Examples 9 in Ratios and Rates

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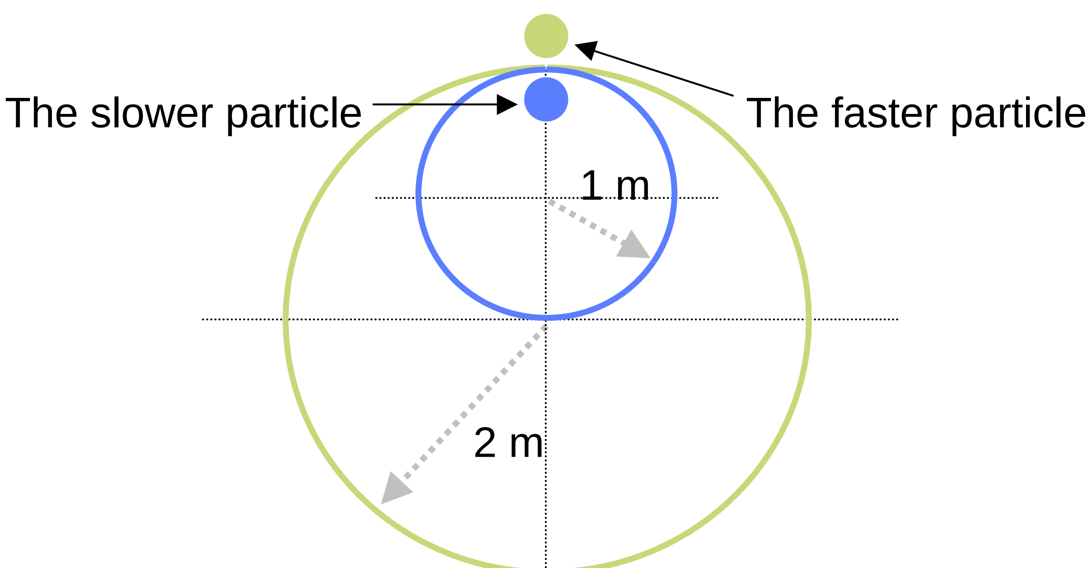
As in Fig. 0 below, one particle is orbiting at 10 m/s along a circular path of the radius 2 m, and the other particle is at 8 m/s along the other path with 1 m in radius. Both move in the same direction. The smaller path is inscribing the larger, so both paths meet at one point. Suppose now both just have started moving at the point where the paths meet. Then, see if the two particles will meet.

Fig. 0



If they are going to meet, explain how they get to meet. Otherwise, how are they not going to meet?

We want to find how the particles meet if they will.
So we are now going to come up with an equation or equations that can give us the answer. To do so, we need to ask some questions ourselves.
What question then do we need to begin with?

We want to see where the particles meet and when they meet. The particles arrive at the same place at the same time if they meet. Where then has to be the same spot?

The meeting will happen at the point where the orbits meet. They begin moving at the same time, and will meet at the same time, of course. So they will travel the same amount of time until they meet.

Then, we want to know the amount of time the particles will travel (or spend) until they meet. The time is unknown.
What then do we want to do first?

We want to name the unknown first. So let’s call it t. And we may want to name the particles, too. So let’s call the slower particle S and the faster one F.

Then, t is the amount of time S and F will travel until they meet. What then is the next question?

We may want to know the amount of distance each particle covers until the meeting happens.
What then is the distance covered?

S and F each will make a lap or laps along their orbits for the time t. The number of laps determines the distance covered by each. So we want to know the number of laps each makes for the time t.

Thus, we may want to name the number of laps.
So let s be the number of laps S makes, and f be the number of laps F makes.
Then, S makes s laps in t, and F makes f laps in t.
What else then do we have in the problem definition?

The speed of F is 10 m/s, and the speed of S is 8 m/s.
The radius of the orbit F travels along is 2 m, and the radius of the orbit S travels along is 1 m. So?

We can again come up with the distance each covers.

We know the distance = the speed x the time spent.

So we want to use some data (info) as follows.

The numbers of the laps: s for S and f for F.

The speeds: 8 m/s for S and 10 m/s for F.

The amount of time until they meet: t for S and F both.

The radii of the orbits: 1 m for S and 2 m for F.

Now, using the data above, we can express the distance that each of S and F covers in t the way as follows.

The distance S covers = the speed of S x the time spent
= 8 x t = 8t, and is also,
= s x m, where s is the number of laps S makes, and m is the length of the track S is on. So we get sm = 8t.

The distance F covers = the speed of F x the time spent
= 10 x t = 10t, and is also,
= f x M, where f is the number of laps F makes, and M is the length of the track F is on. So we get fM = 10t.

What then about the track lengths?

We have $C = \pi D = \pi \times 2 \times r = 2\pi r$, where C is the length (circumference) of a circle, D is the diameter, π is the circular ratio, and r is the radius of the circle.

And we know the radius for F is 2, the radius for S is 1, m is the track for S, and M is the track for F.
So we get $m = 2\pi \times 2 = 2\pi$, and $M = 2\pi \times 2 = 4\pi$.

And we have sm = 8t and fM = 10t.
So we get
$sm = s \times 2\pi = 2\pi s = 8t$, and $fM = f \times 4\pi = 4\pi f = 10t$

Thus, we get $2\pi s = 8t$ for S, and $4\pi f = 10t$ for F.

Now, we have 2 equations, but the equations have 3 unknowns, which means many solutions. The solution is not unique.

In fact, a solution to an equation does not have to be unique.
An equation may have many particular solutions.
How many then?

It may have several up to infinitely many solutions.
What then are the solutions?

The particles will meet when their particular numbers of the laps are completed in time t.
Then, they will meet again when another set of the numbers of the laps is completed in another time t.
Then, we can see that they meet at every time interval of t, every set of their numbers of the laps is completed.

We know that the two distances, $4\pi f$ and $2\pi s$ have to be covered in the same amount of time t.

Each particle will have to make the same number of laps respectively in the time ‘t’ until the next rendezvous.
What then can we say about the numbers of the laps?

We can talk about the ratio between the numbers of the laps of the two particles.
So we want to find the ratio, f:s, and the ratio is the solution to the set of equations, $2\pi s = 8t$ and $4\pi f = 10t$.

We want to get the ratio between f and s.
So we need to extract f and s, that is, we want to solve the equations for f and s.

We have 10t = 4πf and 8t = 2πs.
So we get f = $\frac{10\boldsymbol{t}}{4\pi} = 5 \cdot \frac{\boldsymbol{t}}{2\pi}$ and t = $\frac{8\boldsymbol{t}}{2\pi} = 4 \cdot \frac{\boldsymbol{t}}{2\pi}$.
Thus, we get f:s = 5:8, since $\frac{\boldsymbol{t}}{2\pi}$ is a common divisor
So in every time interval t in which F, the faster particle makes 5 laps, and S, the slower particle makes 8 laps, they get to meet at the point where the two orbits meet.

Let's now take a look at the amount of time interval t, too.

Setting f = 5, we get10t = 4πf = 20π ⇒ t = 2π sec.
So at every 2π (approximately 6.28) seconds, the particles meet at the point where the two orbits meet.
Therefore, the two particles meet every time the faster particle makes 5 laps, and the other makes 8 laps in every 2π seconds.

Now, we may want to ask ourselves if the particles will meet even if they move at any constant speed or on paths with any lengths or shapes.

Of course, when we take an exam and fill up the answer sheet, we can’t put all the procedures stated above.

However, we need to show the basis on which our calculations have been performed to produce any particular numbers. In fact, we should be able to provide a summary on the procedures that we have taken to solve the problem.

The summary needs to have essential facts that can explain the outcomes of the significant subsequent procedures.
The answer sheet should look like as follows, for instance.

Suppose the particles will meet, one is A, and the other is B. They will meet at the same time at the point where the paths meet.
Then, they will have to make laps around their orbits, and have to travel the same amount of time.

Suppose

A = the faster particle.

B = the slower particle.

t = the time the particles travel until they meet.

u = the number of laps made by A in t.

v = the number of laps made by B in t.

P = the distance covered by A in t.

Q = the distance covered by B in t.

W = the length of the orbit A is on.

S = the length of the orbit B is on.

C = the speed of A = 10 m/s

D = the speed of B = 8 m/s

R = the radius of the larger orbit = 2 m

L = the radius of the smaller orbit = 1 m

Then,

$P = C \times t = 10t$, and $Q = D \times t = 8t$.

$P = W \times u$, and $Q = S \times v$.

$W = 2\pi R = 2\pi \times 2 = 4\pi$, and $S = 2\pi \times 1 = 2\pi$.

So $P = 4\pi u$, and $Q = 2\pi v$.

Thus, $4\pi u = 10t$, and $2\pi v = 8t$.

Now, once the particles make the first rendezvous, they repeat the same until the next one.

Therefore, we take the ratio between the numbers of the laps by A and B, that is, u:v.

We have $4\pi u = 10t$, and $2\pi v = 8t$.

So $u = 10t/4\pi = 5t/2\pi$, and $v = 8t/2\pi$.

Thus, u:v = 5:8.

Therefore, in every time interval t in which A makes 5 laps, and B makes 8 laps, they meet at the point where the two orbits meet.

Suppose now, u = 5. Then, $10t = 4\pi u = 20\pi \Rightarrow t = 2\pi$ sec.

So at every 2π (approximately 6.28) seconds, the particles meet at the point where the two orbits meet.

Therefore, the two particles meet every time the faster particle makes 5 laps, and the other makes 8 laps in every 2π seconds.