance between the two trains is 195 Km. Suppose that the distance between the trains = C = 195 Km. Now then, A = 3S = 3(D/4) = 3D/4

B = 3T = 3(5D/22) = 15D/22

C = 195



Then, D = A + B – C = 3D/4 + 15D/22 – 195. → D = 33D/44 + 30D/44 – 195 = 63D/44 – 195

→ 63D/44 – D = 195 → (63 – 44)D/44 = 19D/44 = 195 → D = 195 x 44/19 Km.