Examples 6 in Ratios and Rates

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1. There are two kinds of salt water. One of the two has 25% in density of salt, and the amount of water in it is 15 Kg more than that of salt. In the other, there is 17 and 2/5 Kg of salt, and the amount of water is 3/5 more of the amount of the salt. If you mix the two kinds of liquids, what is the percentage in the density of salt in the water after the mix?

2. Robert had an amount of fund in his account for his science project. He spent 1/10 of the fund on the first day. On the second day, he spent 1/9 of the fund left from the first day. On the third day, he spent 1/8 of the leftover. He kept spending the fund in the same manner; therefore, on the ninth day, he spent half the leftover. Suppose that there was 100 dollars in the account for the fund in the end of the ninth day. Then, how much was the fund in the beginning?

3. If two workers, A and B work together, a project can be completed in 20 days. The two have the same productivity. They worked together for the first 5 days. Then, B worked alone, and completed it after 20 more days. How many days does one worker need to complete the same entire project?

4. Two buses run between two cities. It takes 8 hours for one bus to get to one city from the other city. However, the other bus can cover the same route in 6 hours. The two buses left each city at the same time, and met at a point 32 Km away from the midpoint between the two cities. How far are the cities apart?

5. A group of 10 people can complete a project in 16 days if they work 8 hours a day. Another group of 16 people can complete the same project in 9 days if they work 10 hours a day. The other group of 4 people can complete the same project in 6 days if they work 12 hours a day.

5.A. If all the groups work altogether, how many days do they need to complete the project if they work 6 hours a day?

5.B. If 4 people from the first group, 10 people from the second group, and 2 people from the last group work together, how many days do they need to complete the project if they work 9 hours a day?

Suggestions or Solutions
To The Problem 1 In Examples 6

1. There are two kinds of salt water. One of the two has 25% in density of salt, and the amount of water in it is 15 Kg more than that of salt. In the other, there is 17 and 2/5 Kg of salt, and the amount of water is 3/5 more of the amount of the salt. If you mix the two kinds of salt water, what is the percentage in the density of salt in the salt water after the mix?

First, what is the problem is asking?

It's asking what % of the mixture is salt.

Suppose now, W is the amount of water, S is the amount of salt in the salt water, and D is the density of salt in %.

Then, D% of the salt water is salt, and, we get D = $\frac{100S}{W+S}$,

where W + S is the amount of salt water, of course.

What then is the amount of water in the salt water W + S?

Since the density of salt is D%, the amount of water is (100 – D)%.
How then can we get the amount of water W?

Taking the product of the salt water and the ratio of the water to the salt water, we get W, the amount of water.

What then is the ratio of the water to the salt water?

Since the density, that is, the percentage of salt to the salt water is D, the percentage of water to the salt water is 100 – D.
And a percentage is 100 times a ratio.
So dividing a percentage by 100, we get the ratio.
Thus, the ratio of water to the salt water is $\frac{100 - D}{100}$.
So taking now the amount of water in the salt water, we get
$\frac{100 - D}{100}$(W + S).

This problem is asking what % of the mixture is salt.

So we want to find first, the ratio of the salt to the mixture. What then do we need to get?

We need to get the amount of salt and that of water's in the mixture. What then is the mixture?

We get the mixture mixing two salt waters with different density of salt.
One has 25% in density of salt, and the amount of water in it is 15 Kg more than that of salt's.
In the other, there is 17 and 2/5 Kg of salt, and the amount of water is 3/5 more of the amount of salt.

Let's now start with the first one.

Suppose W is the amount of water, and S is the amount of salt. Then first, the amount of water in the salt water is 15 Kg more than that of salt's, so we get W = 15 + S.
Next, the density of salt is 25%, so we get
$S = \frac{25}{100}(W + S)$, since 25% = 0.25.

Thus, we have W = 15 + S and S = $\frac{25}{100}$(W + S)

So we get S = $\frac{25}{100}$(15 + S + S) = $\frac{25}{100}$(15 + 2S)

$\Rightarrow$ S = $\frac{25}{100}$(15 + 2S)

$\Rightarrow$ 100S = 25(15 + 2S) = 25 x 15 + 50S $\Rightarrow$ $\Rightarrow$ 50S = 25 x 15

$\Rightarrow$ 10S = 5 x 15 $\Rightarrow$ 2S = 15 $\Rightarrow$ S = $\frac{15}{2}$ Kg.

Next, we want to get the amount of water.

One salt water has 25% in density of salt, and the amount of water in it is 15 Kg more than that of salt's.

So since W = 15 + S, we get W = 15 + $\frac{15}{2}$

= 15(1 + $\frac{1}{2}$) = 15 x $\frac{3}{2}$ = $\frac{45}{2}$.

Thus, W = $\frac{45}{2}$ Kg, and S = $\frac{15}{2}$ Kg in the salt water that has 25% in density of salt.

Next, in the other salt water, there is 17 and 2/5 Kg of salt, and the amount of water is 3/5 more of the amount of the salt.

So assuming again, W is the amount of water, and S is the amount of salt in the salt water, we get S = 17 + 2/5, since there is 17 and 2/5 Kg of salt in the salt water.

And we get W = S + (3/5)S, since the amount of water is 3/5 more of the amount of salt in the salt water.

So we have S = 17 + 2/5 and W = S + (3/5)S.

We can have S = 17 + $\frac{2}{5}$ = $\frac{87}{5}$.

So we get W = S(1 + $\frac{3}{5}$) = $\frac{8}{5}$S = $\frac{8}{5}$ x $\frac{87}{5}$.

And we have W = $\frac{45}{2}$ Kg, and S = $\frac{15}{2}$ Kg in the salt water that has 25% in density of salt.

Thus, the total amount of salt = $\frac{15}{2} + \frac{87}{5} = \frac{249}{10}$.

And the total amount of water = $\frac{45}{2} + \frac{8}{5} \times \frac{87}{5} = \frac{2517}{50}$.

So assuming T is the sum of the salt and the water above,

we get T = $\frac{249}{10} + \frac{2517}{50} = \frac{3762}{50}$.

And for instance, the ratio of X to Y is $\frac{X}{Y} = X \div Y$.

Thus, assuming R is the ratio of the salt to the mixture, we

get R = $\frac{249}{10} \div \frac{3762}{50} = \frac{249}{10} \times \frac{50}{3762} = \frac{1245}{3762}$.

And a percentage is 100 times a ratio.

So assuming now D is the percentage of salt to the mixture,

we get D = 100 x R = $\frac{124500}{3762}$%, which is $(33 + \frac{3324}{3762})$%

Suggestions or Solutions
To The Problem 2 In Examples 6

2. Robert had an amount of fund in his account for his science project. He spent 1/10 of the fund on the first day. On the second day, he spent 1/9 of the fund left from the first day. On the third day, he spent 1/8 of the leftover. He kept spending the fund in the same manner; therefore, on the ninth day, he spent 1/2 the leftover. Suppose that there was 100 dollars in the account for the fund in the end of the ninth day.
Then, how much was the fund in the beginning?

Suppose M is the original amount of fund.

Then, the amount spent on the first day is $\frac{1}{10}$M.

Then, the amount left is M – $\frac{1}{10}$M = M(1 – $\frac{1}{10}$) = $\frac{9}{10}$M.

Then, the amount spent on the second day is

$\frac{1}{9} \times \frac{9}{10}$M = $\frac{1}{10}$M.

Then, the amount left is $\frac{9}{10}$M – $\frac{1}{10}$M = M($\frac{9}{10}$ – $\frac{1}{10}$) = $\frac{8}{10}$M

Then, the amount spent on the third day is

$\frac{1}{8}$ x $\frac{8}{10}$M = $\frac{1}{10}$M.

So we can see that he spent $\frac{1}{10}$M everyday for the past 9 days.

Thus, the total amount spent out of the fund is

9 x $\frac{1}{10}$M = $\frac{9}{10}$M.

So the final leftover is M – $\frac{9}{10}$M = M(1 – $\frac{9}{10}$) = $\frac{1}{10}$M = 100.

Therefore, M = 1000.

Suggestions or Solutions
To The Problem 3 In Examples 6

3. If two workers, A and B work together, a project can be completed in 20 days. The two have the same productivity. They worked together for the first 5 days. Then, B worked alone, and completed it after 20 more days. How many days does one worker need to complete the same entire project?

The two workers can complete a project in 20 days if they work together. So they can complete 1/20 of the entire work in the project a day.

We can assume that each works at the same speed everyday throughout the whole work.
However, we cannot assume that it will have to take twice as many days for each worker to complete the entire work if each works alone. That's because working together may cause some factors that may affect their productivity itself even if their productivities are still the same.

Suppose now, W is the entire amount of the work in the project.

Then, they can complete $\frac{1}{20}$W a day if they work together.

They worked 5 days together for the same project.

So 5 x$\frac{1}{20}$W = $\frac{1}{4}$W has been done, and $\frac{3}{4}$ of W is left to be done.

Now, B worked alone, and completed the rest, which is $\frac{3}{4}$W in additional 20 days.

So B completed $\frac{1}{20}$ of $\frac{3}{4}$W a day.

In other words, the amount of work B can complete a day is

$\frac{1}{20}$ x $\frac{3}{4}$W = $\frac{3}{80}$W.

So $\frac{3}{80}$W is the speed of work at which B can work on the project.

Then, we can find the number of days it would have taken for B to work alone on the entire project the way as follows.

$$W \div \frac{3}{80}W = W \text{ x} \frac{80}{3W} = \frac{80}{3} = \frac{66+14}{3} = 22 + 4 + \frac{2}{3} = 26 + \frac{2}{3}.$$

So it would have taken 26 and two-thirds days for B to complete the same work in the project.
It would have taken the same for A, too.

Therefore, we can see that two workers' working together does not necessarily double the productivity in all cases.

Suppose the two workers need to complete two same projects. Then, the manager can expect two different results based on the data found above. If they work together on each project, and complete the two projects, then it will take 40 days. However, if each worker works on each project, both projects can be completed in 26 and two-thirds days. Thus, the manager will probably ask them to work alone on each project.

Suggestions or Solutions
To The Problem 4 In Examples 6

4. Two buses run between two cities. It takes 8 hours for one bus to get to one city from the other city. However, the other bus can cover the same route in 6 hours. The two buses left each city at the same time, and they met at a point 32 Km away from the middle between the two cities. How far then are the cities apart?

Suppose D is the distance between the two cities, bus A can cover D in 8 hours, and bus B can cover D in 6 hours.
So bus B is faster.
Assuming U is the speed of A, and V is the speed of B, we get D = 8U = 6V, since both buses cover the same distance.
So we get U/V = 6/8 = 3/4. Thus, U:V = 3:4.

We know that both buses will spend the same amount of time until they meet. Suppose T is the amount of time it takes for them to meet. Then, UT is the distance A covered, and VT is the distance B covered.

We know the position at which they meet is 32 kilometers away from the middle between the two cities, D/2 is the distance between the midpoint and each of the cities, and bus B is faster.

Also, UT is the distance A covered, and VT is the distance B covered.

So assuming M = D/2,

we get M – 32 = UT, and M + 32 = VT.

And we have U/V = 3/4. So we get $\frac{UT}{VT} = \frac{M-32}{M+32} = \frac{3}{4}$

Thus, we get

$4(M - 32) = 3(M + 32) \Rightarrow M = 7 \times 32 = 224$.

We know M = D/2. So we get D/2 = 224, and D = 448.

Therefore, the distance between the two cities is 448 Km.

Suggestions or Solutions
To The Problem 5.A In Examples 6

5. A group of 10 people can complete a project in 16 days if they work 8 hours a day. Another group of 16 people can complete the same project in 9 days if they work 10 hours a day. The other group of 4 people can complete the same project in 6 days if they work 12 hours a day.

5.A. If all groups work altogether, how many days does it take to complete the project if they work 6 hours a day?

Suppose A is the group of 10 people, B is the group of 16 people, and C is the group of 4 people

We put all the groups together.
No other information is given for any of the groups.

So we may want to find the individual hourly productivities for the groups, and put them together.

We know the people in A work 8 hours a day for 16 days.

The people in B work 10 hours a day for 9 days.

The people in C work 12 hours a day for 6 days.

So assuming W is the amount of the work, we get these:

The productivity of A = $\frac{W}{16 \times 8}$ per hour

The productivity of B = $\frac{W}{9 \times 10}$ per hour

And the productivity of C = $\frac{W}{6 \times 12}$ per hour

Assuming thus, S is the sum of all the productivities, we get

$S = \frac{W}{16 \times 8} + \frac{W}{9 \times 10} + \frac{W}{6 \times 12}$. And we have

$16 \times 8 = 2^7$, $9 \times 10 = 2 \times 3^2 \times 5$, and $6 \times 12 = 2^3 \times 3^2$.

So the LCM = $2^7 \times 3^2 \times 5$.

Thus, we get $S = \frac{W}{16 \text{ x } 8} + \frac{W}{9 \text{ x } 10} + \frac{W}{6 \text{ x } 12}$

$= W(\frac{1}{16 \text{ x } 8} + \frac{1}{9 \text{ x } 10} + \frac{1}{6 \text{ x } 12})$

$= W \text{ x } \frac{3^2 \text{ x } 5 + 2^6 + 2^4 \text{ x } 5}{2^7 \text{ x } 3^2 \text{ x } 5} = \frac{189}{5760}W$

Thus, taking the total number of hours to get the work done,

we take $\frac{\text{the amount of work}}{\text{the productivity}}$, so we get

$\frac{W}{S} = \frac{5760}{189} = 30 + \frac{90}{189}$ hours. And converting $\frac{90}{189}$ to

minutes, we get $\frac{90}{189} \text{ x } 60 = \frac{5400}{189} = (28 + \frac{108}{189})$ minutes.

So they need to work 30 hours and $(28 + \frac{108}{189})$ minutes.

We know that they work 6 hours a day.

So they need to work 5 days, and $(28 + \frac{108}{189})$ minutes more

on the sixth day. Therefore, it takes 6 days.

They would rather however, finish the project in 5 days

doing the work worth $(28 + \frac{108}{189})$ minutes on the fifth day.

Suggestions or Solutions
To The Problem 5.B In Examples 6

5. A group of 10 people can complete a project in 16 days if they work 8 hours a day. Another group of 16 people can complete the same project in 9 days if they work 10 hours a day. The other group of 4 people can complete the same project in 6 days if they work 12 hours a day.

5.B. If 4 people from the first group, 10 people from the second group, and 2 people from the last group work together, how many days does it take for them to complete the project if they work 9 hours a day?

We need to get the productivity in individual level.

Suppose A is the group of 10 people, B is the group of 16 people, and C is the group of 4 people.
We know the people in A work 8 hours a day for 16 days.
The people in B work 10 hours a day for 9 days.
The people in C work 12 hours a day for 6 days.

So assuming W is the amount of the work, we get these:

The productivity of A = $\frac{W}{16 \text{ x } 8}$ per hour

The productivity of B = $\frac{W}{9 \text{ x } 10}$ per hour

And the productivity of C = $\frac{W}{6 \text{ x } 12}$ per hour

Thus, we get these:

An individual's productivity in A = $\frac{W}{16 \text{ x } 8 \text{ x } 10}$ per hour

An individual's productivity in B = $\frac{W}{9 \text{ x } 10 \text{ x } 16}$ per hour

An individual's productivity in C = $\frac{W}{6 \text{ x } 12 \text{ x } 4}$ per hour

We know among all the people in the new team, 4 people are from A, 10 people are from B, and 2 people are from C.

We need to add all their hourly productivities together.

So assuming S is the sum of all the productivities, we get

$$S = \frac{4W}{16 \text{ x } 8 \text{ x } 10} + \frac{10W}{9 \text{ x } 10 \text{ x } 16} + \frac{2W}{6 \text{ x } 12 \text{ x } 4}$$

$$= \left(\frac{1}{4 \text{ x } 8 \text{ x } 10} + \frac{1}{9 \text{ x } 16} + \frac{1}{3 \text{ x } 12 \text{ x } 4}\right)W$$

$$= \left(\frac{1}{2^6 \text{ x } 5} + \frac{1}{2^4 \text{ x } 3^2} + \frac{1}{2^4 \text{ x } 3^2}\right)W$$

$$= W(3^2 + 2^2 \text{ x } 5 + 2^2 \text{ x } 5)\frac{1}{2^6 3^2 5} = \frac{49W}{2880}$$

Thus, taking the total number of hours to get the work done,

we take $\frac{\text{the amount of work}}{\text{the productivity}}$, so we get

$\frac{W}{S} = \frac{2880}{49}$ = 58 + $\frac{38}{49}$ hours. And converting $\frac{38}{49}$ to

minutes, we get $\frac{38}{49}$ x 60 = $\frac{2280}{49}$ = (46 + $\frac{26}{49}$) minutes.

So they need to work 58 hours and (46 + $\frac{26}{49}$) minutes.

We know that they work 9 hours a day.

So they need to work 6 days, and 4 hours and (28 + $\frac{108}{189}$)

minutes more on the seventh day. Therefore, it takes 7 days.