Examples C in Ratios and Rates

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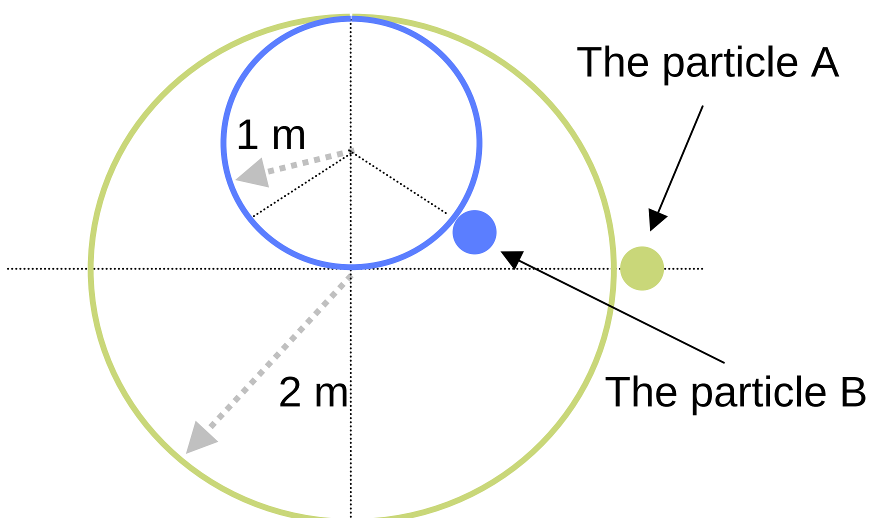
Examples C in Ratios and Rates

As in Fig. 0 below, one particle is orbiting at 10 m/s along a circular path of the radius 2 m, and the other particle is at 8 m/s along the other path with 1 m in radius. They are moving clockwise. The smaller path is inscribing the larger, so both paths meet at one point. Suppose A starts at the point a quarter past to the right of the point where the tracks meet, and B starts at the point a third past to the right of the point where the tracks meet. Then, see if the two particles will meet. If not, what can we do to make the particles meet?

Fig. 0



Let’s take a close look at first what this problem is made of. We may want to begin with making questions as follows.

Where can the particles meet?
When can they meet?
When do they begin orbiting?
Where do they begin moving?
In what direction do they orbit?
Then, the possible answers can be as follows.

The meeting will have to happen at the point where the orbits meet.
They will meet at the same time, of course.
They begin orbiting at the same time.
The faster particle starts at the point a quarter past to the right of the point where the orbits meet, but the other starts at the point a third past to the right of the point where the orbits meet, and of course, starts in its orbit. And both orbit clockwise.
Suppose now that the point where the orbits meet is the target point.

Then, if the particles meet, they both get to the target point at the same time. How then do they get there concurrently?

Keeping track of movements of the particles, we can say when one particle arrives at the target point, if the other is not there, it has to keep moving until it meets the other.
The particles move at their speeds, but meet at the same time if they meet, of course. They spend the same amount of time until the meeting. At different speeds though. Then, the particles cover different distances.
How then do we get the distances?

We can express the distance each particle covers along the orbit until it meets the other.
What then does the distance covered have to do with?

The speed and the time taken.
So for each particle, we can express the distance covered using the speed and the time taken until the meeting.
There is another way to express the distance in this case.
Each particle covers the distance the way as follows, too.

It moves along the track, gets to the target point, and if the other is not there, it continues moving, and gets to the target point again. And it moves along the same path over and over until the meeting.   What then does the particle make?

Laps. And of course, the same is true to the other particle, too. So the particles keep making laps while orbiting.

Thus, we want to find how many laps each particle makes until the rendezvous, and then, we can express the distance. How then can we get the numbers of laps?

First, we need to set up objects representing the number of the laps to be made. So suppose now,
the number of the laps A makes = u,
and the number of the laps B makes = v.

Then, A begins orbiting at the point a quarter past to the right of the target point, so it should make three quarters of a complete lap to get to the target point.

And B begins at the point a third past to the right of the target point, and of course begins in its orbit, so it should make two thirds of a complete lap to get to the target point.

So each particle has a different partial lap. Thus, it looks like we have a little problem with the laps the particles will have to make. What laps then?

We want to find a particular lap that has to satisfy both particles' needs. So we need to come up with a lap common to both particles. In fact, we have been working with a lap common to both particles in all the previous 3 examples. What lap was common to both particles in the example 9?

A complete lap, of course, because both particles start at the target point where the two orbits meet. What lap then was common to both particles in the example A?

A half of a complete lap, because one particle starts at the point opposite of the target point, so it has to make a half lap to get there.

And what’s common between one and a half is a half, since one has two halves, and a half has one half. What lap then was common to both particles in the example B?

A quarter of a complete lap, because one particle starts at the point a quarter past to the right of the target point, so it has to make a three quarters of a complete lap to get there. And what’s common between one and a quarter is a quarter, since one has four quarters, and a quarter has one quarter. What lap then is common to both particles in this example?

A starts at the point a quarter past to the right of the target point, so A is three quarters of a complete lap away form the target point, and B starts at the point a third past to the right of the target point, so B is two thirds of a complete lap away from the target point. What then is common between a quarter and a third?

A twelfth, because a quarter has three twelfths and a third has four twelfths, so a twelfth of a complete lap is common to both particles.

Then, A can get to the point of rendezvous if it makes 9 twelfths of a complete lap, and B can get to the target point if it makes 8 twelfths of a complete lap.

So the lap common to A and B is a twelfth of a complete lap. A complete lap requires 12 twelfths of a complete lap. We can think thus, both particles make twelfth laps until they meet at the target point.

Then, both particles will have to make their particular numbers of laps until they meet for the first time, and each lap is a twelfth of a complete lap.

And the length of the lap is a twelfth of the length (circumference) of the orbit, which is a circle.
So we take a twelfth of the length of the circle for the length of the lap for each particle.
The radius of the track for A is 2 meters, and the radius of the tract for B is 1 meter. So we get these:
The length of the lap for A = $2\pi r/12 = 2\pi \times 2/12 = \pi/3$.
The length of the lap for B = $2\pi r/12 = 2\pi \times 1/12 = \pi/6$.

And we set the speeds the way as follows.
The speed of A = 10 m/s
The speed of B = 8 m/s

And we can express the distance to be covered by each particle until the rendezvous the way as follows.
the speed x the time taken = the length of a lap x the number of laps.

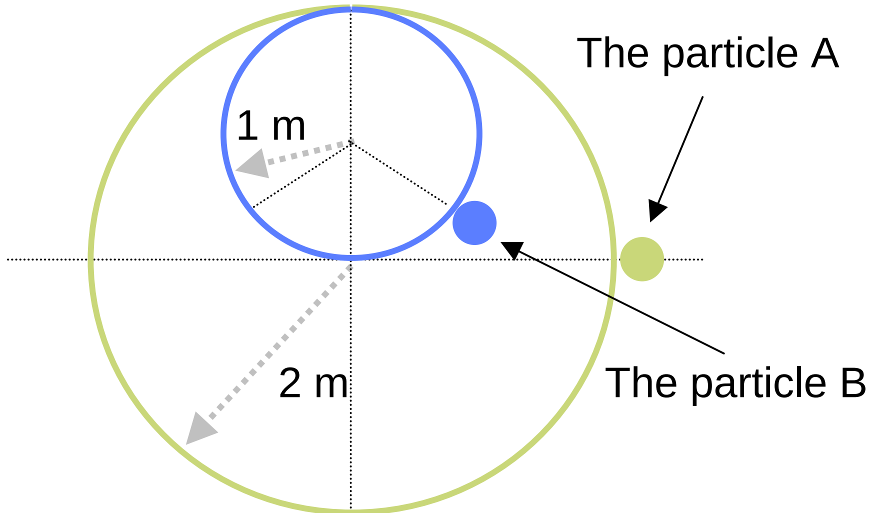
Thus, we get $10t = \pi u/3$ for A, and $8t = \pi v/6$ for B.

So taking the ratio between the numbers of the laps, we get
$10t:8t = (\pi u/3):(\pi v/6) \Rightarrow 5:4 = (u/3):(v/6) = u:(v/2)$
$\Rightarrow 5:4 = u:(v/2) \Rightarrow 5:8 = u:v$.

So the ratio is $u:v = 5:8$.

Now, we need to see if the ratio works.
Let's now begin with finding the number of the laps each particle has to make to get to the target point for the first time from its initial point.

Fig. 0



As shown in Fig. 0 above, initially, A is 3 quarters of the length of its complete lap away from the target point, and B is 2 thirds of the length of its complete lap away from the target point.

And each lap each particle makes is 1/12 of its complete lap. So the number of the laps A will have to make to get to the target point for the first time is as follows.

(3/4) / (1/12) = (3/4) x 12 = 36/4 = 9, in other words,

$\frac{\frac{3}{4}}{\frac{1}{12}} = \frac{3}{4} \text{ x } 12 = \frac{36}{4} = 9$. Thus, A will need to make 9 laps.

And the number of the laps B will have to make to get to the target point for the first time is as follows.
(2/3) / (1/12) = (2/3) x 12 = 24/3 = 8, in other words,
$\frac{\frac{2}{3}}{\frac{1}{12}} = \frac{2}{3}$ x $12 = \frac{24}{3} = 8$. So B will have to make 8 laps.

Thus, it will take 9 laps for A to get to the target point for the first time. And it will take 8 laps for B to get to the target point for the first time.

And a complete lap is 12 times each of the laps above.
So we can see the facts as follows.

Fact 1.
The number of the laps A has to make is 9 + 12n so that A can meet B, where n is the number of complete laps.

Fact 2.
The number of the laps B has to make is 8 + 12m so that B can meet A, where m indicates the number of complete laps.

In accordance with the speeds, the lengths of the orbits, and the initial positions given in the problem, we found that the ratio between u and v is u:v = 5:8.

Thus, we need to find two integers n and m such that (9 + 12n):(8 + 12m) = 5:8. If there are the two, the particles can meet; otherwise, they cannot.

So to begin with, we can set (9 + 12n)/(8 + 12m) = 5/8.

Then, we can get

$3(3 + 4n)/4(2 + 3m) = 5/8 \Rightarrow 3(3 + 4n)/(2 + 3m) = 5/2$

$\Rightarrow 2 \times 3(3 + 4n)/(2 + 3m) = 5 \Rightarrow 6(3 + 4n) = 5(2 + 3m)$

$\Rightarrow 18 + 24n = 10 + 15m \Rightarrow 15m - 24n = 18 - 10 = 8$

$\Rightarrow 3(5m - 8n) = 8.$

We know 5m – 8n is an integer.
So we can see that there are no integers for n and m such that 3(5m – 8n) = 8, since 3 does not divide 8. In other words, 8 is not a multiple of 3.

Thus, there are no integers that m and n can be so that they can satisfy the ratio. So the particles cannot meet.
What then can we do to make them meet?

We have the facts as follows.
Fact 1.
The number of the laps A has to make is $9 + 12n$ so that A can meet B, where n is the number of complete laps.
Fact 2.
The number of the laps B has to make is $8 + 12m$ so that B can meet A, where m indicates the number of complete laps.

So we can use the two facts above. How?

We can make some choices on the numbers of complete laps the particles have to make until the first meeting. So if making the choices, we can set n and m equal to some nonnegative integers. Setting thus, for the simplest instance, $n = 0$ and $m = 0$, we get these:
$u = 9 + 12n = 9$ and $v = 8 + 12m = 8$.

So the ratio is u:v = 9:8, which should work.

Let's now reset the speeds.

Then, suppose first, that the speed of the particle A is p, and that the speed of B is q.

Then, we can use the equation as follows.

The speed of each particle x t = the length of the lap x the number of the laps

Then, we can see that for A, pt = πu/3, where π/3 is the length of its lap and u is the number of its laps.

And for B, qt = πv/6, where π/6 is the length of its lap and v is the number of its laps.

So we get pt = πu/3 $\Rightarrow$ u = 3pt/π, and qt = πv/6 $\Rightarrow$ v = 6qt/π

Thus, we get u/v = p/2q $\Rightarrow$ u:v = p:2q.

And we want the ratio p:2q to be 9:8, since u:v = 9:8.

Then, we get p:2q = 9:8 $\Rightarrow$ 2p:2q = 18:8 = 9:4 = p:q

So the ratio p:q = 9:4.

Thus, setting for instance, the speed of A = 9 m/s, and the speed of B = 4 m/s, we get 2 equations as follows.

$9t = \pi u/3$, and $4t = \pi v/6 \Rightarrow 8t = \pi v/3$

So we get u:v = 9:8 as expected.

Thus, after the particle A makes 9 of its laps, and the particle B makes 8 of its laps, they will meet for the first time.

And checking to see if they really meet, we get these:

9 x 1/12 = 3/4 and 8 x 1/12 = 2/3.

So A covers 3 quarters of a complete lap and B covers two thirds of a complete lap, and thus, they meet.

Let’s now get the amount of time it takes for the first meeting to happen. Then, setting first, u = 9, we get

$9t = \pi u/3 = 9\pi/3 = 3\pi \Rightarrow t = \pi/3$ sec.

And next, setting v = 8, we get

$4t = \pi v/6 = 8\pi/6 \Rightarrow t = \pi/3$ sec, too.

Therefore, after $\pi/3$ (approximately 1.04) seconds, the particles will meet for the first time.

How about setting the speeds the way as follows?

The speed of A = 18 m/s   The speed of B = 8 m/s

So we double the speeds.

Then, we get $18t = \pi u/3$, and $8t = \pi v/6 \Rightarrow 16t = \pi v/3$.

So we get u:v = 18:16 = 9:8 as expected.

Thus, after the particle A makes 9 of its laps, and the particle B makes 8 of its laps, they meet. So the numbers of the laps don't change. What then is the difference?

They get to the target point faster if we double the speeds. Of course, they will be twice as fast.
So it should take half the amount of time for the first meeting to happen.

Let's now get the amount of time it will take for the particles to meet for the first time.
We have $18t = \pi u/3$ for A, and $8t = \pi v/6$ for B.

Setting first therefore, $u = 9$, we get $18t = \pi u/3 = 9\pi/3 = 3\pi$, so we get $t = 3\pi/18 = \pi/6 = (\pi/3) \times (1/2)$, half the time.

And setting next, $v = 8$, we get $8t = \pi v/6 = 8\pi/6 = 4\pi/3$, so we get $t = 4\pi/24 = \pi/6 = (\pi/3) \times (1/2)$, half the time, too.

Thus, they will meet twice as fast, so it will take half the time. Twice the speed, half the time, of course.

Let's now take another example using again Fact 1 and Fact 2 stated above. And the two facts are as follows.

Fact 1.

The number of the laps A has to make is $9 + 12n$ so that A can meet B, where n is the number of complete laps.

Fact 2.

The number of the laps B has to make is $8 + 12m$ so that B can meet A, where m indicates the number of complete laps.

We can make some choices on the numbers of complete laps the particles have to make until the first meeting. So if making the choices, we can set n and m equal to some nonnegative integers. Setting thus, for another instance,

$n = 1$ and $m = 2$, we get these:

$u = 9 + 12n = 9 + 12 = 21$, and $v = 8 + 12m = 8 + 24 = 32$.

So the ratio is $u:v = 21:32$, which should work, too.

Let's now reset the speeds.

Then, assuming again the speed of A is p, and the speed of B is q, we get $u:v = p:2q$, which was found above.

And we want the ratio $p:2q$ to be $21:32$, since $u:v = 21:32$.

Then, we get $p:2q = 21:32 \Rightarrow p:q = 21:16$.

Thus, setting for instance, the speed of A = 21 m/s, and the speed of B = 16 m/s, we get 2 equations as follows.

$21t = \pi u/3$, and $16t = \pi v/6 \Rightarrow 32t = \pi v/3$

So we get $u:v = 21:32$ as expected.

Thus, after A makes 21 of its laps, and B makes 32 of its laps, they will meet for the first time.

And checking to see if they really meet, we get

$21 \times 1/12 = 7/4 = 1 + 3/4$ and $32 \times 1/12 = 8/3 = 2 + 2/3$.

So A makes 1 complete lap and three quarters of a complete lap, and B makes 2 complete laps and two thirds of a complete lap, and thus, they meet.

Let's now get the amount of time it takes for the first meeting to happen. Then, setting first, $u = 21$, we get

$21t = \pi u/3 = 21\pi/3 = 7\pi \Rightarrow t = \pi/3$ sec.

And next, setting $v = 32$, we get

$16t = \pi v/6 = 32\pi/6 \Rightarrow t = \pi/3$ sec, too.

Therefore, after $\pi/3$ (approximately 1.04) seconds, the particles will meet for the first time.

You may want to try more examples taking some different combinations of n and m, and try thinking about some exceptional cases, too. And you may want to reset the lengths of the tracks and see if the particles meet.

Let's now use the numbers of the complete laps.
Then, to begin with, the initial setting is as follows.
The speed of A is 9 m /sec, and the speed of B is 4 m/sec.
A has to make a number of laps and 3 quarters of a lap to get to the target point.
B has to make a number of laps and 2 thirds of a lap to get to the target point.
The radius of the orbit for A is 2 meters, and the radius of the orbit for B is 1 meter.

The number of the complete laps A has to make until the first meeting is u, the number of the complete laps B has to make until the first meeting is v, and of course, the length of the complete lap is the circumference of the orbit.
So A makes (u + 3/4) of its laps, and B makes (v + 2/3) of its laps to get to the target point at the same time.

Then, we get $9t = 2\pi \times 2 \times (u + 3/4)= 4\pi(u + 3/4)$,

and $4t = 2\pi \times 1 \times (v + 2/3) = 2\pi(v + 2/3) \Rightarrow 8t = 4\pi(v + 2/3)$.

So we get $9t = 4\pi(u + 3/4)$ and $8t = 4\pi(v + 2/3)$

Thus, we get

$4\pi(u + 3/4):4\pi(v + 2/3) = (u + 3/4):(v + 2/3) = 9t:8t = 9:8$

$\Rightarrow (u + 3/4)/(v + 2/3) = 9/8 \Rightarrow 8(u + 3/4) = 9(v + 2/3)$

$\Rightarrow 8u + 6 = 9v + 6 \Rightarrow 8u = 9v \Rightarrow u:v = 9:8$.

So we get $(u + 3/4):(v + 2/3) = 9:8$ and $8u = 9v$, where u and v are nonnegative integers.

And we know A makes $(u + 3/4)$ of its laps, and B makes $(v + 2/3)$ of its laps to get to the target point at the same time.

So we can set $u = 0$ and $v = 0$. Then, A has to make 3/4 of its lap, and B has to make 2/3 of its lap so that they can meet for the first time. And each lap is a complete lap.

What then about the next meeting?

We have $8u = 9v$, so we can get this:

$8u = 9v \Rightarrow u/v = 9/8 \Rightarrow u:v = 9:8 = 18:16 = 27:24 = \ldots$

So we can set u = 9 and v = 8 for the second meeting, we can set u = 18 and v = 16 for the third meeting, and so forth.

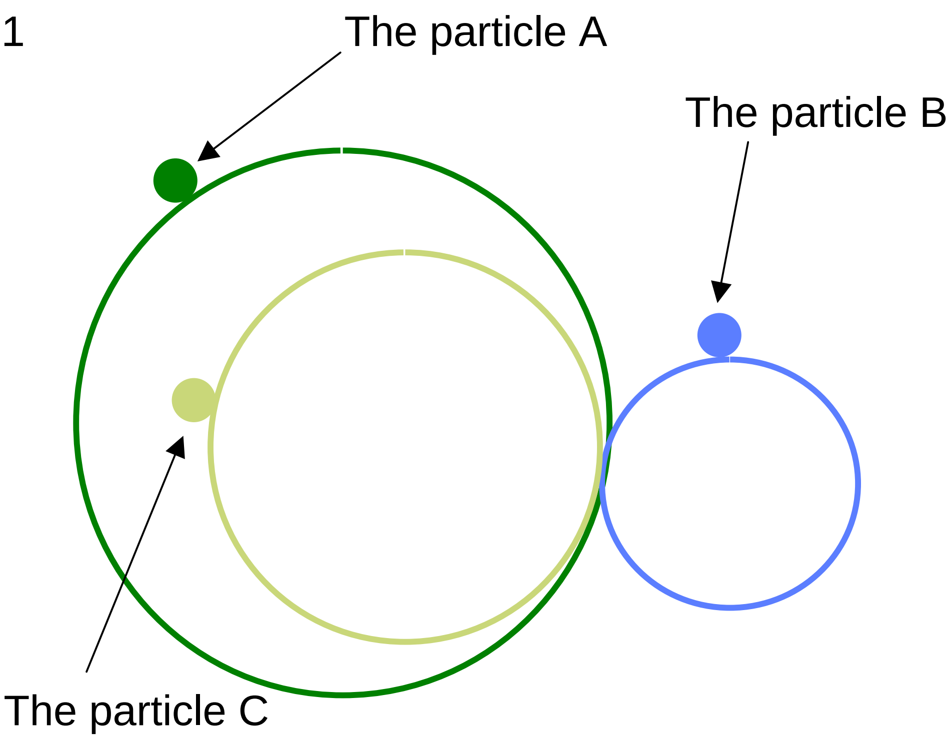
Thus, after the first meeting, A has to make 9 of its laps, and B has to make 8 of its laps so that they can meet the second time. And after the second meeting, A has to make 9 of its laps again, and B has to make 8 of its laps again so that they can meet the third time.

How about three particles?
Suppose three particles begin orbiting on their circular tracks at the same time as shown in Fig. 1 below.
All the particles are moving at their constant speeds on their tracks. They are moving clockwise. The tracks are tangent to each other at one point.
Suppose now that the particles start on their tracks at the tangent point. The particle A starts at the point two thirds past to the right of the tangent point, the particle B starts at the point a half past to the right of the tangent point, and the particle C starts at the point a quarter past to the right of the tangent point.

The speed of A is 10 m/sec, the speed of B is 8 m/sec, and the speed of C is 4 m/sec.

The radius of the track for A is 3 meters, the radius of the tract for B is 2 meters, and the radius of the tract C is 1 meter. Then, see if the particles will meet. If not, what can we do to make the particles meet?

Fig. 1



The particles meet at the tangent point if they meet.

We know the particles keep making laps while orbiting.

Thus, we want to find how many laps each particle makes until the rendezvous, and then, we can express the distance. How then can we get the numbers of laps?

First, we need to set up objects representing the number of the laps to be made. So suppose now,
the number of the laps A makes = u,
the number of the laps B makes = v,
and the number of the laps C makes = w.

Then, A begins orbiting at the point two thirds past to the right of the tangent point, so it should make a third of a complete lap to get to the tangent point.

Next, B begins in its orbit at the point a quarter past to the right of the tangent point, so it should make three quarters of a complete lap to get to the tangent point.

And C begins in its orbit at the point half past to the right of the tangent point, so it should make half a complete lap to get to the tangent point.
So each particle has a different partial lap.

We want to find thus, a particular lap that has to satisfy all the three particles' needs. So we need to come up with a lap common to three particles. What lap then is common to all the three particles?

A twelfth, because a quarter has three twelfths, a third has four twelfths, and a half has six twelfths, so a twelfth of a complete lap is common to all the three particles.
Then, A can get to the point of rendezvous if it makes 4 twelfths of a complete lap, B can get to the tangent point if it makes 9 twelfths of a complete lap, and C can get to the tangent point if it makes 6 twelfths of a complete lap.

So the lap common to A, B, and C is a twelfth of a complete lap. A complete lap requires 12 twelfths of a complete lap. We can think thus, all the particles make twelfth laps until they meet at the tangent point.
Then, all particles will have to make their particular numbers of laps until they meet for the first time, and each lap is a twelfth of a complete lap.
And the length of the lap is a twelfth of the length

(circumference) of the orbit, which is a circle.
So we take a twelfth of the length of the circle for the length of the lap for each particle.

The radius of the track for A is 3 meters, the radius of the tract for B is 2 meters, and the radius of the tract C is 1 meter. So we get these:
The length of the lap for A = $2\pi r/12 = 2\pi \times 3/12 = \pi/2$.
The length of the lap for B = $2\pi r/12 = 2\pi \times 2/12 = \pi/3$.
The length of the lap for C = $2\pi r/12 = 2\pi \times 1/12 = \pi/6$.

And we set the speeds the way as follows.
The speed of A = 10 m/s
The speed of B = 8 m/s
The speed of C = 4 m/s

And we can express the distance to be covered by each particle until the rendezvous the way as follows.
the speed x the time taken = the length of a lap x the number of laps.
So $10t = \pi u/2$ for A, $8t = \pi v/3$ for B, and $4t = \pi w/6$ for C.

Taking thus, the ratio between the numbers of the laps, we get 10t:8t:4t = (πu/2):(πv/3):(πw/6)

⇒ 5:4:2 = (u/2):(v/3):(w/6)

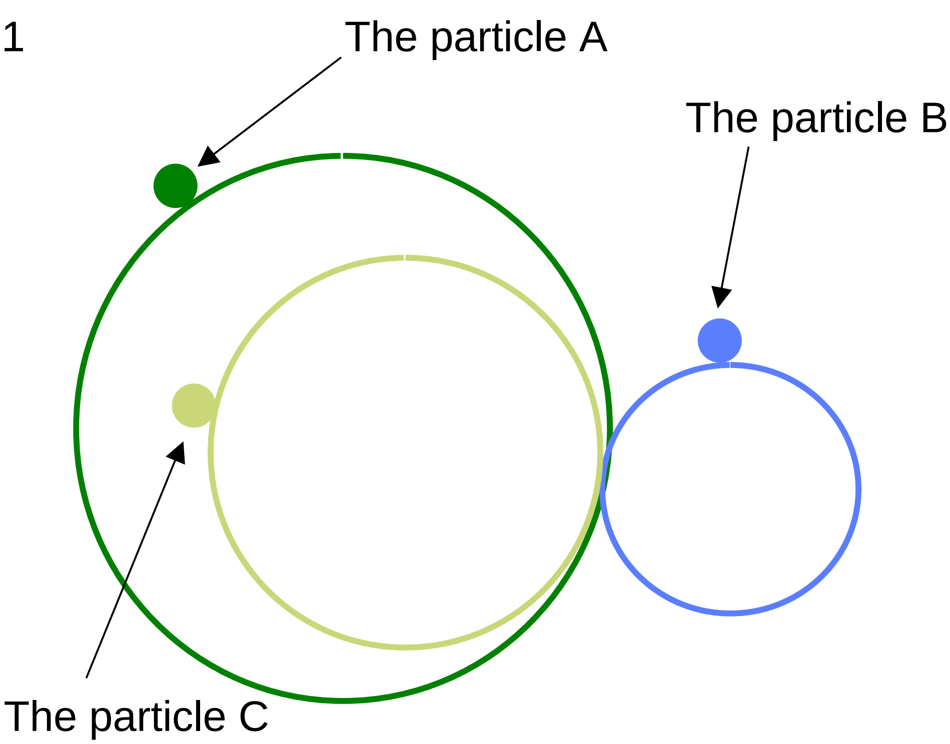
⇒ 10:12:12 = u:v:w

So the ratio is u:v:w = 5:6:6.

Now, we need to see if the ratio works.

Let's now begin with finding the number of the laps each particle has to make to get to the target point for the first time from its initial point.

Fig. 1



As shown in Fig. 1 above, initially, A is a third of the length of its complete lap away from the tangent point, B is 3 quarters of the length of its complete lap away from the tangent point, and C is half the length of its complete lap away from the tangent point.
And each lap each particle makes is 1/12 of its complete lap.

So the number of the laps A will have to make to get to the tangent point for the first time is as follows.
(1/3) / (1/12) = (1/3) x 12 = 12/3 = 4, in other words,
$\frac{\frac{1}{3}}{\frac{1}{12}} = \frac{1}{3} \text{ x } 12 = \frac{12}{3} = 4$. Thus, A will need to make 4 laps.

Next, the number of the laps B will have to make to get to the tangent point for the first time is as follows.
(3/4) / (1/12) = (3/4) x 12 = 36/4 = 9, in other words,
$\frac{\frac{3}{4}}{\frac{1}{12}} = \frac{3}{4} \text{ x } 12 = \frac{36}{4} = 9$. So B will have to make 9 laps.

And the number of the laps C will have to make to get to the tangent point for the first time is as follows.

(1/2) / (1/12) = (1/2) x 12 = 12/2 = 6, in other words,

$\frac{\frac{1}{2}}{\frac{1}{12}} = \frac{1}{2}$ x $12 = \frac{12}{2} = 6$. So C will have to make 6 laps.

Thus, it will take 4 laps for A to get to the target point for the first time. It will take 9 laps for B to get to the target point for the first time. And it will take 6 laps for C to get to the target point for the first time.

And a complete lap is 12 times each of the laps above.

So we can see the facts as follows.

Fact 1.

The number of the laps A makes is 4 + 12n so that A can meet B and C, where n is the number of complete laps.

Fact 2.

The number of the laps B makes is 9 + 12m so that B can meet A and C, where m indicates the number of complete laps.

Fact 3.

The number of the laps C makes is 6 + 12k so that C can meet A and B, where k is the number of complete laps.

In accordance with the speeds, the lengths of the orbits, and the initial positions given in the problem, we found that the ratio between u, v, and w is u:v: = 5:6:6.

Thus, we need to find three integers n, m, k such that $(4 + 12n):(9 + 12m):(6 + 12k) = 5:6:6$. If there are the three, the particles can meet; otherwise, they cannot.

In the ratio, we have $(9 + 12m):(6 + 12k) = 6:6$ and m and k are integers, so there is no solution for m and k.

There are no integers that m and n can be so that they can satisfy the ratio $(9 + 12m):(6 + 12k) = 6:6 = 1:1$.

That's because the ratio means $9 + 12m = 6 + 12k$, and there are no integers for m and k. So the particles cannot meet.

What then can we do to make them meet?

We have the facts as follows.

Fact 1.
The number of the laps A makes is 4 + 12n so that A can meet B and C, where n is the number of complete laps.
Fact 2.
The number of the laps B makes is 9 + 12m so that B can meet A and C, where m indicates the number of complete laps.
Fact 3.
The number of the laps C makes is 6 + 12k so that C can meet A and B, where k is the number of complete laps.

So we can use the three facts above. How?

We can make some choices on the numbers of complete laps the particles have to make until the first meeting. So if making the choices, we can set n, m, and k equal to some nonnegative integers. Setting thus, for the simplest instance, n = 0, m = 0, and k = 0, we get these:
u = 4 + 12n = 4, v = 9 + 12m = 9, and w = 6 + 12k = 6.

So the ratio is u:v:w = 4:9:6, which should work.

Let's now reset the speeds. Then, suppose first, the speed of the particle A is p, the speed of B is q, and the speed of C is r. Then, we can use the equation as follows.

The speed of each particle x t = the length of the lap x the number of the laps

Then, we can see that for A, $pt = \pi u/2$, where $\pi/2$ is the length of its lap and u is the number of its laps.
For B, $qt = \pi v/3$, where $\pi/3$ is the length of its lap and v is the number of its laps.
And for C, $rt = \pi w/6$, where $\pi/6$ is the length of its lap and w is the number of its laps.

So we get $pt = \pi u/2 \Rightarrow u = 2pt/\pi$, $qt = \pi v/3 \Rightarrow v = 3qt/\pi$, and $rt = \pi w/6 \Rightarrow w = 6rt/\pi$.
Thus, we get $u:v:w = 2p:3q:6r$.

And we want the ratio $2p:3q:6r$ to be $4:9:6$,
since $u:v:w = 4:9:6$.
Then, we get $2p:3q:6r = 4:9:6 \Rightarrow p:q:r = 2:3:1$.

So the ratio p:q:r = 2:3:1.

Thus, setting for instance, the speed of A = 2 m/s, the speed of B = 3 m/s, and the speed of C = 1 m/s, we get 3 equations as follows.

$2t = \pi u/2 \Rightarrow 4t = \pi u$, $3t = \pi v/3 \Rightarrow 9t = \pi v$,
and $t = \pi w/6 \Rightarrow 6t = \pi w$.
So we get u:v:w = 4:9:6 as expected.

Thus, after the particle A makes 4 of its laps, the particle B makes 9 of its laps, and the particle C makes 6 of its laps, they will meet for the first time.

And checking to see if they really meet, we get these:
$4 \times 1/12 = 1/3$, $9 \times 1/12 = 3/4$, and $6 \times 1/12 = 1/2$.

So A covers a third of a complete lap, B covers three quarters of a complete lap, and C covers half a complete lap, and thus, they meet.

Let's now get the amount of time it takes for the first meeting to happen. Then, setting first, $u = 4$, we get

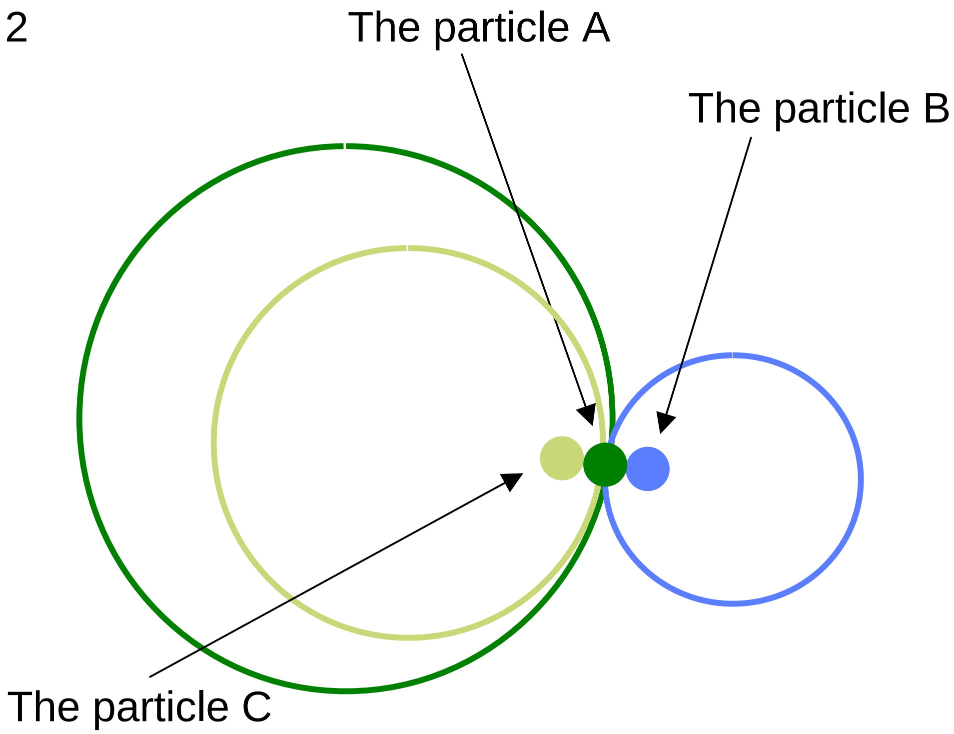
$2t = \pi u/2 = 4\pi/2 \Rightarrow t = \pi$ sec.

Next, setting $v = 9$, we get $3t = \pi v/3 = 9\pi/3 \Rightarrow t = \pi$ sec, too.

And next, setting $w = 6$, we get $t = \pi w/6 = 6\pi/6 \Rightarrow t = \pi$ sec, as well.

Therefore, after π (approximately 3.14) seconds, the particles will meet for the first time.

Fig. 2



Setting next, for another instance, $n = 1$, $m = 2$, and $k = 3$, we get these:

$u = 4 + 12n = 16$, $v = 9 + 12m = 33$, and $w = 6 + 12k = 42$.

So the ratio is $u:v:w = 16:33:42$, which should work, too.

Let's now reset the speeds.

Then, assuming again the speed of A is p, and the speed of B is q, we get $u:v:w = 2p:3q:6r$, which was found above.

And we want the ratio $2p:3q:6r$ to be $16:33:42$,

since $u:v:w = 16:33:42$.

Then, we get $2p:3q:6r = 16:33:42 \Rightarrow p:q:r = 8:11:7$.

Thus, setting for instance, the speed of A = 16 m/s, the speed of B = 22 m/s, and the speed of C = 14 m/s, so doubling the entries, we get 3 equations as follows.

$16t = \pi u/2 \Rightarrow 32t = \pi u$, $22t = \pi v/3 \Rightarrow 66t = \pi v$,

and $14t = \pi w/6 \Rightarrow 84t = \pi w$.

So we get $u:v:w = 32:66:84 = 16:33:42$ as expected.

Thus, after A makes 16 of its laps, B makes 33 of its laps, and C makes 42 of its laps, they will meet for the first time.

And checking to see if they really meet, we get

$16 \times 1/12 = 16/12 = 1 + 1/3$, $33 \times 1/12 = 33/12 = 2 + 3/4$,

and $42 \times 1/12 = 42/12 = 3 + 1/2$.

So A makes 1 complete lap and a third of a complete lap, B makes 2 complete laps and three quarters of a complete lap, and C makes 3 complete laps and half a complete lap, and thus, they meet for the first time.

Let's now get the amount of time it takes for the first meeting to happen. Then, setting first, $u = 16$, we get

$16t = \pi u/2 = 16\pi/2 = 8\pi \Rightarrow t = \pi/2$ sec.

Next, setting $v = 33$, we get

$22t = \pi v/3 = 33\pi/3 = 11\pi \Rightarrow t = \pi/2$ sec, too

And next, setting $w = 42$, we get

$14t = \pi w/6 = 42\pi/6 = 7\pi \Rightarrow t = \pi/2$ sec, as well.

Therefore, after $\pi/2$ (approximately 1.57) seconds, the particles will meet for the first time.

What then about the second meeting and all the subsequent meetings?

After the first meeting, all the particles start at the tangent point at the same time, so their laps are all complete laps. So we get 3 equations as follows.

For A, $8t = 2\pi ru$, for B, $11t = 2\pi rv$, and for C, $7t = 2\pi rw$.

And getting the ratio between the numbers of laps, we will probably get $u:v:w = 16:33:42$.

That's because we just apply the same scale factor of 12 to the length of a lap for each particle, so the ratio does not change.

Checking to see if it is the case, we get first

$8t = 2\pi ru = 6\pi u$, since $r = 3 \Rightarrow u = 4t/3\pi$,

$11t = 2\pi rv = 4\pi v$, since $r = 2 \Rightarrow v = 11t/4\pi$, and

$7t = 2\pi rw = 2\pi w$, since $r = 1 \Rightarrow w = 7t/2\pi$

So next, we get $u:v:w = (4/3):(11/4):(7/2) = 16:33:42$

Thus, we get $u:v:w = 16:33:42$ as expected

So after the first meeting, every time A makes 16 complete laps, B makes 33 complete laps, and C makes 42 complete laps, their meeting happens.

And getting the time for each subsequent meeting to happen, we get these:

For A, $8t = 2\pi ru = 2\pi \times 3 \times 16 \Rightarrow t = 12\pi$ sec.

For B, $11t = 2\pi rv = 2\pi \times 2 \times 33 \Rightarrow t = 12\pi$ sec, too.

For C, $7t = 2\pi rw = 2\pi \times 1 \times 42 \Rightarrow t = 12\pi$ sec, as well.

Thus, A, B, and C meet every 12π seconds.

You may want to try more examples taking some different combinations of n and m, and try thinking about some exceptional cases, too. And you may want to reset the lengths of the tracks and see if the particles meet.

Let's now use the numbers of the complete laps.
Then, to begin with, the setting is as follows.

The speed of A is 16 m /sec, the speed of B is 22 m/sec, and the speed of C is 14 m/sec.

The radius of the orbit for A is 3 meters, the radius of the orbit for B is 2 meter, and the radius of the orbit for C is 1 meter.

A has to make a number of laps and a third of a lap to get to the tangent point.
B has to make a number of laps and 3 quarters of a lap to get to the tangent point.
C has to make a number of laps and half a complete lap to get to the tangent point.

The number of the complete laps A has to make until the first meeting is u, the number of the complete laps B has to make until the first meeting is v, the number of complete laps C has to make until the first meeting is w, and of course, the length of the complete lap is the circumference of the orbit.

So A makes (u + 1/3) of its laps, B makes (v + 3/4) of its laps, and C makes (w + 1/2) of its laps to get to the tangent point at the same time. Then, we get

$16t = 2\pi \times 3 \times (u + 1/3) = 6\pi(u + 1/3) \Rightarrow (8/3)t = \pi(u + 1/3)$,

$22t = 2\pi \times 2 \times (v + 3/4) = 4\pi(v + 3/4) \Rightarrow (11/2)t = \pi(v + 3/4)$.

and $14t = 2\pi \times 1 \times (w + 1/2) = 2\pi(w + 1/2) \Rightarrow 7t = \pi(w + 1/2)$.

So we get

$(u + 1/3):(v + 3/4):(w + 1/2) = (8/3)t:(11/2)t:7t = 16:33:42$

$\Rightarrow (u + 1/3):(v + 3/4):(w + 1/2) = 16:33:42$

where u and v are nonnegative integers.

And we can put the ratio above this way, too:

$(u + 4/12):(v + 9/12):(w + 6/12) = 16:33:42$

Then, it can be the case $u = 1$, $v = 2$, and $w = 3$. That is to say that we can have this: $(u + 4/12):(v + 9/12):(w + 6/12)$

$= (1 + 4/12):(2 + 9/12):(3 + 6/12) = (16/12):(33/12):(42/12)$

$= 16:33:42$

And $(1 + 4/12):(2 + 9/12):(3 + 6/12)$ means that after A makes 1 complete lap and a third of a complete lap, B makes 2 complete laps and 3 quarters of a complete lap, and C makes 3 complete laps and half a complete lap, they meet at the tangent point for the first time.

And practically, $(16/12):(33/12):(42/12)$ means the ratio above, and is saying that a lap is a twelfth of a complete lap, and after A makes 16 laps, B makes 33 laps, and C makes 42 laps, they meet for the first time.

What then about the subsequent meetings?

After the first meeting, all the particles start at the tangent point at the same time, so they all make complete laps only so that they can meet at the tangent point.
And the ratio between the laps is u:v:w = 16:33:42.
So after the first meeting, since all the particles start at the tangent point, they make complete laps only, with the ratio above, for every subsequent meeting at the tangent point.

Thus, after the first meeting, every time A makes 16 complete laps, B makes 33 complete laps, and C makes 42 complete laps, they meet at the tangent point concurrently.

Let’s now check to see if it really is the case.
We have (u + 4/12):(v + 9/12):(w + 6/12) = 16:33:42
For the first meeting, we get
(1 + 1/3):(2 + 3/4):(3 + 1/2) = (1 + 4/12):(2 + 9/12):(3 + 6/12)
(16/12):(33/12):(42/12) = 16:33:42.
So after A makes 1 complete lap and a third of a complete lap, B makes 2 complete laps and three quarters of a complete lap, and C makes 3 complete laps and half a complete lap, the first meeting happens.

For the second time on, all the particles make complete laps only, with this ratio: u:v:w = 16:33:42.

So for the second meeting, we get

{(1 + 16) + 1/3}:{(2 + 33) + 3/4}:{(3 + 42) + 1/2}

= (17 + 4/12):(35 + 9/12):(45 + 6/12)

= (208/12):(429/12):(546/12) = 208:429:546

= (16 x 13):(33 x 13):(42 x 13) = 16:33:42

After the first meeting thus, after A makes 16 complete laps, B makes 33 complete laps, and C makes 42 complete laps, the second meeting happens. And after the second meeting, A has to make 16 of its laps again, B has to make 33 of its laps again, and C has to make 42 of its laps again so that they can meet the third time.