Examples 8 in Ratios and Rates

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Examples 8 in Ratios and Rates

A clock usually has three hands rotating about their axes. In such a clock, we can easily see the hour hand and the minute hand making angles as they rotate.

The angle between the two hands depends on the time the clock indicates. In each example, we’ll find the time it takes for the hands to make a particular angle called the target angle from the current time given, and will find the time in hours, minutes, and seconds. Also, we’ll find the first moment of time when the target angle happens.

We’ll specify the moment of time in such a manner as ‘hh:mm:ss’ using the 24-hour system where ‘hh’ stands for the hour, ‘mm’ for the minute, and ‘ss’ for the second. For instance, if the target angle happens at 9 o’clock 17 minutes 32 + 5/11 seconds in the morning, the answer should be in such a form as follows. 09:17:32 + 5/11.

0. In each of the two cases below, find the amount of time it takes for the hands to make the target angle, and find the first moment of time the hands make the target angle.

0.0. The target angle is 180^o, and the current time is 00:12:37. The target angle has to happen before 01:00:00.

0.1. The target angle is 5^o, and the current time is 00:15:42. The target angle has to happen before 01:00:00.

1. In each of the five cases below, find the first moment of time when the hands make the target angle if the present time is each of every hour on the hour. So we need to set the present time to each of 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00., and find the first moment of time when the hands make the target angle..

1.0. The target angle is 90^o.
1.1. The target angle is 60^o.
1.2. The target angle is 45^o.
1.3. The target angle is 145^o.
1.4. The target angle is 270^o.

Suggestions or Solutions
To The Problem 0.0 In Examples 8

0.0. If the current time is 00:12:37, find the first moment of time when the hands make a target angle 180^o, and the time it takes for the hands to make the target angle.
Note that the target angle has to happen before 01:00:00.

In this problem, the current time is 00:12:37, and the target angle is 180^o.
What if however, the current time is anytime between the midnight (00:00:00) and 1 o'clock (01:00:00)?

Of course, the answer depends on both the target angle and the current time. So it depends on the positions of the two hands at the beginning, and also, depends on the size of the target angle. However, the problem solving procedure hardly changes. Some adjustments may be needed. The most important fact to understand is that the hands are moving at their own constant speeds.

We know the target angle is 180^o, and the current time is 00:12:37. Then, the answer is still the same as the one in the problem 0 in the Examples 7 in Ratios and Rates.

So the target angle will happen at 00:32:43+7/11.

Next, taking the amount of time it takes for the target angle to happen, we only need to find the difference between the current time and the time when the target angle happens.

In seconds, 43 + 7/11 – 37 = 6 + 7/11 (sec.)
In minutes, 32 – 12 = 11 (min.)

Therefore, 180^o will happen in 11 minutes and 6 + 7/11 seconds from 00:12:37. In fact, in case where the current time is in the time span of [00:00:00 ~ 00:32:43+7/11], the solution for the moment of time is the same. Of course, if the current time is different, the amount of time it takes for the target angle to happen is different, too, though.

Suggestions or Solutions
To The Problem 0.1 In Examples 8

0.1. If the current time is 00:15:42, find the first moment of time when the hands make a target angle 5^o, and the time it takes for the hands to make the target angle.
Note that the target angle has to happen before 01:00:00.

It won't happen before 1 o'clock.
That's because at the current time, 00:15:42, the angle between the hands is more than 5^o, so $M – H > 5^o$, and the minute hand is ahead of the hour hand.
Also, the minute hand is moving faster than the hour hand, and the angle between the hands gets larger as time proceeds until the angle becomes 180^o. We know that the angle will be 180^o when it is 00:32:43+7/11 from the previous problem.
After 00:32:43+7/11, the angle between the hands will get smaller, but the angle will still be bigger than 30^o until 01:00:00. Therefore, the target angle will not happen before 01:00:00. Of course, it'll happen soon after 01:00:00.

Also, if the current time is any moment past 00:32:43+7/11 and before 01:00:00, a target angle of 180^o will not happen before 01:00:00. So some target angles depend on the choice of current time in a time span.

Assuming though, the current time is 00:15:42, and finding the time it takes for the target angle 5^o to happen for the first time after 1 o'clock, we can get it the way as follows.

Assuming the current time is 1 o'clock, H is the angle made by the hour hand, M is the angle made by the minute hand, d is the angle between the two hands, and t is the time taken, we get H = 30 + 0.5t, M = 6t, and d = H – M, since at 1 o'clock, the hour hand is 30^o ahead of the minute hand, and H is bigger than M when 5^o happens the first time after 1 o'clock.

So we get d = 30 + 0.5t – 6t = 5 $\Rightarrow$ 5.5t = 25
$\Rightarrow$ t = 25/5.5 = 5/1.1 = 50/11 = 4 + 6/11 minutes.
And in seconds, we get 60 x 6/11 = 360/11 = 32 + 8/11 seconds. Therefore, from 1 o'clock, it takes 4 minutes and 32 + 8/11 seconds for 5^o to happen the first time.

Now, the actual current time given is 00:15:42. So it takes 44 minutes and 18 seconds to be 1 o'clock. And we need to add 4 minutes and 32 + 8/11 seconds to the time above.

Then, we get 44 + 4 = 48, and 18 + 32 + 8/11 = 50 + 8/11. Thus, from 00:15:42, it takes 48 minutes and 50 + 8/11 seconds for the target angle 5^o to happen the first time.

Suggestions or Solutions
To The Problem 2 In Examples 8

1.0. Find the first moment of time when the hands make 90^o if the present time is each of every hour on the hour. So we need to set the present time to each of 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00., and find the moment of time when the hands make 90^o. Note that the target angle has to happen before the next hour on the hour.

This problem is no other than the previous problems except the present time is every hour on the hour.
So suppose now again, H is the angle made by the hour hand, M is the angle made by the minute hand, d is the angle between the two hands, and t is the time taken.

Then, if the present time is midnight, 00:00:00, since M is bigger than H when the target angle happens, we get
$H = ht$, $M = mt$, and $d = M – H$, where h is the time rate of change in angle for the hour hand, and is 0.5, and m is the time rate of change in angle for the minute hand, and is 6.

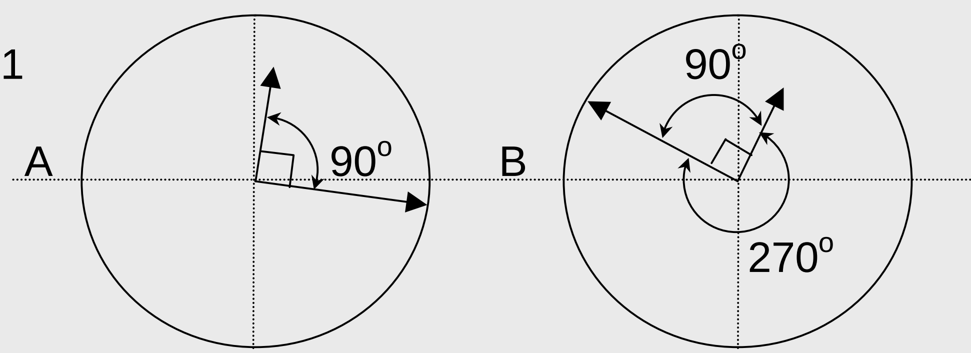
So we get d = M – H = 6t – 0.5t = 90 $\Rightarrow$ 5.5t = 90

Thus, we get t = 90/5.5 = 18/1.1 = 180/11 = 16 + 4/11.

So we get t = 16 + $\frac{4}{11}$ minutes.

Thus, the target angle happens at 0 o'clock 16 + 4/11 minutes. Also, the target angle can happen one more time before 1 o'clock as shown in Fig. 1 B below.

Fig. 1



The Fig. 1 shows another possible case where the target angle 90° can happen before 01:00:00, and the case is B. So we have two cases as follows.

Case A: the time it takes for the hands to make 90°

Case B: the time it takes for the hands to make 270°.

And in the case B, 270° is the difference between 360° and the target angle 90°.

So A's already been done, and now doing B, we reset the target angle to 270^o, and find the time it takes for 270^o to happen, that is, the time it takes for the hands to make 270^o.

M is bigger than H when 270^o happens. So we use
M – H = d. Thus, we get d = 6t – 0.5t = 5.5t = 270
$\Rightarrow$ t = 270/5.5 = 2700/55 = 49 + 5/55 = 49 + 1/11 minutes, which should be 3 times the time it takes for 90^o to happen the first time. Why 3 times?

We know $270^o = 90^o$ x 3, it takes 16 + 4/11 minutes for 90^o to happen the first time, and the hands move at their own constant speeds. So we can expect
49 + 1/11 = (16 + 4/11) x 3.

Checking thus, to see if it's the case, we get (16 + 4/11) x 3 = 48 + 12/11 = 49 + 1/11. So it is the case.
Thus, the hands make 90^o at both 0 o'clock 16 + 4/11 minutes and 0 o'clock 49 + 1/11 minutes.
And of course, we can convert the fraction of a minute to seconds multiplying it by 60.

• Let's now move on to the current time of 01:00:00.

At 01:00:00, the hour hand is at the 1 o'clock marker, and the minute hand is at the 12 o'clock marker. So the hour hand is 30^o ahead of the minute hand at the beginning.

Thus, H = 30 + 0.5t. M is however, greater than H when 90^o happens. So the calculations go the way as follows.

$M – H = d \Rightarrow 6t – (30 + 0.5t) = 90 \Rightarrow 5.5t = 120.$

$\Rightarrow t = 120/5.5 = 1200/55 = 240/11 = 21 + 9/11$ minutes.

Thus, 90^o gets made at 1 o'clock 21 + 9/11 minutes.

Now, what about the time it takes for the target angle 90^o to happen the second time before 2 o'clock?

In this case, we do not just multiply the time by 3. Why not?

That's because the hour hand is 30^o ahead of the minute hand at the beginning. So it takes more time than the time it takes when the current time is midnight, because both hands start at the same spot when the current time is midnight. Why?

If the current time is 1 o'clock, the hour hand is 30^o ahead of the minute hand. So it takes time for the minute hand to catch up and overlap the hour hand. Thus, adding the time until the overlap to the time it takes for 90^o to happen when the current time is midnight, we get the time for 90^o to happen when the current time is 1 o'clock.

So let's see now, how long it takes for the minute hand to catch up and overlap the hour hand.

We know M is the angle made by the minute hand, and H is the angle made by the hour hand. At the beginning thus, the difference between M and H is 30^o, because the hour hand is 30^o ahead of the minute hand.

The difference in angle will be covered at the difference between the speeds of the hands.
And the difference is $6 – 0.5 = 5.5$.
So we get $5.5t = 30 \Rightarrow t = 30/5.5$ minutes, which is the time until the overlap, that is, the time for 30^o to be covered.
We know the minute hand is faster.

Thus, after 30/5.5 minutes, the minute hand overlaps and overtakes the hour hand. And we get

t = 30/5.5 = 300/55 = 60 x 5/55 = 60/11 = (55 + 5)/11

= 5 + 5/11 minutes.

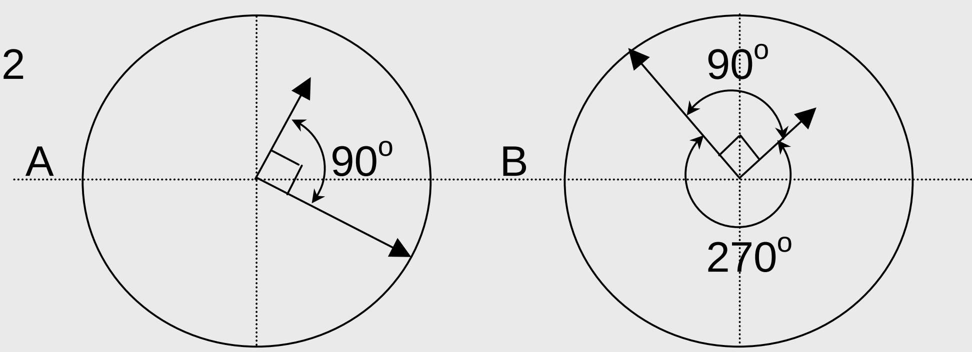
So now, subtracting 5 + 5/11 minutes from 21 + 9/11 minutes, we get

21 + 9/11 – (5 + 5/11) = 21 – 5 + 9/11 – 5/11 = 16 + 4/11,

which is indeed, the time it takes for 90^o to happen when the current time is midnight.

Now, the target angle 90^o happens one more time before 2 o'clock as shown in Fig. 2 B below.

Fig. 2



So as in the case of the current time of 00:00:00 in Fig. 1, after the case A happens, the case B gets made. Thus, the hands make 270^o so that the target angle 90^o happens the second time as shown in Fig. 2 B.

We know $270^o = 3 \times 90^o$. And 90^o is worth 16 + 4/11 minutes. So we just need to add the time until the overlap to the product of 3 and 16 + 4/11 minutes. Thus first, we get

$3 \times (16 + 4/11) = 48 + 12/11 = 49 + \frac{1}{11}$ minutes.

So it takes (5 + 5/11) + (49 + 1/11) = 54 + 6/11 minutes for the target angle 90^o to happen for the second time.

Of course, way at the beginning, we've already noticed that it's nothing but the case where the target angle is 270^o when the current time is 1 o'clock. So let's see now if it is the case.

Then, checking to see if it is the case, we get

$M - H = 270 \Rightarrow 6t - (30 + 0.5t) = 270 \Rightarrow 5.5t = 270 + 30$

$\Rightarrow t = (270 + 30)/5.5 = (3 \times 90 + 30)/5.5 = (3 \times 18 + 6)/1.1$

$= (3 \times 180 + 60)/11 = 3 \times 180/11 + 60/11$

$= 3 \times (16 + 4/11) + (5 + 5/11) = (49 + 1/11) + (5 + 5/11)$

= 54 + 6/11 minutes. So it is the case.

Thus, 90^o happens twice before 2 o'clock, and happens at both 1 o'clock 21 + 9/11 minutes and 1 o'clock 54 + 6/11 minutes.

During the calculation above though, why not just do this:
t = 300/5.5, but why do this: t = (270 + 30)/5.5?

It shows what the calculation is doing. We don't just crunch numbers learning and doing math. We can see a pattern in the calculation. Seeing patterns, we are doing math. Seeing patterns, we can build solutions. That's what we do in math.

• Now, let's move on to the current time of 02:00:00.

At 02:00:00, the hour hand is at the 2 o'clock marker, and the minute hand is at the 12 o'clock marker. So the hour hand is 2 x 30^o = 60^o ahead of the minute hand at the beginning. And M is larger than H when 90^o happens. Thus, the calculations go the way as follows.

$d = M – H \Rightarrow d = 6t – (30 \times 2 + 0.5t) = 90$

$\Rightarrow 6t – 30 \times 2 – 0.5t = 5.5t – 30 \times 2 = 90$

$\Rightarrow 5.5t = 90 + 30 \times 2.$

So we get t = (90 + 2 x 30)/5.5 = 18/1.1 + 2 x 6/1.1

= 180/11 + 2 x 60/11 = 16 + 4/11 + 2 x (5 + 5/11)

= 16 + 4/11 + 10 + 10/11 = 26 + 14/11 = 27 + 3/11 minutes.

Now, how about the time for 90^o happens the second time?

In this case, there is no second time before 3 o'clock.
At 03:00:00 on the hour, the angle between the hands is the very 90^o.
So in fact, if we remove the time until the overlap from the time we've just found above, multiply the difference by 3, and add back the time until the overlap to the product, then the sum will be exactly 60 minutes.
And if we repeat the same story above using numbers, we can put it the way as follows.
If we subtract 10 + 10/11 from 27 + 3/11, we get 16 + 4/11.
Next, if we multiply 16 + 4/11 by 3, we get 49 + 1/11.
And next, if we add back 10 + 10/11 minutes to 49 + 1/11 minutes, then the result will produce exactly 60 minutes.

• Let's now move on to the current time of 03:00:00.

The target angle 90^o happens twice in the time span of [03:00:00 ~ 04:00:00].

At 3 o'clock, the angle between the hands is 90^o, so the target angle has already happened.
There is another moment of time when 90^o happens after 3 o'clock, of course.

The angle between the hands is already 90^o right at the beginning, and the hour hand is at the 3 o'clock marker, and thus, is 90^o ahead of the minute hand. So H = 90 + 0.5t, but when the target angle 90 happens, M is bigger than H.
Finding thus, the second moment of time when the target angle happens, we get
$M – H = d \Rightarrow 6t – (90 + 0.5t) = 90 \Rightarrow 5.5t = 90 + 90 = 2 \times 90$
$\Rightarrow t = 2 \times 90/5.5 = 2 \times (16 + 4/11) = 32 + 8/11$ minutes

So it takes twice the time it takes for the target angle 90^o to happen for the first time if the current time is midnight.
It's because the hour hand is $30^o \times 3 = 90^o$ ahead, since each hour marker is 30^o away from the next hour marker.

- Let's now move on to the current time of 04:00:00.

In this case, at 4 o'clock, the angle between the hands, that is, the difference d is bigger than the target angle.
The difference d is 4 x 30^o = 120^o > the target angle = 90^o.

So the minute hand doesn't have to overtake the hour hand.

In this case, H stays greater than M.
Thus, the equation is H – M = d. And H = 4 x 30 + 0.5t.
So we get (4 x 30 + 0.5t) – 6t = 90 $\Rightarrow$ (120 + 0.5t) – 6t = 90
$\Rightarrow$ 5.5t = 30 $\Rightarrow$ t = 30/5.5 = 5 + 5/11 minutes.

Therefore, the hands make 90^o at 5 + 5/11 minutes past 4 o'clock for the first time.
Let's now find the second moment of time when the target angle 90^o happens.
It will happen after the sum of two amounts of time from 4 o'clock. One is the time until the overlap to cover 120^o, and the other is the time it takes for 90^o to happen.

Note that it is way much better to get the time from 4 o'clock than to get the time from 4 o'clock 5 + 5/11 minutes.

So first, finding the time until the overlap, we get

$M - H = 120 \Rightarrow 6t - 0.5t = 5.5t = 120$

$\Rightarrow t = 120/5.5 = 4 \times 30/5.5$, which means 4 times the time until the overlap to cover 30^o.

And we know 30/5.5 = 5 + 5/11.

Thus, we get $t = 4 \times (5 + 5/11) = 20 + 20/11 = 20 + 1 + 9/11$ $= 21 + 9/11$ minutes until the overlap to cover 120^o.

Next, we know the time it takes for the hands to make 90^o after the time until the overlap is 16 + 4/11 minutes.

So taking the sum of the two amounts of time, we get

(21 + 9/11) + (16 + 4/11) = 37 + 13/11 = 38 + 2/11 minutes.

Of course, it is just another way of saying the calculations as follows.

$M - H = d \Rightarrow 6t - (4 \times 30 + 0.5t) = 90 \Rightarrow 5.5t = 90 + 4 \times 30$

$\Rightarrow t = (90 + 4 \times 30)/5.5 \Rightarrow t = 90/5.5 + 4 \times 30/5.5$

$= 16 + 4/11 + 21 + 9/11 = 38 + 2/11$

The calculations above are for the time for 90^o to happen when the current time is 4 o'clock.

Therefore, the hands make 90^o at both 5 + 5/11 minutes past 4 o'clock and 38 + 2/11 minutes past 4 o'clock.

Now, we can see a pattern, which is saying that the same method applies when the current time is each hour starting from 5 o'clock to 8 o'clock.

We know d is the angle between the hands, that is, the difference between M and H.

So the target angle happens twice if d at the current time is bigger than the target angle, so one is before the minute hand overtakes the hour hand, and the other is after the minute hand overtakes the hour hand.

Thus, we'll use two equations A and B.
The equation A is for the time for the target angle 90^o to happen for the first time,
and the equation B includes the time until the overlap, and thus, is for the second moment of time.

So the equation A is $H - M = 90$, where $M = 6t$,
and $H = n \times 30 + 0.5t$, where $n = 4, 5, 6, 7$, and 8.

The equation B is $t = n \times 30/5.5 + (16 + 4/11)$,
where $n = 4, 5, 6, 7$, and 8.

And rewriting A, we get
$H - M = 90 \Rightarrow n \times 30 + 0.5t - 6t = 90 \Rightarrow 5.5t = 30(n - 3)$
$\Rightarrow t = 30(n - 3)/5.5 = (n - 3)(5 + 5/11)$,
since $30/5.5 = 5 + 5/11$. So we get
$t = (n - 3)(5 + 5/11)$ minutes, where $n = 4, 5, 6, 7$, and 8.

And next, rewriting B, we get
$t = n \times 30/5.5 + (16 + 4/11) = 30n/5.5 + 90/5.5$
$= (n + 3)30/5.5 = (n + 3)(5 + 5/11)$. So we get
$t = (n + 3)(5 + 5/11)$, where $n = 4, 5, 6, 7$, and 8.

Note that if the current time is 08:00:00 (or 20:00:00), the target angle 90^o happens for the second time at 9 o'clock or 21 o'clock. So we don't need to use B. If however, using B, we still get $t = 60$, and thus, can still use B.

Thus, in sum, it takes

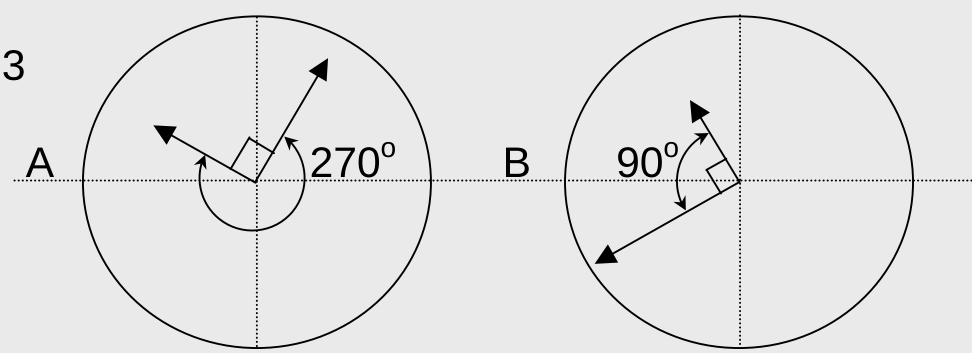
$t = (n - 3)(5 + 5/11)$ minutes, where $n = 4, 5, 6, 7$, and 8, for the target angle 90^o to happen for the first time, and it takes

$t = (n + 3)(5 + 5/11)$ minutes, where $n = 4, 5, 6, 7$, and 8, for the target angle 90^o to happen for the second time.

Now, what if the current times are 9, 10, and 11 o'clock?

The hands make the target angle of 90^o twice if the current time is 9, 10, or 11 o'clock, and the target angle 90^o happens in two cases A and B as shown in Fig. 3 below.

Fig. 3



In the case A, when the hands make 270^o, the target angle 90^o happens the first time as shown in Fig. 3 A.

And in the case B, when the hands make 90^o, the target angle 90^o happens the second time as shown in Fig. 3 B.

Thus, we'll use two equations A and B as follows.
The equation A is for the time for the target angle 90^o to happen for the first time,
and the equation B is for the second time.

In the case A, the hour hand is n x 30^o ahead of the minute hand at the current time, where n = 9, 10, and 11, and H is bigger than M when the target angle 270^o happens.

So the equation A is H – M = 270, where M = 6t,
and H = n x 30 + 0.5t, where n = 9, 10, and 11.

In the case B, the hour hand is also, n x 30^o ahead of the minute hand at the current time, where n = 9, 10, and 11, and H is bigger than M when the target angle 90^o happens.

So the equation B is H – M = 90, where M = 6t,
and H = n x 30 + 0.5t, where n = 9, 10, and 11.

And rewriting A, we get

$H – M = 270 \Rightarrow n \times 30 + 0.5t – 6t = 270 \Rightarrow 5.5t = 30(n – 9)$

$\Rightarrow t = 30(n – 9)/5.5 = (n – 9)(5 + 5/11)$,

since $30/5.5 = 5 + 5/11$. So we get

$t = (n – 9)(5 + 5/11)$ minutes, where $n = 9, 10$, and 11.

Next, rewriting B, we get

$H – M = 90 \Rightarrow n \times 30 + 0.5t – 6t = 90 \Rightarrow 5.5t = 30(n – 3)$

$\Rightarrow t = 30(n – 3)/5.5 = (n – 3)(5 + 5/11)$,

since $30/5.5 = 5 + 5/11$. So we get

$t = (n – 3)(5 + 5/11)$ minutes, where $n = 9, 10$, and 11.

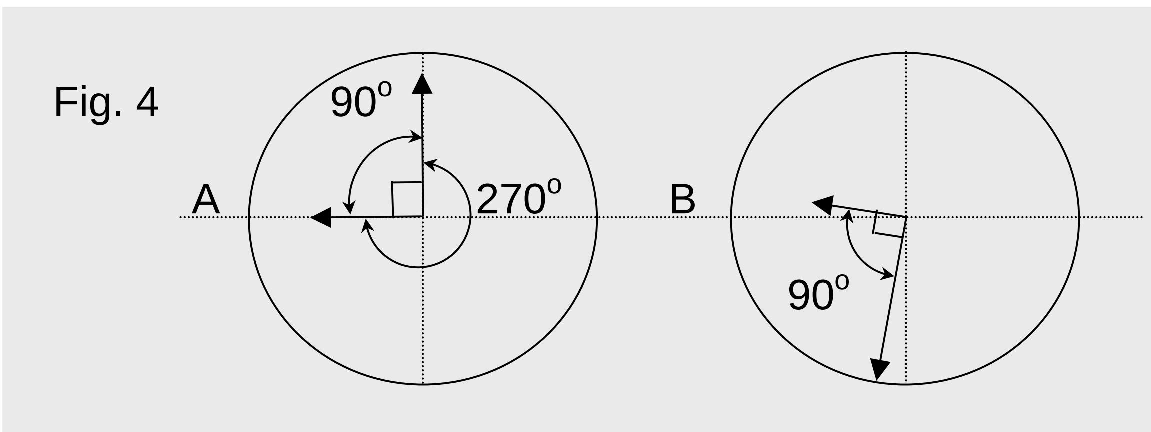
So in sum, it takes

$t = (n – 9)(5 + 5/11)$ minutes, where $n = 9, 10$, and 11, for the target angle 90^o to happen for the first time, and it takes

$t = (n – 3)(5 + 5/11)$ minutes, where $n = 9, 10$, and 11, for the target angle 90^o to happen for the second time.

Note that n is the current time, which is one of 9, 10, and11 o'clock. So for instance if n = 9, the current time is 9 o'clock.

At 9 o'clock or 21 o'clock, the angle between the hands is already 90^o. And the target angle 90^o happens one more time before 10 o'clock as shown in Fig. 4 B below.



So if n = 9, using the equation A, we get t = (n – 9)(5 + 5/11) = 0 minute, so it takes literally no time for the target angle 90^o to happen the first time.

And also, if n = 9, using the equation B, we get

t = (n – 3)(5 + 5/11) = 6(5 + 5/11) = 30 + 30/11 = 32 + 8/11 minutes, so the target angle 90^o happens the second time at 9 o'clock 32 minutes 43 + 7/11 seconds, 09 : 32 : 43 + 7/11, since 8/11 minute = 43 + 7/11 seconds.

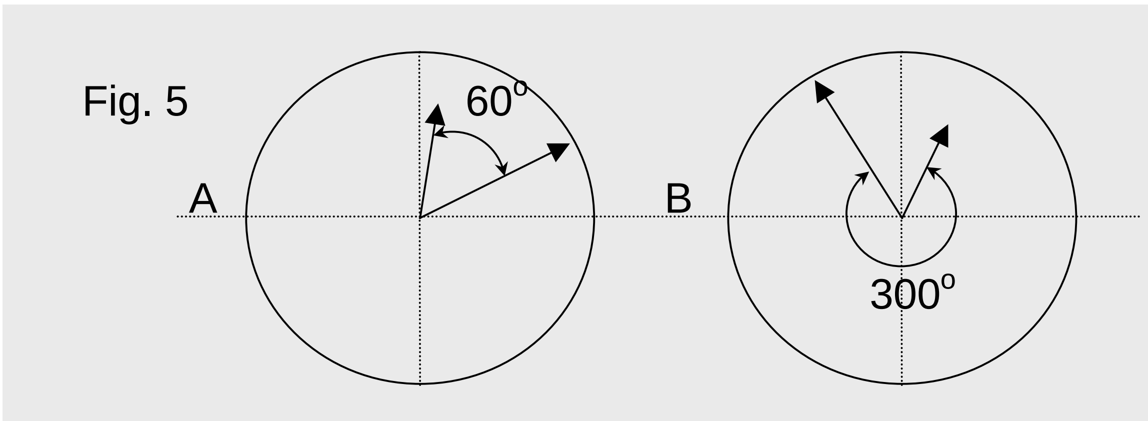
Suggestions or Solutions
To The Problem 1.1 In Examples 8

1.1. Find the first moment of time when the hands make 60^o if the present time is each of every hour on the hour. So we want to set the present time to 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00, and find the first moment of time.

This problem is no other than the previous problem.
So suppose again, H is the angle made by the hour hand, M is the angle made by the minute hand, d is the angle between the two hands, and t is the time taken.

Then, if the current time is 00:00:00, we get $H = ht$, and $M = mt$, where h is the speed of the hour hand, and is 0.5, and m is the speed of the minute hand, and is 6. And M is bigger than H when 60^o happens, so we get $d = M – H = 60$.
Thus, we get $d = 6t – 0.5t = 60 \Rightarrow 5.5t = 60$
And $t = 60/5.5 = 2 \times 30/5.5 = 2(5 + 5/11) = 10 + 10/11$.
So we get $t = 10 + 10/11$ minutes. Thus, the target angle happens at 0 o’clock 10 + 10/11 minutes.

Also, the target angle can happen one more time before 1 o'clock.



Showing the clock when the target angle happens, we can put it the way in Fig. 5 above. Though the drawings are not so accurate, it can show us another possible case where the target angle can happen before 01:00:00, and the case is B. So we have two cases as follows.

A: the time it takes for the target angle 60^o to happen
B: the time it takes for 300^o to happen, where 300^o is the difference between 360^o and the target angle 60^o.

So we've done the case A already, and now, doing the case B, we can reset the target angle to 300^o, and find the time it takes for the new target angle 300^o to happen.
In the case B, M is bigger than H. So we use M – H = d.

Thus, we get $d = 6t – 0.5t = 5.5t = 300$

$\Rightarrow t = 300/5.5 = 10 \times 30/5.5 = 10(5 + 5/11) = 50 + 50/11$

$= 54 + 6/11$ minutes, which should be 5 times the time it takes for 60^o to happen in the case A.

Why?

We know $300^o = 60^o \times 5$, and the hands move at their own constant speeds.

So we can expect $54 + 6/11 = (10 + 10/11) \times 5$.

We get $(10 + 10/11) \times 5 = 50 + 50/11 = 54 + 6/11$.

Thus, the hands make 60^o at both 0 o'clock $10 + 10/11$ minutes and 0 o'clock $54 + 6/11$ minutes.

- The next is the current time of 01:00:00.

At 01:00:00, the hour hand is at the 1 o'clock marker, and the minute hand is at the 12 o'clock marker. So the hour hand is 30^o ahead of the minute hand at the beginning. Thus, $h = 30 + 0.5t$, but M is bigger than H when the target angle 60^o happens.

So the calculations go the way as follows.

$M - H = d \Rightarrow 6t - (30 + 0.5t) = 60$

$\Rightarrow 6t - 30 - 0.5t = 5.5t - 30 = 60 \Rightarrow 5.5t = 60 + 30.$

$\Rightarrow t = 3 \times 30/5.5 = 3(5 + 5/11) = 16 + 4/11.$

Thus, 60^o happens the first time after 16 + 4/11 minutes from 1 o'clock.

What then about the time it takes for the target angle to happen the second time before 2 o'clock?

From the beginning, we've already noticed that it's nothing but the case where the new target angle is 300^o.

We know M is bigger than H when 300^o happens. So finding the time for the new target angle 300^o to happen, we get

$M - H = 300 \Rightarrow 6t - (30 + 0.5t) = 300 \Rightarrow 5.5t = 300 + 30$

$\Rightarrow t = 330/5.5 = 66/1.1 = 60.$

Thus, the hands make 60^o once only before 2 o'clock, and 60^o gets made at 1 o'clock 16 + 4/11 minutes. And it takes 60 minutes for 60^o to happen from 1 o'clock.

At 2 o'clock, the angle between the hands is 60^o.

• Now, let's move on to the current time of 02:00:00.

At 2 o'clock, the angle between the hands is 60^o. So let's now get the time for the target angle to happen for the second time.

At 02:00:00, the hour hand is at the 2 o'clock marker, and the minute hand is at the 12 o'clock marker. So the hour hand is $2 \times 30^o = 60^o$ ahead of the minute hand at the beginning. And M is bigger than H when the target angle 60^o happens the second time.

Thus, the calculations go the way as follows.

$d = M – H \Rightarrow d = 6t – (30 \times 2 + 0.5t) = 60$

$\Rightarrow 6t – 30 \times 2 – 0.5t = 5.5t – 30 \times 2 = 60$

$\Rightarrow 5.5t = 60 + 30 \times 2.$ So we get

$t = 4 \times 30/5.5 = 4(5 + 5/11) = 20 + 20/11 = 21 + 9/11.$

Thus, the target angle 60^o happens for the second time at 2 o'clock and 21 + 9/11 minutes.

• Let's now move on to the current time of 03:00:00.

In this case, the hour hand is at the 3 o'clock marker, so the difference d is bigger than the target angle.
The difference d is $3 \times 30^o = 90^o >$ the target angle $= 60^o$.

Thus, the minute hand doesn't have to overtake the hour hand. In this case, $H = 3 \times 30 + 0.5t$, and H stays greater than M. So the equation is $H – M = d$. Thus, we get

$(3 \times 30 + 0.5t) – 6t = 60 \Rightarrow (90 + 0.5t) – 6t = 60$

$\Rightarrow 5.5t = 30 \Rightarrow t = 30/5.5 = 5 + 5/11$ minutes.

So the hands make 60^o at 5 + 5/11 minutes past 3 o'clock for the first time.
Let's now find the second moment of time when 60^o happens.
We can find the second moment of time using $M – H = d$.
It's because M is bigger than H when the target angle happens.
Finding thus, the time for target angle to happen, we get

$M – H = d \Rightarrow 6t – (3 \times 30 + 0.5t) = 60 \Rightarrow 5.5t = 60 + 3 \times 30$

$\Rightarrow t = (60 + 3 \times 30)/5.5 \Rightarrow t = 60/5.5 + 3 \times 30/5.5$

$= 10 + 10/11 + 16 + 4/11 = 27 + 3/11$ minutes.

Thus, the hands make 60^o at both 5 + 5/11 minutes past 3 o’clock and 27 + 3/11 minutes past 3 o’clock.

Now, we can see a pattern, which is saying that we can apply the same method to a current time of each hour from 3 o’clock until 9 o’clock.

So the target angle can happen at two moments of time, one is before the minute hand overtakes the hour hand, and the other is after the minute hand overtakes the hour hand. Thus, we’ll use two equations A and B.
The equation A is for the time for the target angle 60^o to happen for the first time,
and the equation B includes the time until the overlap, and thus, is for the second moment of time.

So the equation A is H – M = 60, where M = 6t,
and H = n x 30 + 0.5t, where n = 3, 4, …, and 9.

The equation B is t = n x 30/5.5 + (10 + 10/11),
where n = 3, 4, …, and 9.

And rewriting A, we get

$H - M = 60 \Rightarrow n \times 30 + 0.5t - 6t = 60 \Rightarrow 5.5t = 30(n - 2)$

$\Rightarrow t = 30(n - 2)/5.5 = (n - 2)(5 + 5/11)$,

since $30/5.5 = 5 + 5/11$. So we get

$t = (n - 2)(5 + 5/11)$ minutes, where $n = 3, 4, \ldots$, and 9.

And next, rewriting B, we get

$t = n \times 30/5.5 + (10 + 10/11) = n \times 30/5.5 + 60/5.5$

$= (n + 2)30/5.5 = (n + 2)(5 + 5/11)$. So we get

$t = (n + 2)(5 + 5/11)$, where $n = 3, 4, \ldots$, and 9.

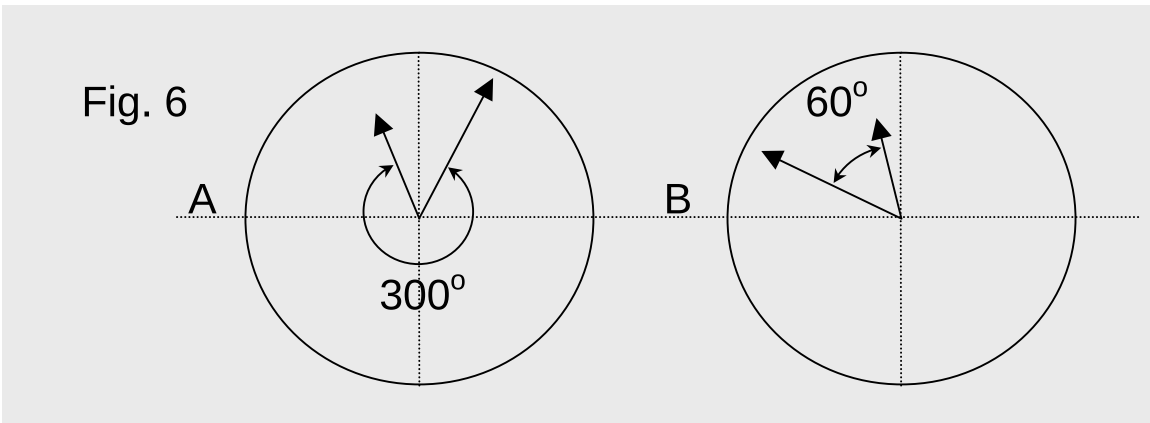
Note that if the current time is 09:00:00 (or 21:00:00), the target angle 60^o happens for the second time at 10 o'clock or 22 o'clock. So we don't need to use B. If however, using B, we still get $t = 60$, and thus, can still use B.

Thus, in sum, it takes

$t = (n - 2)(5 + 5/11)$ minutes, where $n = 3, 4, \ldots$, and 9, for the target angle 60^o to happen for the first time, and it takes

$t = (n + 2)(5 + 5/11)$ minutes, where $n = 3, 4, \ldots$, and 9, for the target angle 60^o to happen for the second time.

Now, what if the current times are 10 and 11 o'clock?



The hands make the target angle of 60^o twice if the current time is 10 or 11 o'clock, and the target angle 60^o happens in two cases A and B as shown in Fig. 6 above.

In the case A, when the hands make 300^o, the target angle 60^o happens the first time as shown in Fig. 6 A.
And in the case B, when the hands make 60^o, the target angle 60^o happens the second time as shown in Fig. 6 B.

Thus, we'll use two equations A and B as follows.
The equation A is for the time for the target angle 60^o to happen for the first time,
and the equation B is for the second time.

In the case A, the hour hand is n x 30^o ahead of the minute hand at the current time, where n = 10 and 11, and H is bigger than M when the target angle 300^o happens.

So the equation A is H – M = 300, where M = 6t,
and H = n x 30 + 0.5t, where n = 10 and 11.

In the case B, the hour hand is also, n x 30^o ahead of the minute hand at the current time, where n = 10 and 11, and H is bigger than M when the target angle 60^o happens.

So the equation B is H – M = 60, where M = 6t,
and H = n x 30 + 0.5t, where n = 10 and 11.

And rewriting A, we get

$H - M = 300 \Rightarrow n \times 30 + 0.5t - 6t = 300 \Rightarrow 5.5t = 30(n - 10)$
$\Rightarrow t = 30(n - 10)/5.5 = (n - 10)(5 + 5/11)$,
since 30/5.5 = 5 + 5/11. So we get
t = (n – 10)(5 + 5/11) minutes, where n = 10 and 11.

Next, rewriting B, we get

$H - M = 60 \Rightarrow n \times 30 + 0.5t - 6t = 60 \Rightarrow 5.5t = 30(n - 2)$

$\Rightarrow t = 30(n - 2)/5.5 = (n - 2)(5 + 5/11)$,

since $30/5.5 = 5 + 5/11$. So we get

$t = (n - 2)(5 + 5/11)$ minutes, where $n = 10$ and 11.

So in sum, it takes

$t = (n - 10)(5 + 5/11)$ minutes, where $n = 10$ and 11, for the target angle 60^o to happen for the first time, and it takes

$t = (n - 2)(5 + 5/11)$ minutes, where $n = 10$ and 11, for the target angle 60^o to happen for the second time.

Note that n is the current time, which is one of 10 and11 o'clock.

So if $n = 10$, the current time is 10 o'clock.

And if $n = 10$, using the equation A, we get

$t = (n - 10)(5 + 5/11) = 0$ minute, so it takes 0 minute for the target angle 60^o to happen the first time. And in fact, at 10 o'clock, the angle between the hands is already 60^o.

And also, if n = 10, using the equation B, we get

t = (n – 2)(5 + 5/11) = 8(5 + 5/11) = 40 + 40/11 = 43 + 7/11 minutes, so the target angle 60^o happens the second time at 10 o'clock 43 minutes 38 + 2/11 seconds, 10: 43 : 38 + 2/11, since 7/11 minute = 38 + 2/11 seconds.

Next, if n = 11, the current time is 11 o'clock.

So if n = 11, using the equation A, we get

t = (n – 10)(5 + 5/11) = 5 + 5/11 minutes, so it takes 5 + 5/11 minutes for the target angle 60^o to happen the first time.

And also, if n = 11, using the equation B, we get

t = (n – 2)(5 + 5/11) = 9(5 + 5/11) = 45 + 45/11 = 49 + 1/11 minutes, so the target angle 60^o happens the second time at 11 o'clock 49 minutes 5 + 5/11 seconds, 11 : 49 : 5 + 5/11, since 1/11 minute = 5 + 5/11 seconds.

Suggestions or Solutions
To The Problem 1.2 In Examples 8

1.2. Find the first moment of time when the hands make 45^o if the present time is each of every hour on the hour. So we want to set the present time to 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00, and find the first moment of time.

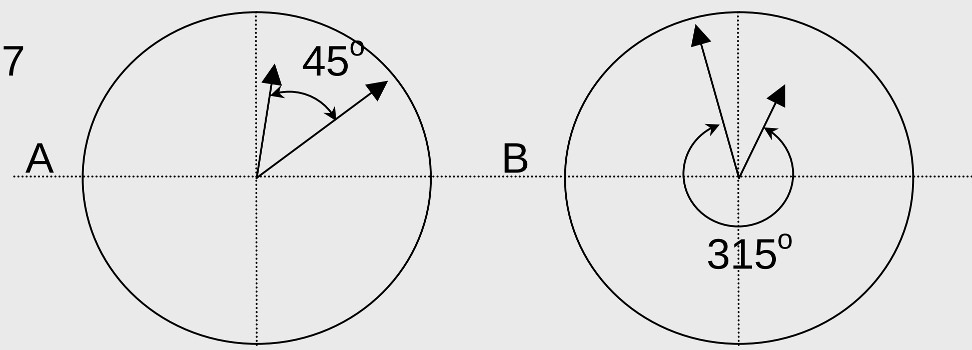
This problem is no other than the previous problem.
So suppose again, H is the angle made by the hour hand, M is the angle made by the minute hand, d is the angle between the two hands, and t is the time taken.

Then, if the current time is 00:00:00, we get H = ht, and M = mt, where h is the speed of the hour hand, and is 0.5, and m is the speed of the minute hand, and is 6. And M is bigger than H when 60^o happens, so we get d = M – H = 60.

Thus, we get $d = 6t - 0.5t = 45 \Rightarrow 5.5t = 45$
$\Rightarrow t = 45/5.5 = (1/2)90/5.5 = (1/2)(16 + 4/11) = 8 + 2/11$.
So we get t = 8 + 2/11 minutes. Thus, the target angle 60^O happens at 0 o'clock 8 + 2/11 minutes.

Also, the target angle can happen one more time before 1 o'clock.

Fig. 7



In Fig. 7 above. the drawings can show us another possible case where the target angle can happen before 01:00:00, and the case is B. So we have two cases as follows.

A: the time it takes for the target angle 45° to happen
B: the time it takes for 315° to happen, where 315° is the difference between 360° and the target angle 45°.

So the case A's been done already, and now, doing the case B, we reset the target angle to 315°, and find the time it takes for the new target angle 315° to happen.

In this case, M is bigger than H. So we use M – H = d.

Thus, we get d = 6t – 0.5t = 5.5t = 315

$\Rightarrow$ t = 315/5.5 = =(270 + 45)/5.5 = (3 x 90 + 90/2)/5.5

= 3.5 x 90/5.5 = 3.5(16 + 4/11) = (7/2)(16 + 4/11)

= 7(8 + 2/11) = 56 + 14/11 = 57 + 3/11 minutes, which should be 7 times the time it takes for 45^o to happen in the case A.   Why?

We know $315^o = 45^o$ x 7, and the hands move at their own constant speeds.
So we can expect 54 + 6/11 = (8 + 2/11) x 7.

We get (8 + 2/11) x 7 = 56 + 14/11 = 57 + 3/11.

Thus, the hands make 45^o at both 0 o'clock 8 + 2/11 minutes and 0 o'clock 57 + 3/11 minutes.

• Let's now move on to the current time of 1 o'clock.

At 01:00:00, the hour hand is at the 1 o'clock marker, and the minute hand is at the 12 o'clock marker.

So the hour hand is 30^o ahead of the minute hand at the beginning. Thus, h = 30 + 0.5t, but M is bigger than H when the target angle 45^o happens. So the calculations go the way as follows.

$M – H = d = 45 \Rightarrow 6t – (30 + 0.5t) = 5.5t – 30 = 90/2$

$\Rightarrow t = (90/2 + 30)/5.5 = (3/2) \times 30/5.5 + 30/5.5$

$= (5/2) \times 30/5.5 = (5/2)(5 + 5/11) = (5/2)(55 + 5)/11$

$= (5/2) \times 60/11 = (5/2)(2 \times 30)/11 = 150/11 = (110 + 40)/11$

$= 10 + (33 + 7)/11 = 13 + 7/11$

Thus, 45^o happens the first time after 13 + 7/11 minutes from 1 o'clock.
What then about the second time before 2 o'clock?

There is no second time before 2 o'clock. If it did happen, the moment of time would be at around 2 o'clock, but the hour hand is almost at the 2 o'clock marker, so the angle between the hands is still noticeably bigger than 45^o, the target angle. So the second time would be after 2 o'clock. We may want to however, try getting the time and see if it is the case.

If it happens, it’s the case B as shown in Fig. 7 above, where the new target angle is 315^o.

We know M is bigger than H when 315^o happens. So finding the time for the new target angle 315^o to happen, we get

$M – H = 315 \Rightarrow 6t – (30 + 0.5t) = 315 \Rightarrow 5.5t = 315 + 30$

$\Rightarrow t = (315 + 30)/5.5 = 345/5.5 = 69/1.1 = 690/11$

$= 62 + 8/11$ minutes.

So the target angle 45^o happens 2 o’clock 2 + 8/11 minutes.

Thus, the target angle cannot happen before 2 o’clock.

Once the minute hand overtakes the hour hand, the hands cannot make the target angle again, because the hour hand is too far away from the 12 o’clock marker as the minute hand approaches the 12 o’clock marker.

• Let’s now move on to the current time of 02:00:00.

At 2 o’clock, the hour hand is at the 2 o’clock marker, so the difference d is bigger than the target angle.

The difference d is $2 \times 30^o = 60^o >$ the target angle $= 45^o$.

Thus, the minute hand doesn't have to overtake the hour hand. In this case, $H = 2 \times 30 + 0.5t$, and H stays greater than M. So the equation is $H - M = d$. Thus, we get

$(2 \times 30 + 0.5t) - 6t = 45 \Rightarrow (60 + 0.5t) - 6t = 45$

$\Rightarrow 5.5t = 15 \Rightarrow t = 15/5.5 = 3/1.1 = 30/11 = 2 + 8/11$ minutes.

So the hands make 45^o at 2 + 8/11 minutes past 2 o'clock for the first time.

Let's now find the second moment of time when 45^o happens.

We can find the second moment of time using $M - H = d$.

It's because the target angle happens after the minute hand overtakes the hour hand.

Finding thus, the time for target angle to happen, we get

$M - H = d \Rightarrow 6t - (2 \times 30 + 0.5t) = 45 \Rightarrow 5.5t = 45 + 2 \times 30$

$\Rightarrow t = (45 + 2 \times 30)/5.5 = 105/5.5 \Rightarrow t = 21/1.1 = 210/11$

$= 19 + 1/11$ minutes.

Thus, the hands make 45^o at both 2 + 8/11 minutes past 2 o'clock and 19 + 1/11 minutes past 2 o'clock.

Now, we can see a pattern, which is saying that we can apply the same method to the cases where the current time is each hour on the hour from 2 o’clock until 9 o’clock.

So the target angle can happen twice, the first one is before the overlap, and the other is after the overlap.
Thus, we’ll use two equations A and B.
The equation A is for the time for the target angle 45^o to happen for the first time,
and the equation B includes the time until the overlap, and thus, is for the second moment of time.

So the equation A is H – M = d = 45, where M = 6t,
and H = n x 30 + 0.5t, where n = 2, 3, …, and 9.

The equation B is M – H = d = 45, where M = 6t,
and H = n x 30 + 0.5t, where n = 2, 3, …, and 9.

And rewriting A, we get
$H - M = 45 \Rightarrow n \times 30 + 0.5t - 6t = 45 \Rightarrow 5.5t = 30n - 45$
$\Rightarrow 11t = 60n - 90.$

So we get $t = 30(2n - 3)/11$, where $n = 2, 3, \ldots$, and 9.

And next, rewriting B, we get

$M - H = 45 \Rightarrow 6t - n \times 30 - 0.5t = 45 \Rightarrow 5.5t = 30n + 45$

$\Rightarrow 11t = 60n + 90$

So we get $t = 30(2n + 3)/11$, where $n = 2, 3, \ldots$, and 9.

Thus, in sum, it takes

$t = 30(2n - 3)/11$ minutes, where $n = 2, 3, \ldots$, and 9, for the target angle 45^o to happen for the first time, and it takes

$t = 30(2n + 3)/11$ minutes, where $n = 2, 3, \ldots$, and 9, for the target angle 45^o to happen for the second time.

So for instance, if $n = 2$, the current time is 2 o'clock, and we get $t = 30(2n - 3)/11 = 30(4 - 3)/11 = 30/11 = 2 + 8/11$, and we get $t = 30(2n + 3)/11 = 30(4 + 3)/11 = 210/11 = 19 + 1/11$.

Thus, the target angle 45^o happens the first time at 2 o'clock 2 + 8/11 minutes, and it happens the second time at 2 o'clock 19 + 1/11 minutes.

Now, what if the current times are 10 and 11 o'clock?

Then, the target angle happens only once before the next hour on the hour, and it happens before the overlap. That's because after the minute hand overtakes the hour hand, the minute hand cannot be 45^o away from the hour hand before the next hour.

Thus, we can use $t = 30(2n – 3)/11$ for $n = 10$ and 11.
So if the current time is 10:00:00, then $n = 10$, and we get

$t = 30(2 \times 10 – 3)/11 = 30 \times 17/11 = 30(1 + 6/11)$
$= 30 + 6 \times 30/11 = 30 + 6(2 + 8/11) = 42 + 48/11$
$= 46 + 4/11$ minutes.

And if the current time is 11:00:00, then $n = 11$, and we get

$t = 30(2 \times 11 – 3)/11 = 30 \times 19/11 = 30(1 + 8/11)$
$= 30 + 8 \times 30/11 = 30 + 8(2 + 8/11) = 46 + 64/11$
$= 46 + (55 + 9)/11 = 51 + 9/11$ minutes.

Suggestions or Solutions
To The Problem 1.3 In Examples 8

1.3. Find the first moment of time when the hands make 145^o if the present time is each of every hour on the hour. So we want to set the present time to 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00, and find the first moment of time.

The time it takes for a target angle to happen depends on the size of the target angle and the current time.
And the current time determines the initial positions of the hour hand and the minute hand.
Those are what matters in these kind of problems.
Based on those data, we redefine the problem.
Once the problem’s been redefined, we can get the solution.
How do we get the solution then?

We need measurements, so we get the measurements.
We need angles and their magnitudes, where and how the hands are positioned, how the hands move, what the target angle is made of and how it gets made, and such and such.

How then do we get the measurements?

The point of reference matters.
The point of reference is where the measurement begins.
Usually, we set the point of reference to 0 in a number line, so we often call it the origin, and thus, denoted by $\boldsymbol{O}$, an italicized boldface English alphabet capital O.
In these problems, we set it to the 12 o'clock marker.
And we take the measurements from that point.
So we take the measurements of the angles made by the hour hand and the minute hand. From the point of reference, of course. What then about the target angle?

The target angle is the difference between the angles made by those two hands. So the point of reference still matters. Doing math, we've gotta know what to measure, how to measure, and most importantly, what the point of reference is and where it is. And we'd better name what we are working with first. Don't say this and that. Name it first. Name it simple, and use it the way it works.

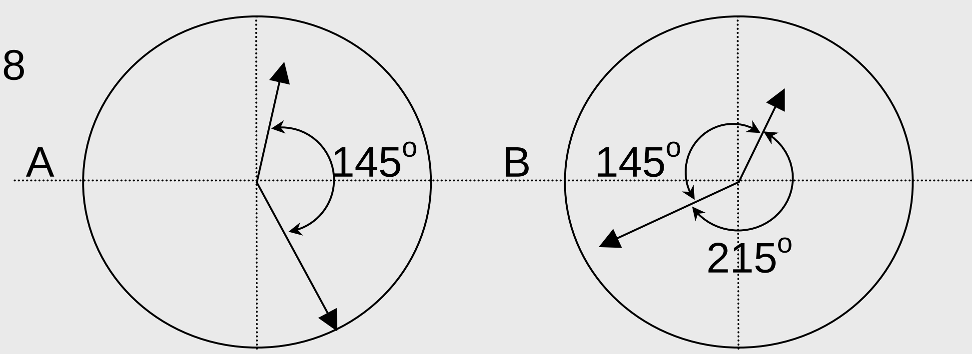
So now, we want to get the time for the target angle 145^o to happen.

Thus again, suppose H is the angle made by the hour hand, M is the angle made by the minute hand, d is the angle between the two hands, and t is the time taken.

Then, if the current time is 00:00:00, we get H = ht, and M = mt, where h is the speed of the hour hand, and is 0.5, and m is the speed of the minute hand, and is 6.

So let's now, put the problem in a figure so that we can see better what they are, what they are doing, and how they are doing it.

Fig. 8



We can see in the case A, that M is bigger than H when the target angle 145^o happens, so we get d = M – H = 145.

And if the current time is n o'clock, the hour hand is n x 30^o ahead of the minute hand, so we can set $H = 30n + 0.5t$. Also, at every hour on the hour, the minute hand is at the 12 o'clock marker, the point of reference, so we can set $M = 6t$. Thus, finding the time it takes for the target angle to happen, we get

$M – H = 145 \Rightarrow 6t – (30n + 0.5t) = 5.5t – 30n = 145$

$\Rightarrow t = (145 + 30n)/5.5 = (90 + 55 + 30n)/5.5$

$= (n + 3)30/5.5 + 55/5.5 = (n + 3)(5 + 5/11) + 10$,

where n = 0, 1, … ?   What then is the question mark for?

If n = 7, that is, the current time is 7 o'clock, we get t > 60, so ? should be 6.

So checking to see if 6 works, we get

$t = (6 + 3)(5 + 5/11) + 10 = 45 + 45/11 + 10 = 59 + 1/11$.

Thus, we get $t = (n + 3)(5 + 5/11) + 10$, where t = 0, 1, …, 6.

Next, in the case B, we can see that M is still bigger than H when the target angle 145^o happens, and that d is not 145 but 215^o, so we get $d = M – H = 215$.

Thus, finding the time it takes for the target angle to happen, we get

$M - H = 360 - 145 \Rightarrow 6t - (30n + 0.5t)$

$= 5.5t - 30n = 360 - 145 \Rightarrow t = (4 \times 90 - 90 - 55 + 30n)/5.5$

$= 3 \times 90/5.5 - 55/5.5 + 30n/5.5 = (n + 9)30/5.5 - 55/5.5$

$= (n + 9)(5 + 5/11) - 10$, where $n = 0, 1, 2, \ldots.$?

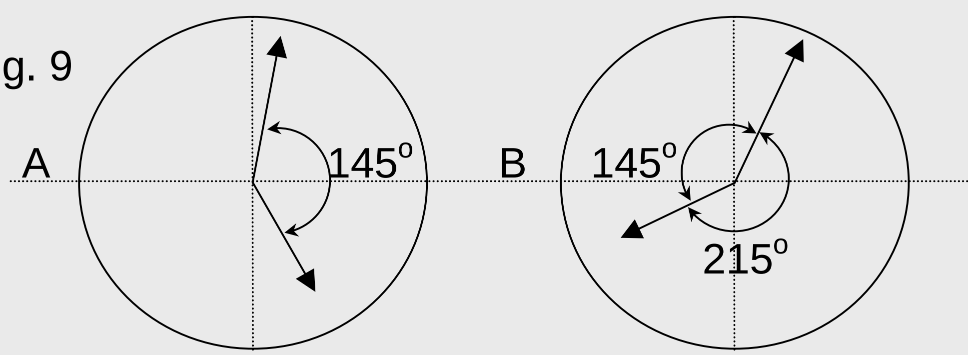
If $n = 4$, we get $t = (4 + 9)(5 + 5/11) - 10 = 65 + 65/11 - 10$

$= 65 + 5 + 10/11 - 10 > 60$.

So the target angle cannot happen before 5 o'clock. Thus, we get $t = (n + 9)(5 + 5/11) - 10$, where $n = 0, 1, 2$, and 3.

What if now, the situation in the clock is reversed the way shown in Fig. 9 below?

Fig. 9



The positions of the hands are exchanged.

We can see in the case A, that H is bigger than A when the target angle 145^o happens, so we get d = H – M = 145.
And if the current time is n o'clock, the hour hand is n x 30^o ahead of the minute hand, so we can set H = 30n + 0.5t.
Also, at every hour on the hour, the minute hand is at the 12 o'clock marker, the point of reference, so we can set M = 6t.
Thus, finding the time it takes for the target angle to happen, we get

$H - M = d \Rightarrow (30n + 0.5t) - 6t = 145 \Rightarrow 5.5t = 30n - 145$

$\Rightarrow t = (30n - 145)/5.5 \Rightarrow t = (30n - 90 - 55)/5.5$

$= (n - 3)30/5.5 - 55/5.5 = (n - 3)(5 + 5/11) - 10$,

where n = 5, 6, …, 11.

So we get t = (n – 3)(5 + 5/11) – 10, where n = 5, 6, …, 11.

And next, we can see in the case B, that H is bigger than A when the target angle 145^o happens, but the angle between the hands is 215^o, so we get d = H – M = 215.

Thus, finding the time it takes for the target angle to happen, we get

$H - M = 360 - 145 \Rightarrow (30n + 0.5t) - 6t = 360 - 145$

$\Rightarrow 5.5t = 30n - 360 + 145 = 30n - 12 \times 30 + 3 \times 30 + 55$

$= (n - 9)30 + 55 \Rightarrow t = (n - 9)30/5.5 + 55/5.5$

$= (n - 9)(5 + 5/11) + 10$

So we get $t = (n - 9)(5 + 5/11) + 10$,

where $n = 8, 9, 10$, and 11.

In sum, we have

$t = (n + 3)(5 + 5/11) + 10$, where $t = 0, 1, \ldots, 6$.

$t = (n + 9)(5 + 5/11) - 10$, where $n = 0, 1, 2$, and 3.

$t = (n - 3)(5 + 5/11) - 10$, where $n = 5, 6, \ldots, 11$.

$t = (n - 9)(5 + 5/11) + 10$, where $n = 8, 9, 10$, and 11.

Suggestions or Solutions
To The Problem 1.4 In Examples 8

1.4. Find the first moment of time when the hands make 270^o if the present time is each of every hour on the hour.
So we want to set the present time to 00:00:00, 01:00:00, …, 22:00:00, and 23:00:00, and find the first moment of time.

This problem is no other than the problem where we need to find the first moment of time when the hands make 90^o if the present time is each of every hour on the hour.
It's because when the hands make 270^o, they are making 90^o, too, because we have $90 + 270 = 360$, and the angle in a circle is 360^o. Therefore, the solution is the same as the one to the problem with the target angle of 90^o.

Quite often, we need to take the other way around working in math. For instance, subtracting, we can add the negative, and dividing, we can multiply by the reciprocal. We are looking for the ways to get to the solutions to problems.

For instance, getting the sum of all the numbers from 1 to 100, we don't want to do this: 1 + 2 + 3 + … + 100.

We know 1 = 100 = 101, 2 + 99 = 101, …, 50 + 51 = 101.

So how many 101s?

50 of course. What then do we get?

50 x 101 = 5050, or we can do the same this way, too:

We know 1 + 2 + 3 + … + 100 + 1 + 2 + 3 + … + 100

= 2 x (1 + 2 + … + 100)

= 1 + 100 + 2 + 99 + 3 + 98 + … + 99 + 2 + 100 + 1

= (1 + 100) + (2 + 99) + (3 + 98) + …+ (100 + 1)

= 100 x 101

= 10100

So 1 + 2 + … + 100 = 10100/2 = (10000 + 100)/2

= 10000/2 + 100/2 = 5000 + 50 = 5050.

Thus, running math, we can do this in no time:

1 + 2 + … + 1000000000000000000000000000000

It's $(10^{30} + 1)10^{30}/2$.